A method for improving time-stepping numerics

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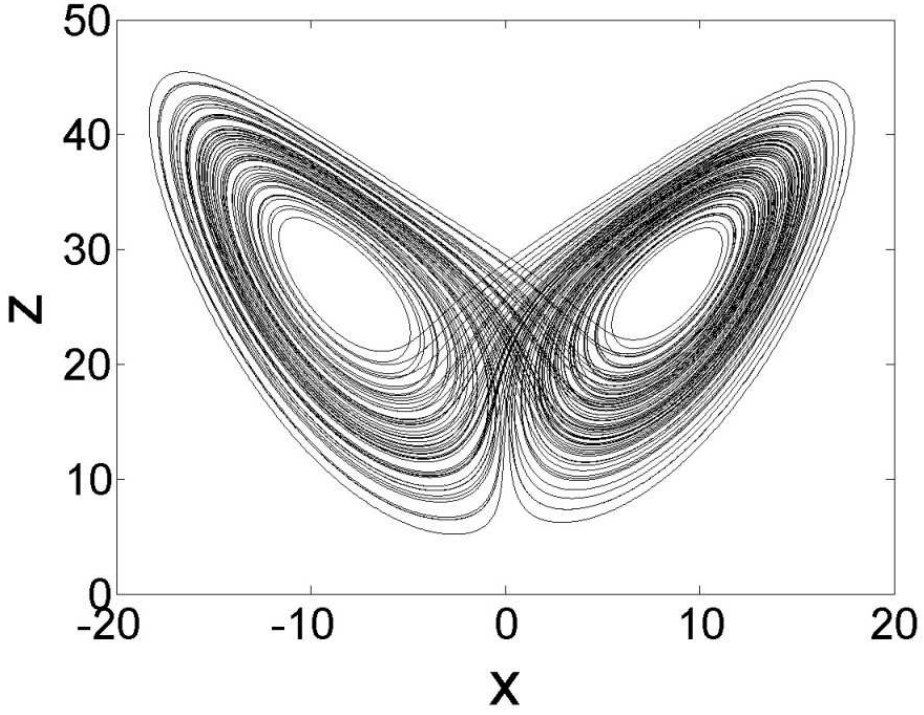
Impact of time stepping in weather and climate prediction

"In the weather and climate prediction community, when thinking in terms of model predictability, there is a tendency to associate model error with the physical parameterizations. In this paper, it is shown that time truncation error can be a substantial part of the total forecast error."

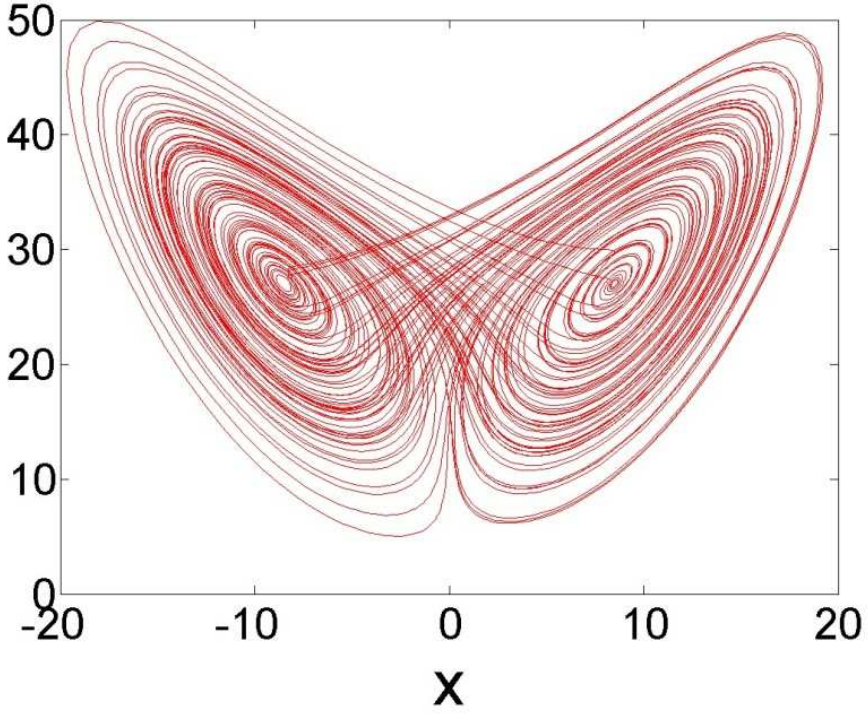
(Teixeira et al. 2007)

