A theoretical framework for the interpretation of karst spring signals

Matt Covington – Karst Research Institute, Slovenia

<u>Collaborators:</u>

Andrew Luhmann – University of Minnesota

Martin Saar – University of Minnesota

Carol Wicks – Louisiana State University

Franci Gabrovšek – Karst Research Institute, Slovenia

(Work supported by an NSF Earth Sciences Postdoctoral Fellowship and an NSF International Research Fellowship)

Interpretation of karst spring signals

One of the key questions in the field of karst hydrogeology concerns the relationship between the **variability in the signals** at karst springs and the **physical structure of the aquifer.**

Shuster and White [1971] used chemical and thermal variability to classify karst aquifers into two types: **diffuse systems**, which display little variation in total hardness, and **conduit systems**, which display large variations in total hardness.

However, there has been significant discussion about this terminology and the root causes for the presence or lack of variations...others have suggested that **most of the differences in responses could be accounted for by considering the fraction of recharge from autogenic versus allogenic sources** [Newson, 1971; Worthington et al., 1992]. Worthington et al. [1992] showed that some systems known to contain **large conduits** displayed **little variability**, suggesting that the terms “diffuse” and “conduit” might not be appropriate.

(text modified from Covington et al., 2012, J. Geophys. Res.)

Two typical approaches to understanding spring signals

Statistical, black box, and time series analysis of observed signals

Strength: broadly applicable to field data from a wide variety of settings. Few parameters.

Weakness: connections to aquifer structure and physical process are frequently uncertain

Process-based numerical simulations of signal transport

Strength: direct connections to physical processes and aquifer structure.

Weakness: computationally expensive, and many unknown parameters.

Two typical approaches to understanding spring signals

Statistical, black box, and time series analysis of observed signals

Strength: broadly applicable to field data from a wide variety of settings. Few parameters.

Weakness: connections to aquifer structure and physical process are frequently uncertain

Process-based numerical simulations of signal transport

Strength: direct connections to physical processes and aquifer structure.

Weakness: computationally expensive, and many unknown parameters.

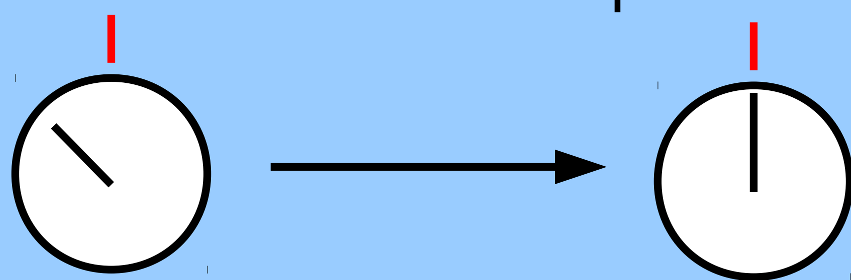
A general theory for spring responses?

Metrics and analytical solutions developed from process-based analysis

Process Length Scales:
A simple idea with many potential applications

Two requirements:

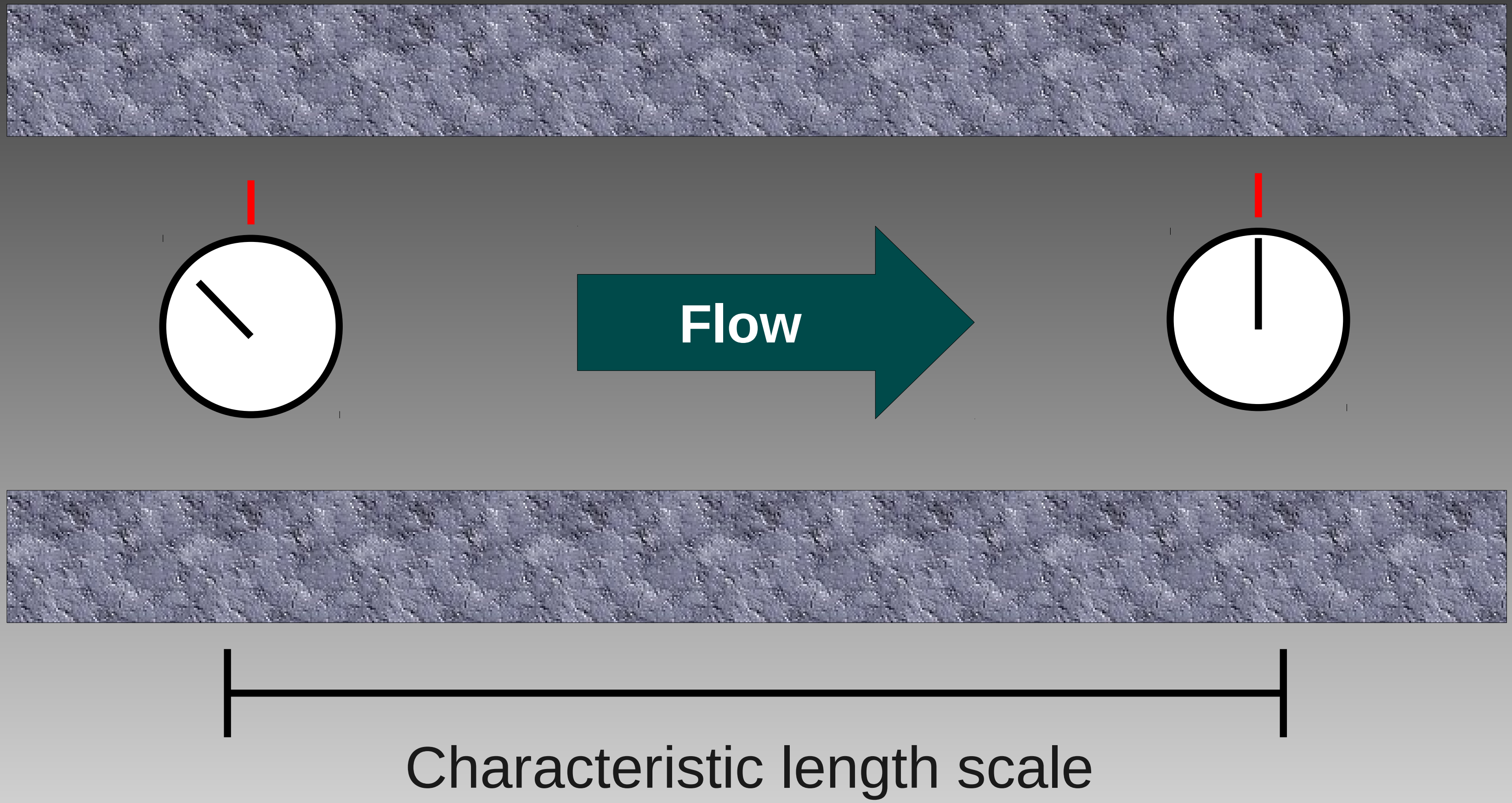
1. A process that occurs over a particular time scale



