Different radial basis functions and their applicability for regional gravity representation on the sphere

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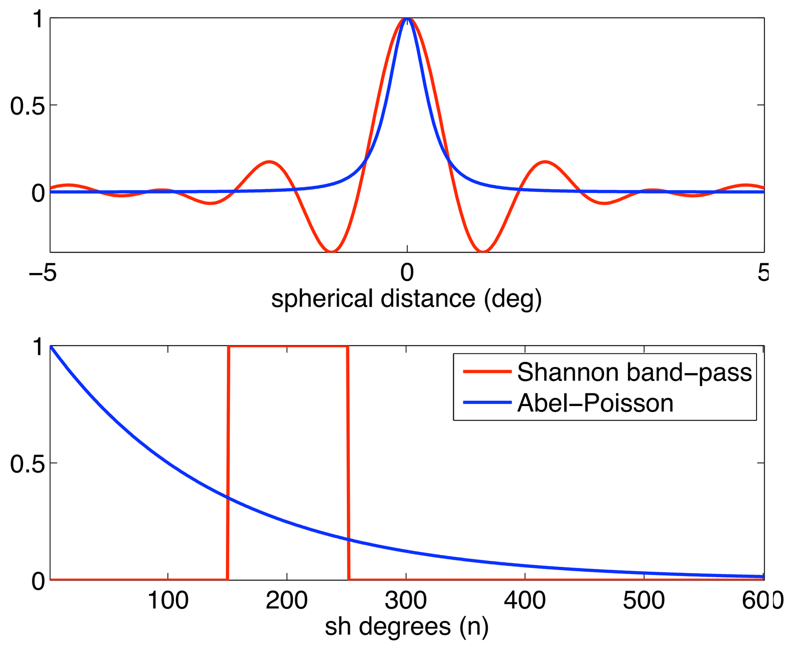
1. Introduction

Global Earth gravity models can be regionally refined by terrestrial and/or airborne measurements. Spherical harmonic basis functions, which are used to represent the global models, are not very appropriate to represent the additional regional data. Instead, basis functions with localizing character are needed to represent the regional signal. The resulting series expansion is then added to the global models. We investigate different types of quasi-localizing radial basis functions on a sphere and their applicability to represent a regional gravity signal.

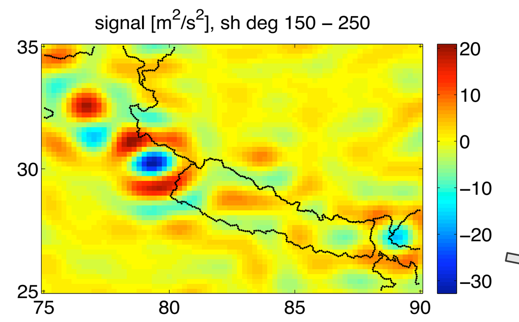
2. Spatial and frequency localization

Shannon kernel (band-pass): Optimal localization of the signal bandwidth in the frequency domain, but globally oscillating in the spatial domain.
Abel-Poisson kernel: Quasi-compact support in the spatial domain, but non-bandlimited in the frequency domain.



top: spatial domain B
bottom: frequency domain B_n

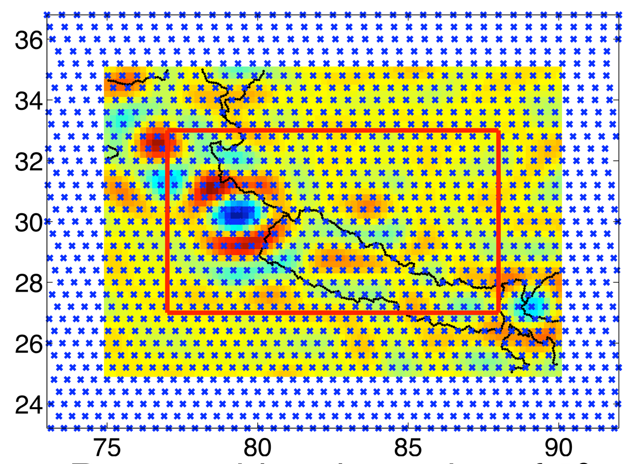
3. Scheme of the simulation



Gravity potential observations on a regular grid with 0.2° spacing in the Himalaya region.