Impact of different time steps on the ‘climate’ of the Lorenz attractor

Δt = 0.001



Δt = 0.01

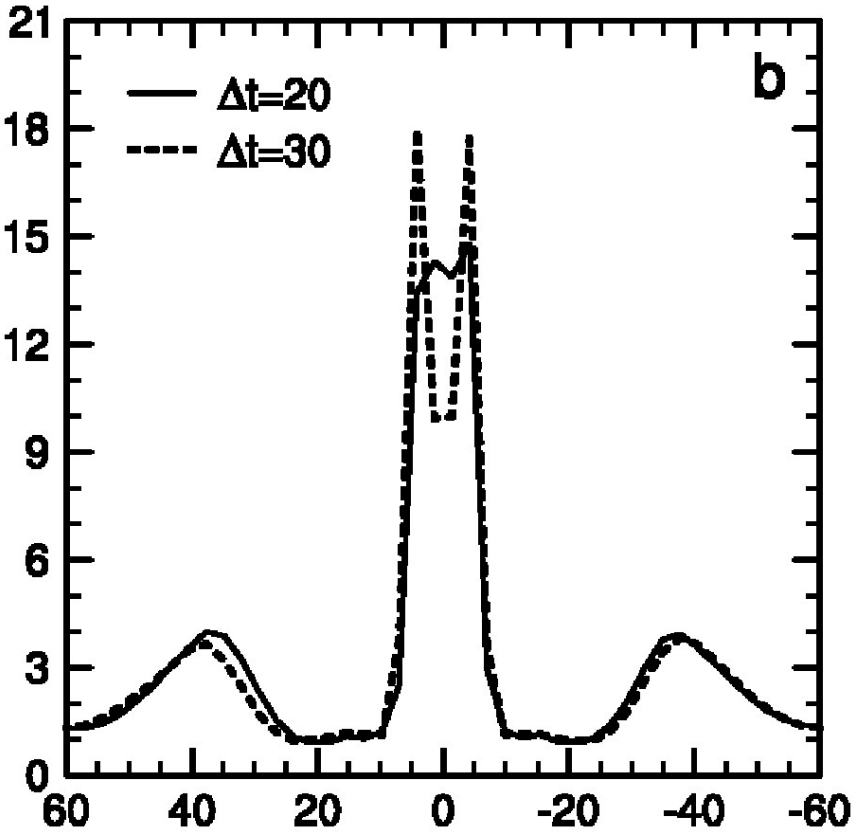


Using the explicit Euler forward scheme

(Williams, in prep)

Impact of different time steps in an aqua-planet atmosphere GCM

zonal-mean precipitation (mm/day)

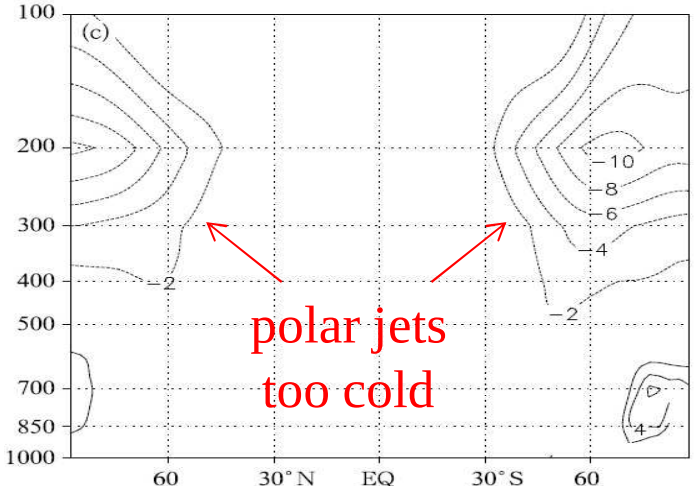


(Williamson & Olson 2003)