2. Flow that carries the process down a conduit (water or air)

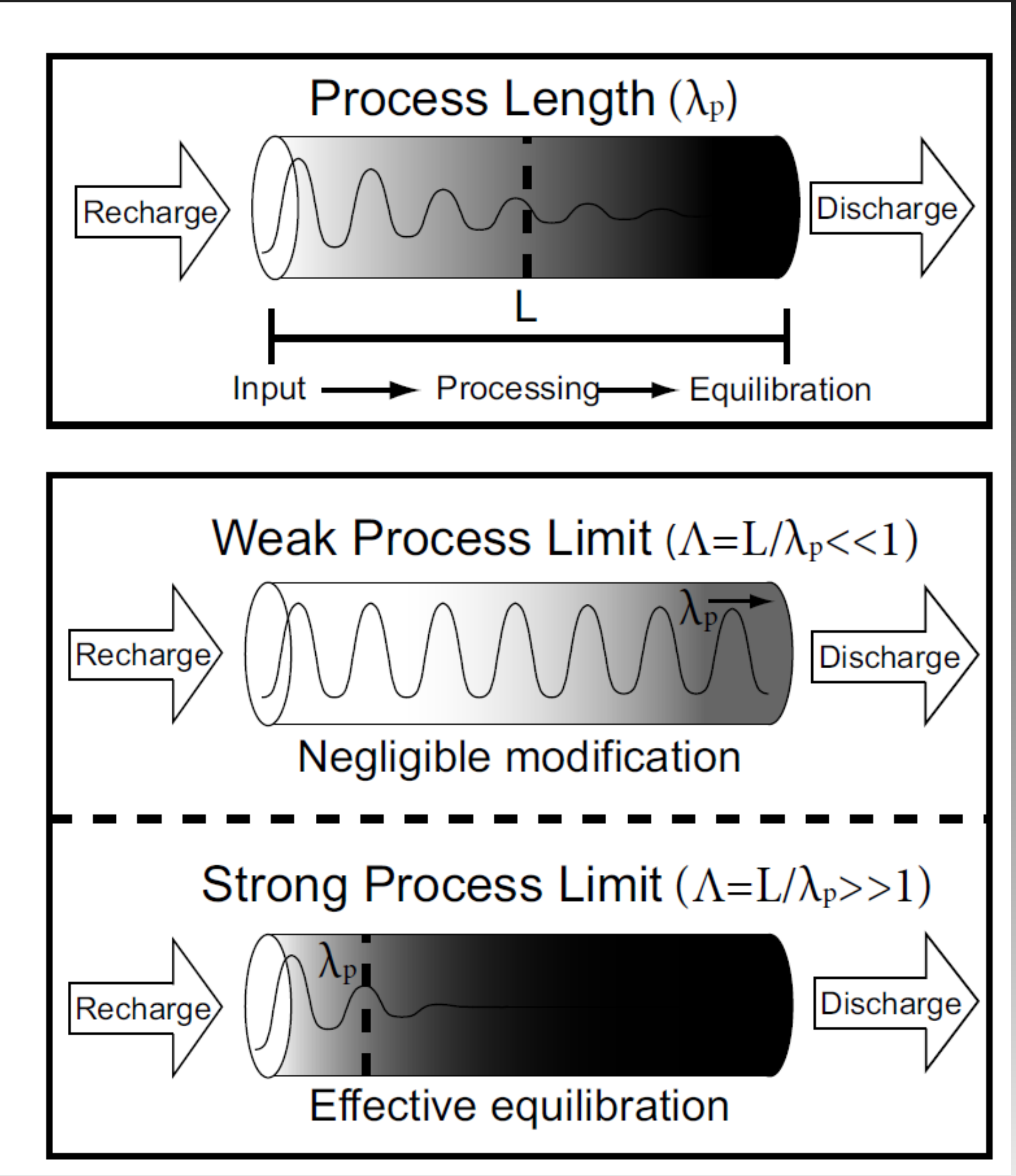
(Covington et al., 2012, J. Geophys. Res.)

A characteristic length scale emerges



Length = (Time Scale) X (Flow Velocity)

Does a given process allow variations at a spring?



(Covington et al., 2012, J. Geophys. Res.)

Conductivity Signals: Dissolutional Length Scales

$$\lambda_{\mathrm{dis}} = \frac{Q}{P\alpha} = \frac{VD_{\mathrm{H}}}{4\alpha},$$

Using laminar flow:

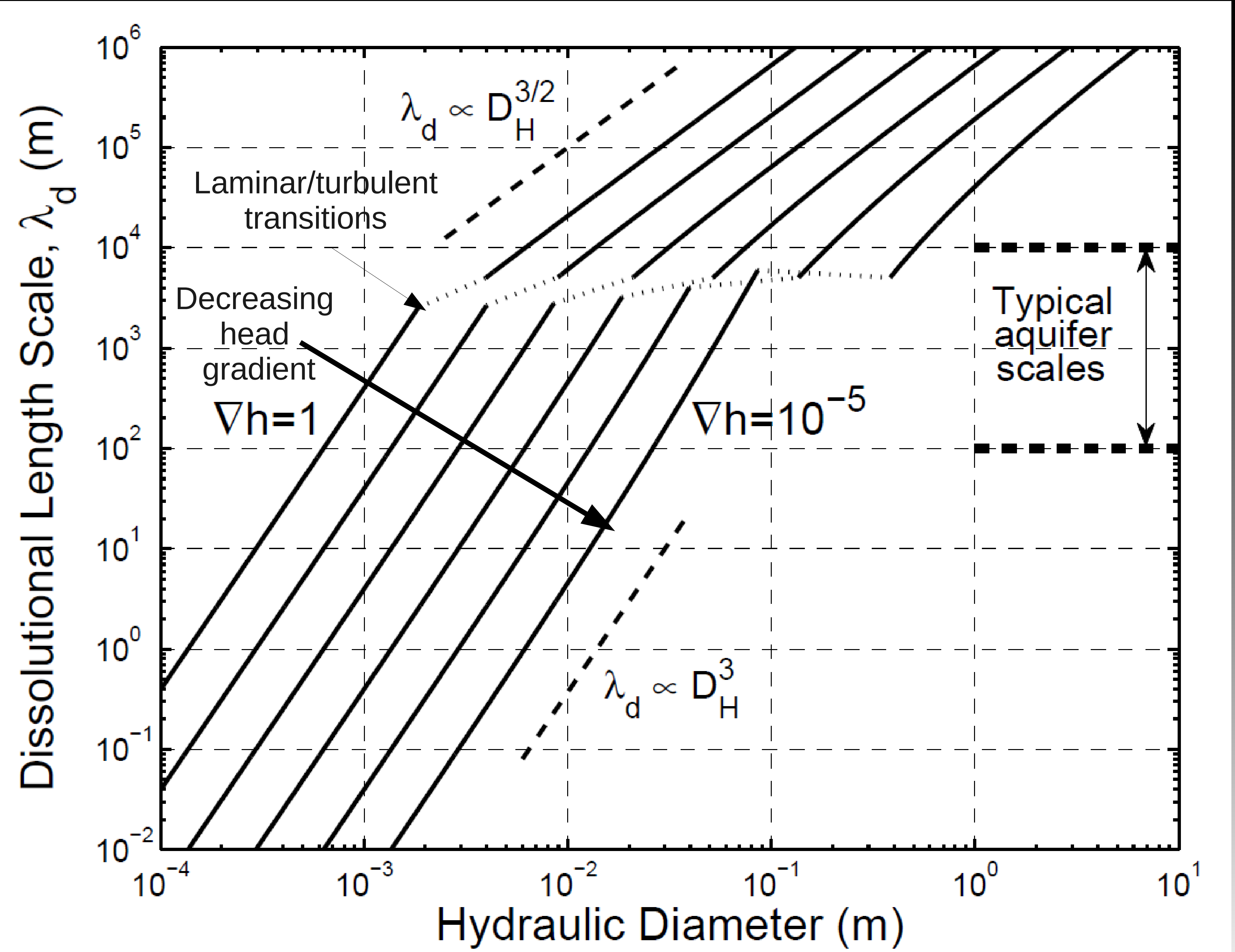
$$\lambda_{\mathrm{dis,lam}} = \frac{\rho g}{128\mu\alpha_{\mathrm{d}}} \nabla h D_{\mathrm{H}}^{3}.$$

Using turbulent flow:

$$\lambda_{\mathrm{dis,turb}} = \frac{1}{4\alpha}\sqrt{\frac{2g\nabla h}{f}} D_{\mathrm{H}}^{3/2}.$$

Longitudinal profiles of concentration are exponential, with e-folding length, λ.

(Covington et al., 2012, J. Geophys. Res.)



(Covington et al., 2012, J. Geophys. Res.)