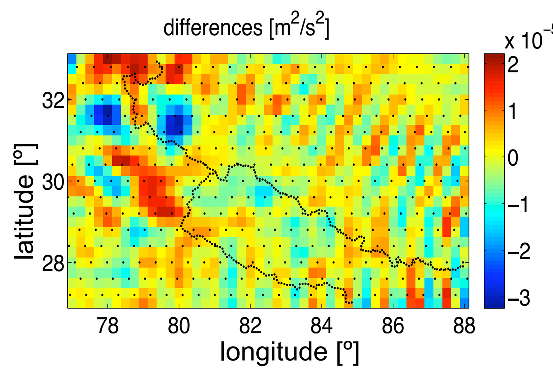
Parameter estimation with pseudoinverse (pinv) and with variance component estimation (vce).

Analysis



Reuter grid and margins of 2°

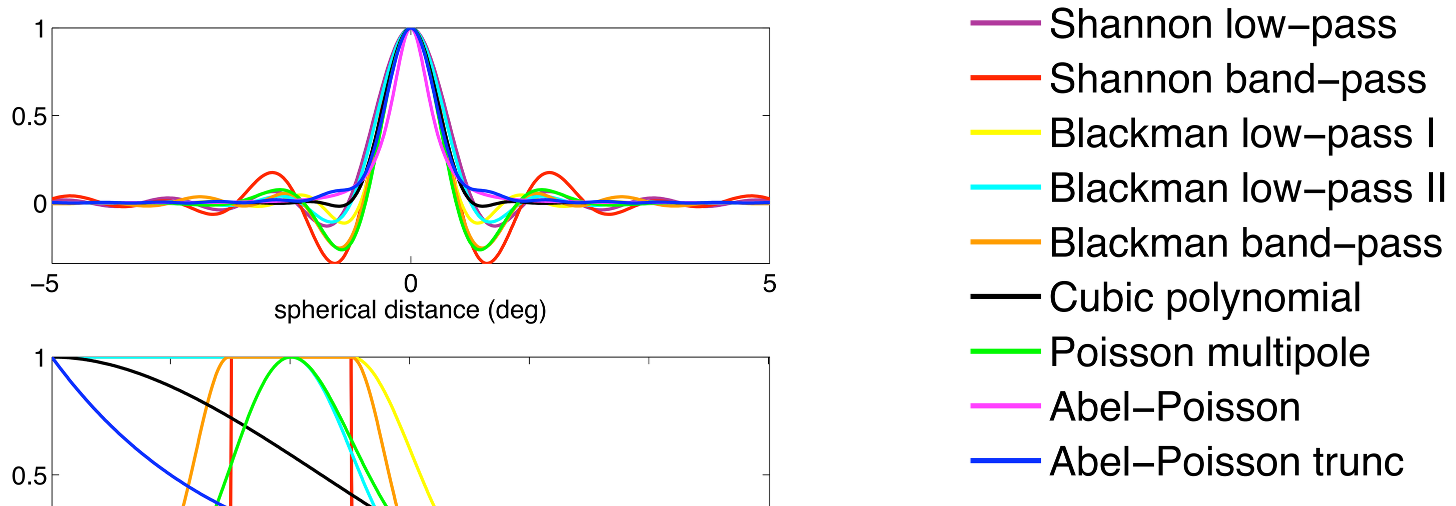
Synthesis



Cross-validation: Comparison to a field from spherical harmonics (sh deg 150 to 250) with 0.25° spacing, plot of the differences in gravity potential.

The simulation gives residuals for each of the radial basis functions. From the residual fields, relative root mean square values are computed and given in the table in section 5. The RMS values for the Shannon kernel and Abel-Poisson kernel, which are given in red, are fairly high. Since these two functions are only optimal in one domain, we investigate in section 4 functions which give a compromise between spatial and frequency localization. They give better RMS values.