Impact of different time-stepping schemes in CAM atmosphere GCM

zonal-mean annual-mean temperature error (°C) relative to ERA40

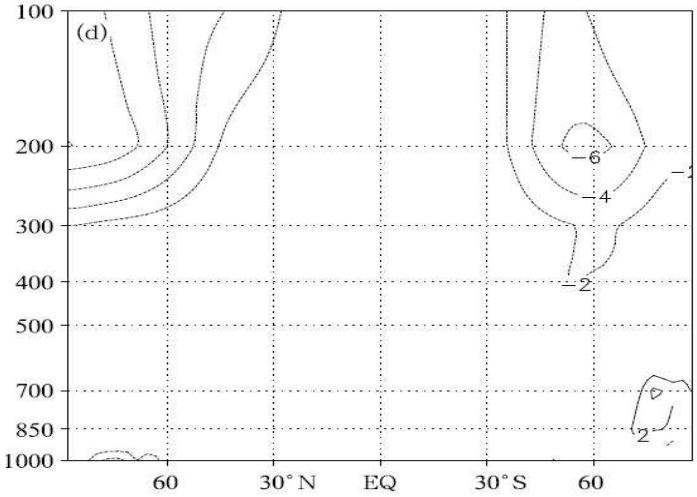
pressure (hPa)



polar jets too cold

Leapfrog with Robert-Asselin filter (1st order)

pressure (hPa)



Adams-Bashforth (2nd order)

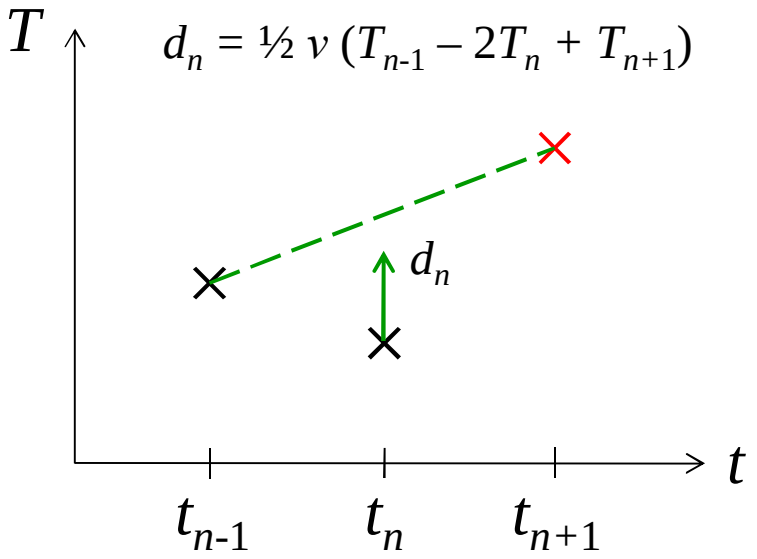
(Zhao & Zhong 2009)

Leapfrog with Robert–Asselin filter

- Widely used in current atmosphere models:
  - ECHAM, MAECHAM, MM5, CAM, MESO-NH, HIRLAM, KMCM, LIMA, SPEEDY, IGCM, PUMA, COSMO, FSU-GSM, FSU-NRSM, NCEP-GFS, NCEP-RSM, NSEAM, NOGAPS, RAMS, CCSR/NIES-AGCM
- Asselin (1972) has received over 450 citations
- Has many problems:
  - "*The Robert-Asselin filter has proved immensely popular, and has been widely used for over 20 years. However, it is not the last word…*" (Lynch 1991)
  - "*Replacement of the Asselin time filter… can be a feasible way to improve the ability of climate models*" (Zhao & Zhong 2009)
  - "*The Robert-Asselin filter can produce slewing frequency as well as the well-known damping and phase errors*" (Thrastarson & Cho 2011)