Propagation of Thermal Signals

Analytical solution for temperature profile including 1D conduction

$$T'_{\mathrm{w}}(x^*, t^*) = T'_0 H(t^* - x^*) erfc\left[\frac{x^*}{\Psi\sqrt{\Theta(t^* - x^*)}}\right]$$

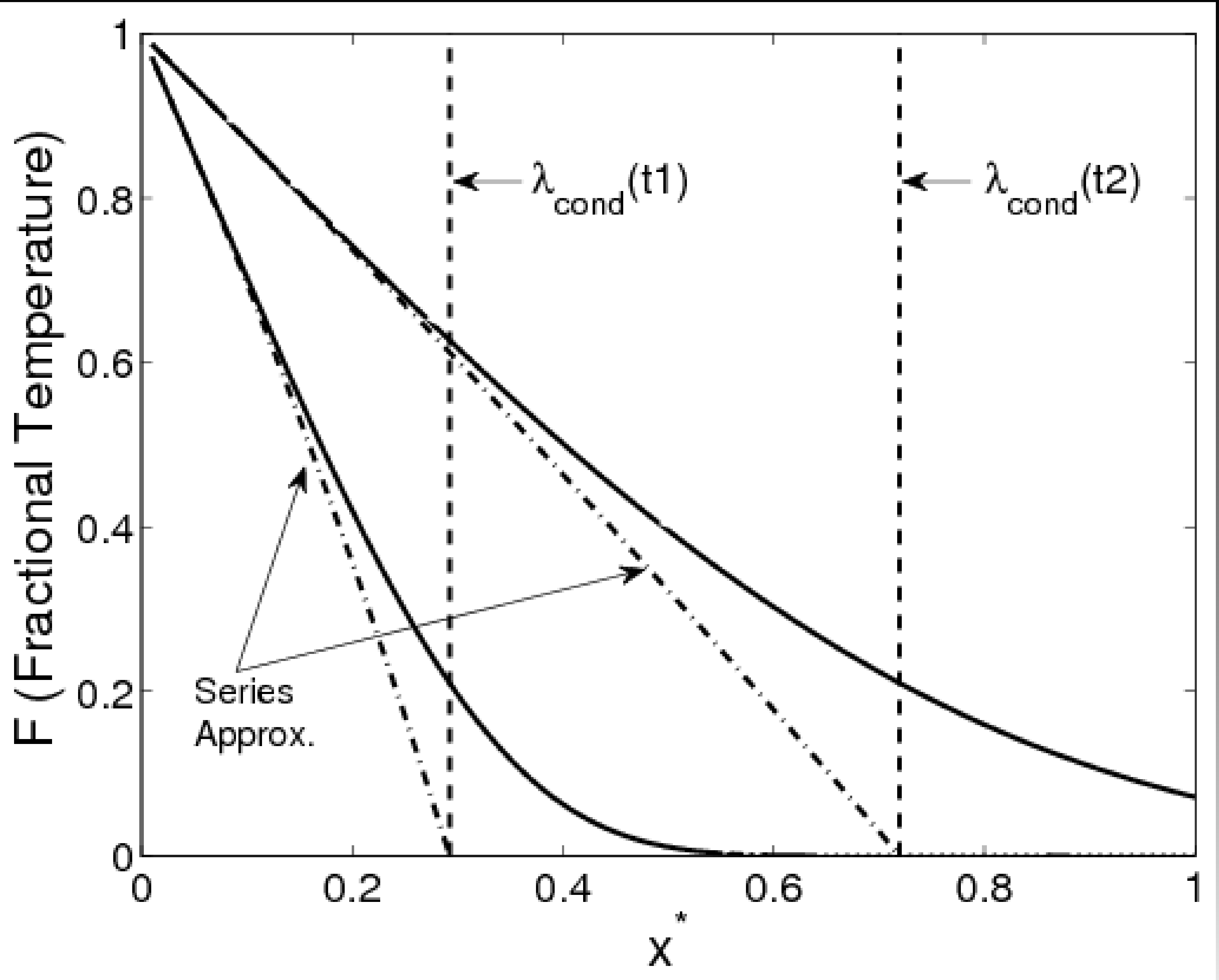
$$\Psi = \frac{\rho_w c_{p,w}}{\rho_r c_{p,r}}$$