4. Normalized kernels in spatial and in frequency domains



top: spatial domain B
bottom: frequency domain B_n

All base functions B, which are used here, are based on an expansion in Legendre polynomials P_n and defined by the choice of the coefficients B_n. For details see [4], [1], or [2] amongst others.

$$B = \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} B_n P_n$$

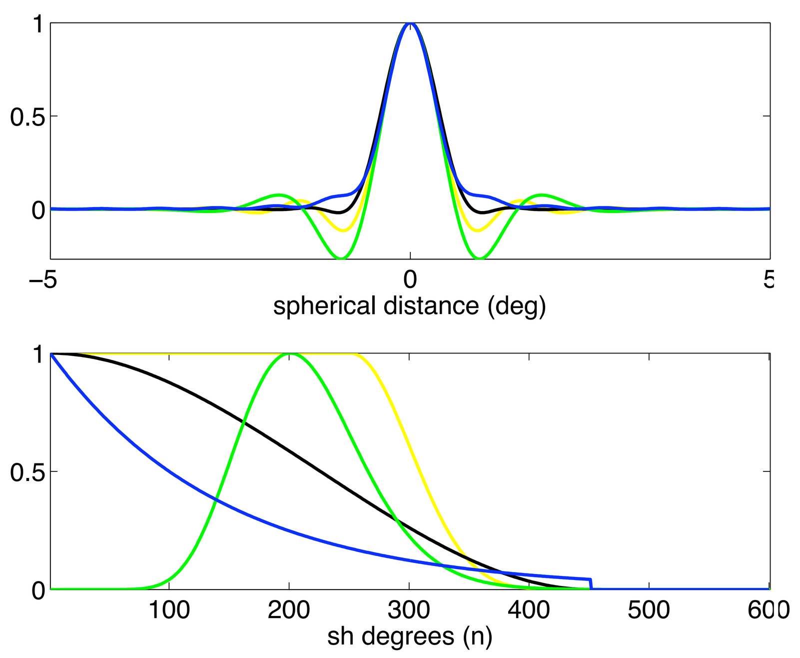
5. Overview of the RMS values of the residuals

Residuals are computed in the cross-validation and σ are relative root mean squares.