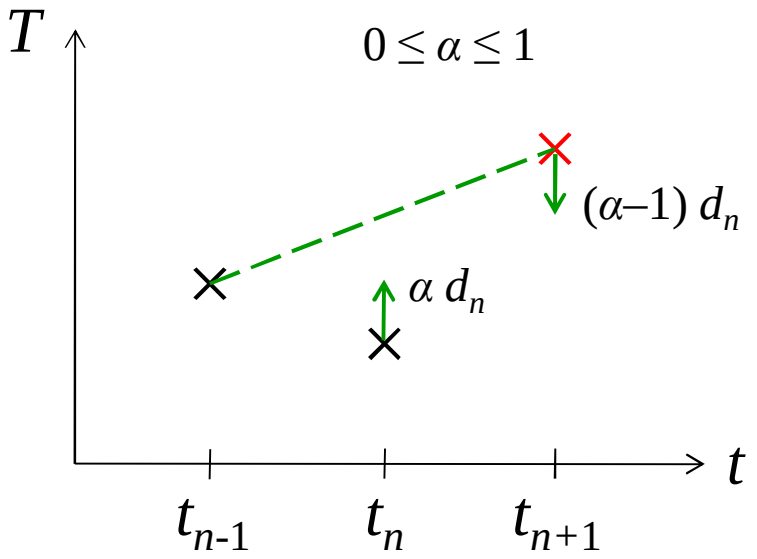
A proposed improvement: RAW filter

LF+RA
(Robert 1966, Asselin 1972)



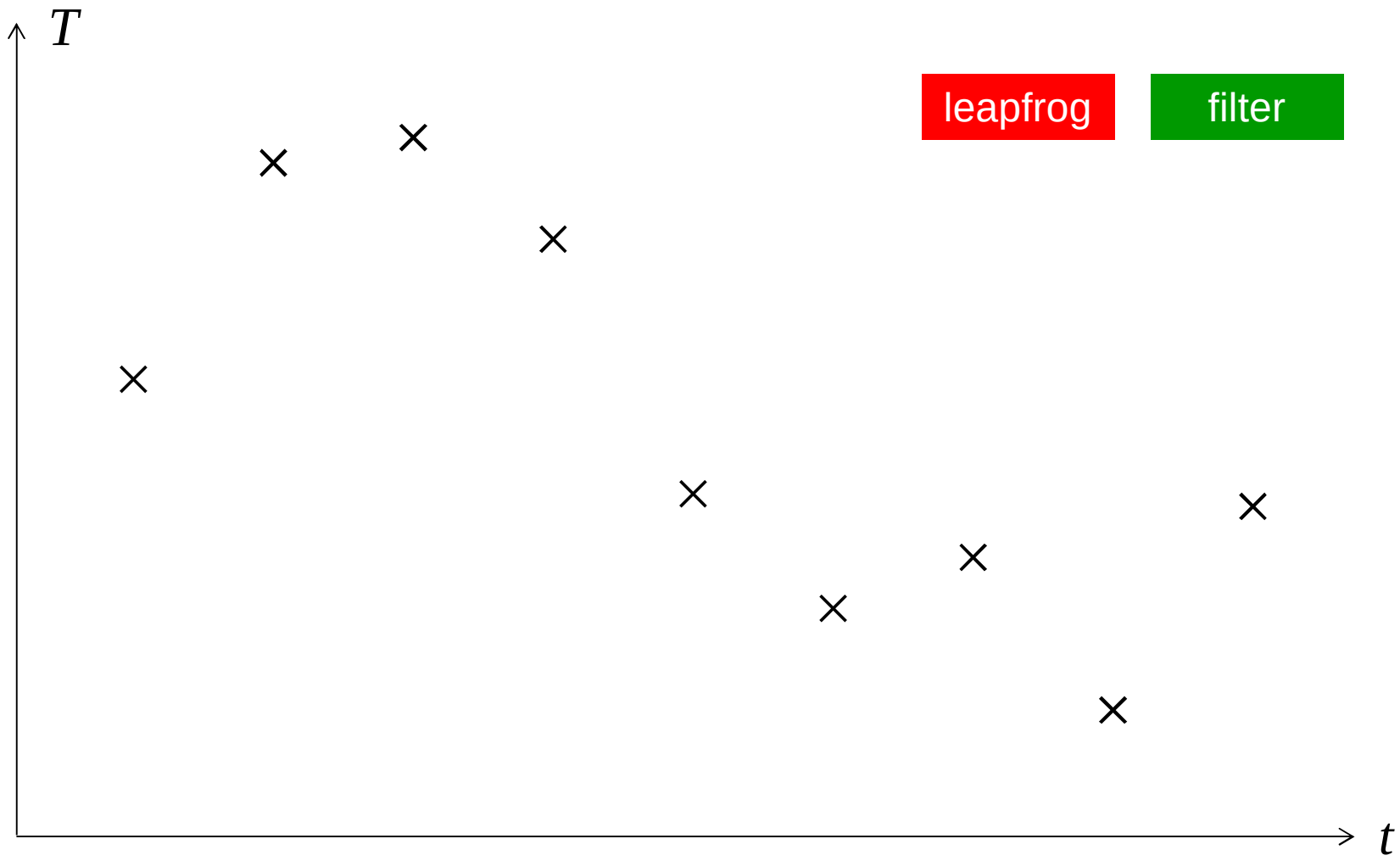
- use leapfrog to calculate T_{n+1}
- RA filter nudges T_n
- reduces curvature but does not conserve mean
- amplitude accuracy is 1st order