$$\theta = \frac{R^2 \bar{V}}{\alpha_r L}$$

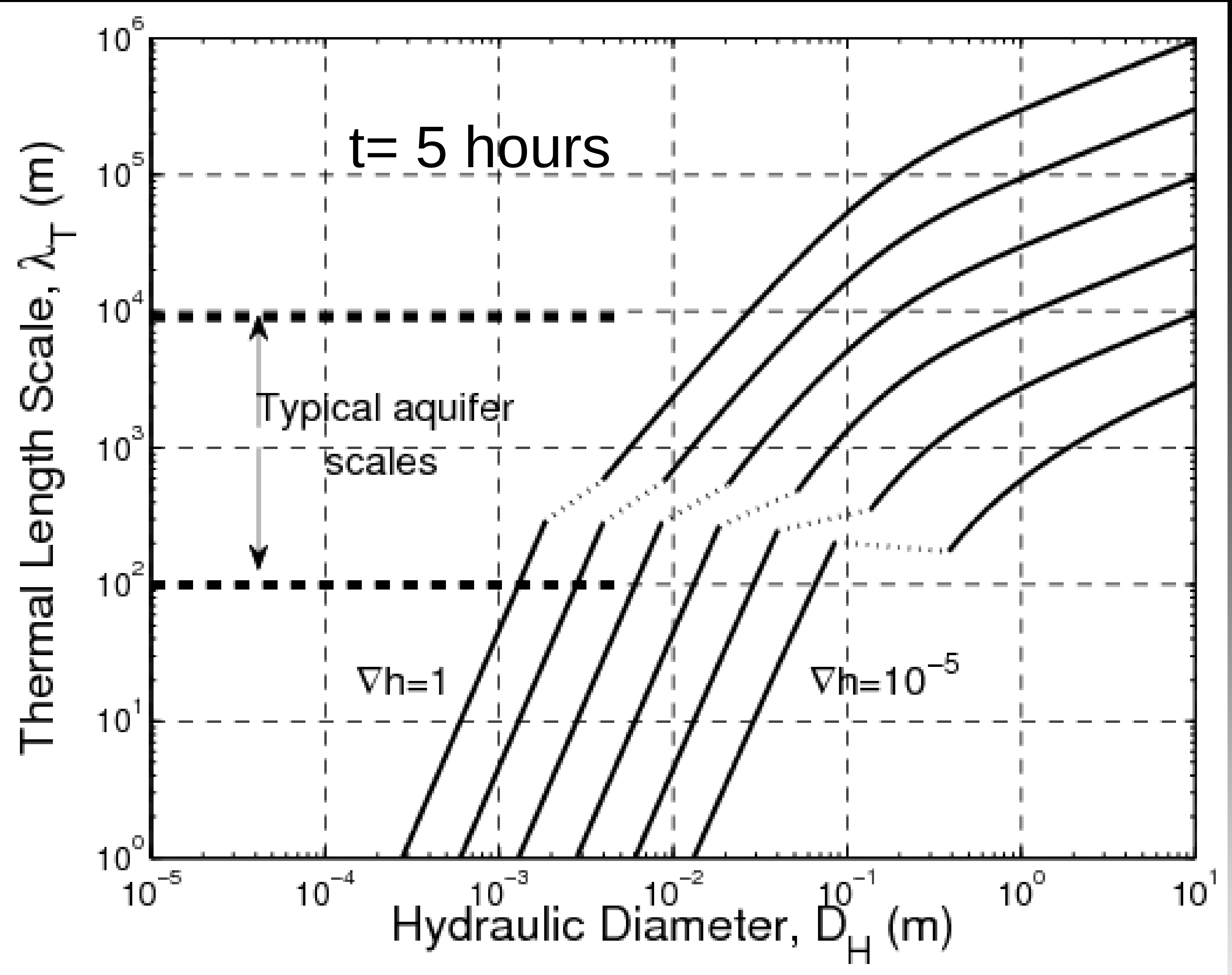
Taylor Series approximation of thermal length scale

$$\lambda_{\mathrm{T,cond}}(t^*) \sim \sqrt{\left(\frac{\pi^2\Psi^4 R^4 \bar{V}^2}{64\alpha_{\mathrm{r}}^2} + \frac{\pi\Psi^2 R^2 \bar{V}^2}{4\alpha_{\mathrm{r}}} t\right)} - \frac{\pi\Psi^2 R^2 \bar{V}}{8\alpha_{\mathrm{r}}}.$$

(Covington et al., 2012, J. Geophys. Res.)