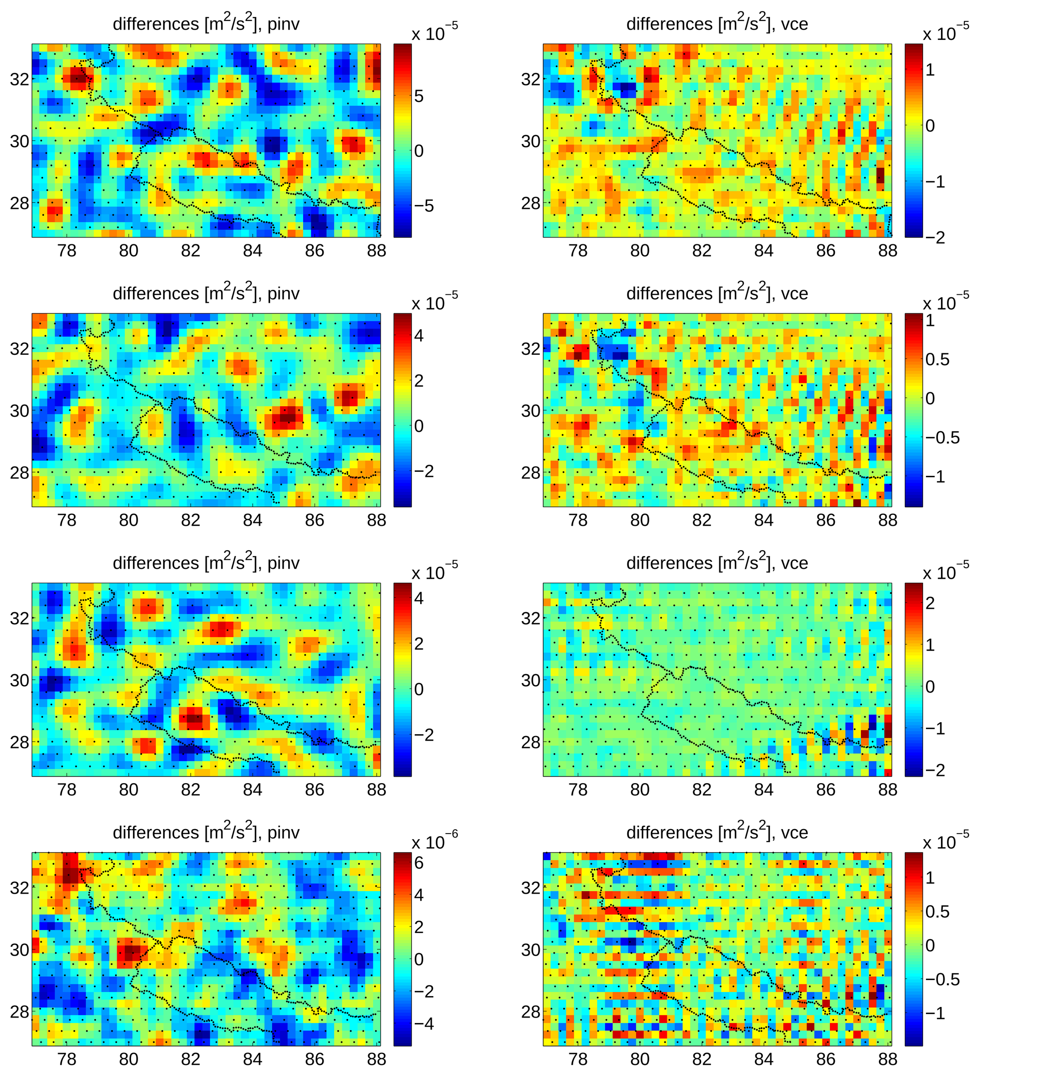
Shannon kernel (low-pass)	σ_{pinv}=4.3e-4	σ_{vce}=0.0017
Shannon kernel (band-pass)	σ_{pinv}=0.0014	σ_{vce}=0.011
Blackman kernel type I (low-pass)	σ_{pinv}=4.1e-4	σ_{vce}=2.7e-5
Blackman kernel type II (low-pass)	σ_{pinv}=2.5e-4	σ_{vce}=7.6e-5
Modified Blackman kernel (band-pass)	σ_{pinv}=3.0e-4	σ_{vce}=4.3e-4
Cubic polynomial kernel	σ_{pinv}=1.7e-4	σ_{vce}=2.4e-5
Poisson multipole kernel	σ_{pinv}=1.4e-4	σ_{vce}=2.1e-5
Abel-Poisson kernel	σ_{pinv}=0.0051	σ_{vce}=0.0050
Abel-Poisson kernel truncated	σ_{pinv}=5.8e-5	σ_{vce}=3.6e-5

6. Residuals

The following four kernels give equally good results in the gravity representation in terms of their RMS values (see table above), but they have very different behaviour in the frequency domain. Thus, they are compared in more detail.



Blackman low−pass I
Cubic polynomial
Poisson multipole
Abel−Poisson trunc



7. Conclusions and outlook

To represent a signal regionally on a sphere is fairly complex, since a lot of different aspects have to be taken into account and boundary effects occur. Besides the type of radial basis functions, their spatial distribution has to be chosen.

The comparison of different radial basis functions shows, that a compromise between spatial and frequency localization is needed. The band-pass Shannon kernel gives an optimal representation in the global case, but not regionally. Our simulation study allows to compare different radial basis functions not only by their spatial and frequency behaviour, but also in their performance in the representation of a regional gravity signal.

Plots of the residuals are given for basis functions with low RMS values. There seem to be resonance structures in the residuals obtained with a variance component estimation. The low-pass Blackman kernel and cubic polynomial kernel show fairly similar spectral behaviour and, thus, their residuals from variance component estimation show similar structures, too.

Further detailed investigations of radial basis functions are needed and will be carried out. Thereby, one of the next steps is to improve the variance component estimation, since this method offers a lot of possibilities. This will also be an issue within an ICCT study group on methodology in regional modeling.

References:

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