LF+RAW
(Williams 2009, 2011)



- use leapfrog to calculate T_{n+1}
- RAW filter nudges T_n and T_{n+1}
- reduces curvature and conserves mean (for α=½)
- amplitude accuracy is 3rd order

A proposed improvement: RAW filter



Simple test integration

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -\omega Y$$
$$\frac{\mathrm{d}Y}{\mathrm{d}t} = +\omega X$$

$\omega = 1 \text{ rad s}^{-1}$

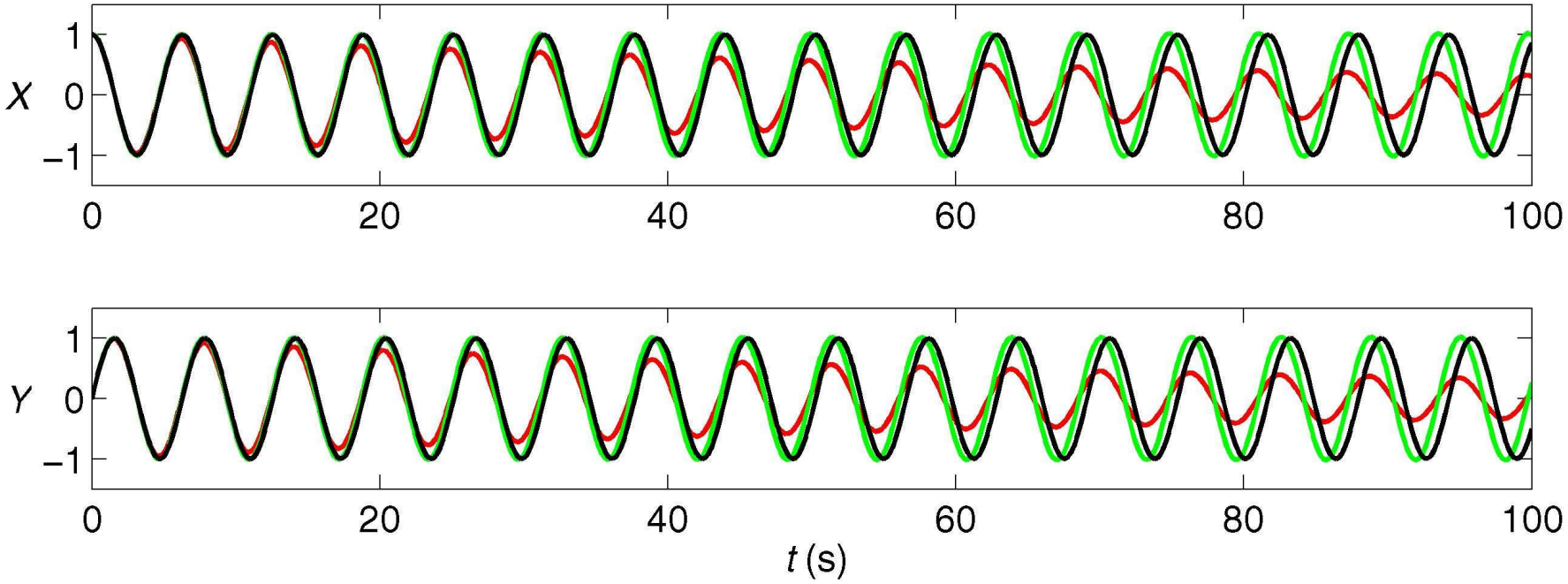
exact

LF+RA

LF+RAW$_{\alpha=1/2}$

$\Delta t = 0.2$ s

$\nu = 0.2$



(Williams 2009)

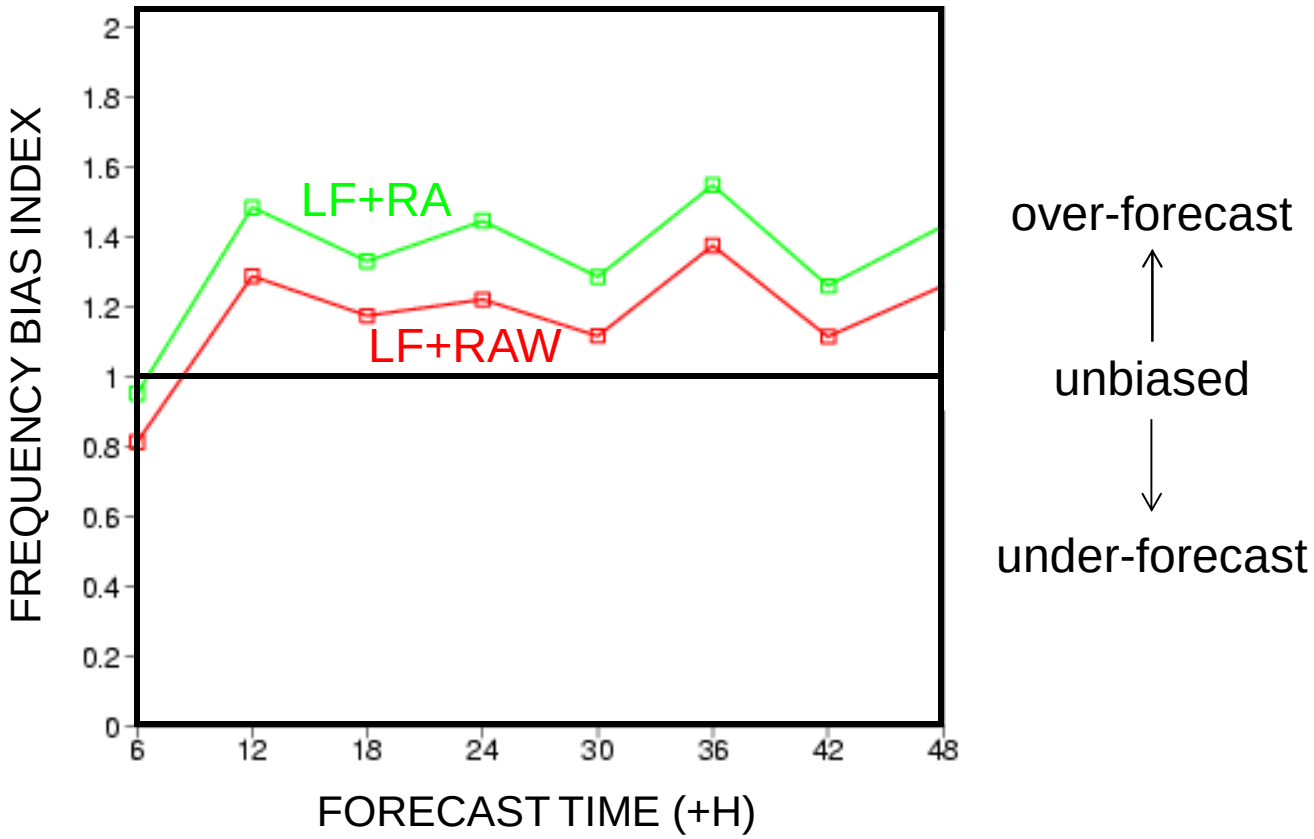
Implementation in existing code

```
! Compute tendency at this time step
tendency = […]

! Leapfrog step
x_next = x_last + tendency*2*delta_t

! Compute filter displacement
d = nu*(x_last – 2*x_this + x_next)/2

! Apply filter
x_this = x_this + d*alpha
x_next = x_next + d*(alpha-1)
```

(Williams 2011)

Implementation in COSMO

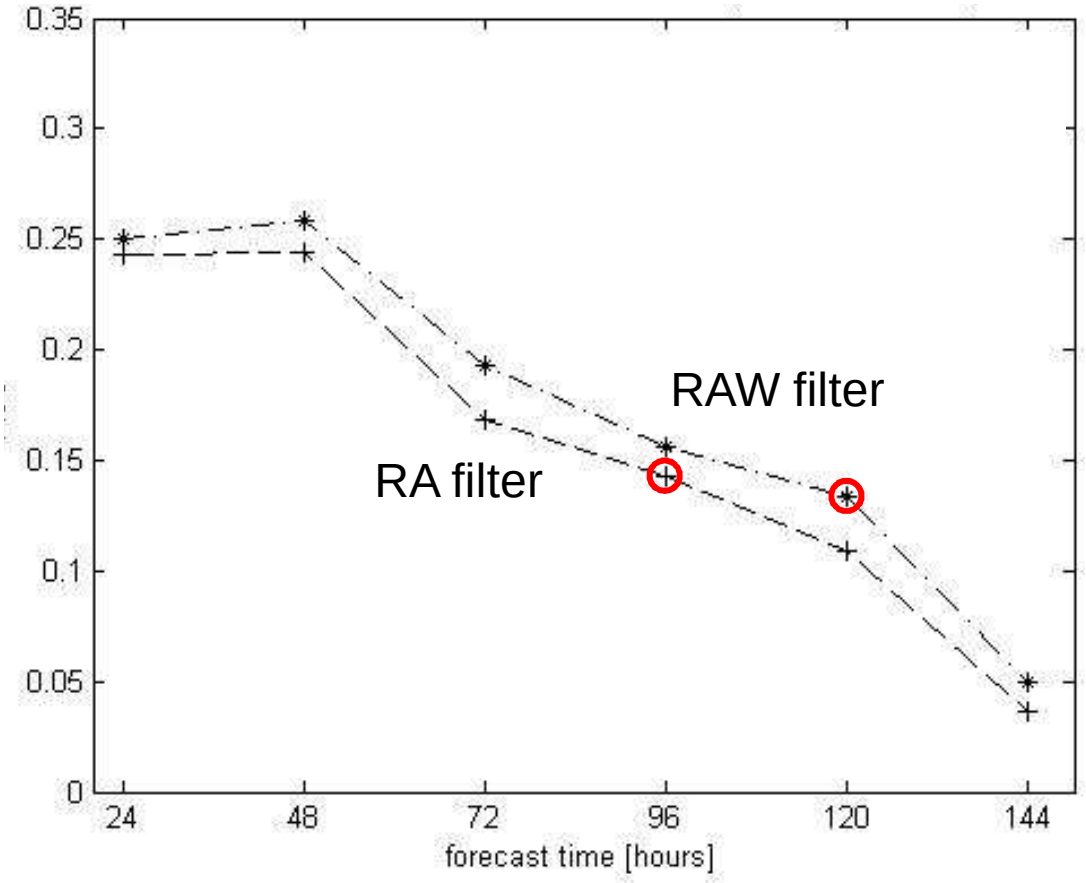
- Statistical analysis: two 48h forecasts per day from 16 Dec 2008 to 18 Jan 2009
- The modified filter significantly improves precipitation forecasts (2-10 mm / 6h)



(Lucio Torrisi, CNMCA, Italy)

Implementation in SPEEDY

ACC for surface pressure in the tropics (25°S-25°N)



5-day forecasts made using the RAW filter have approximately the same skill as 4-day forecasts made using the RA filter

(Amezcua, Kalnay & Williams 2011)

Summary

- Time stepping is an important contributor to model error
- The Robert-Asselin filter is widely used but is dissipative and reduces accuracy
- The RAW filter has approximately the same stability but much greater accuracy
- Implementation in an existing code is trivial, and there is virtually no extra computational cost

Further information

Williams PD (2009) A proposed modification to the Robert-Asselin time filter. *Monthly Weather Review* **137**(8), 2538-2546.

Williams PD (2011) The RAW filter: An improvement to the Robert-Asselin filter in semi-implicit integrations. *Monthly Weather Review* **139**(6), pp 1996-2007.

Amezcua J, Kalnay E, and Williams PD (2011) The effects of the RAW filter on the climatology and forecast skill of the SPEEDY model. *Monthly Weather Review* **139**(2), 608-619.

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