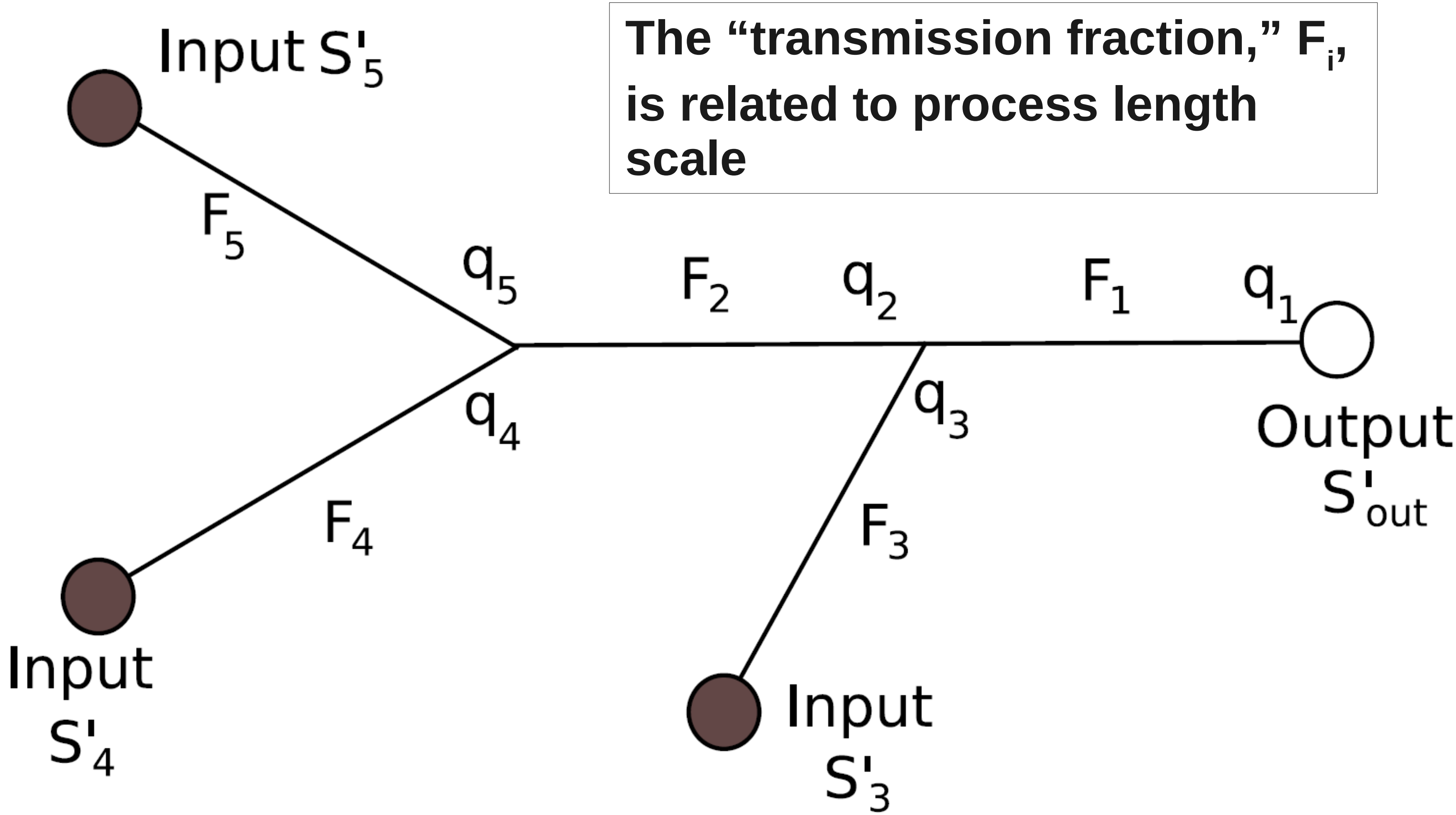


(Covington et al., 2012, J. Geophys. Res.)



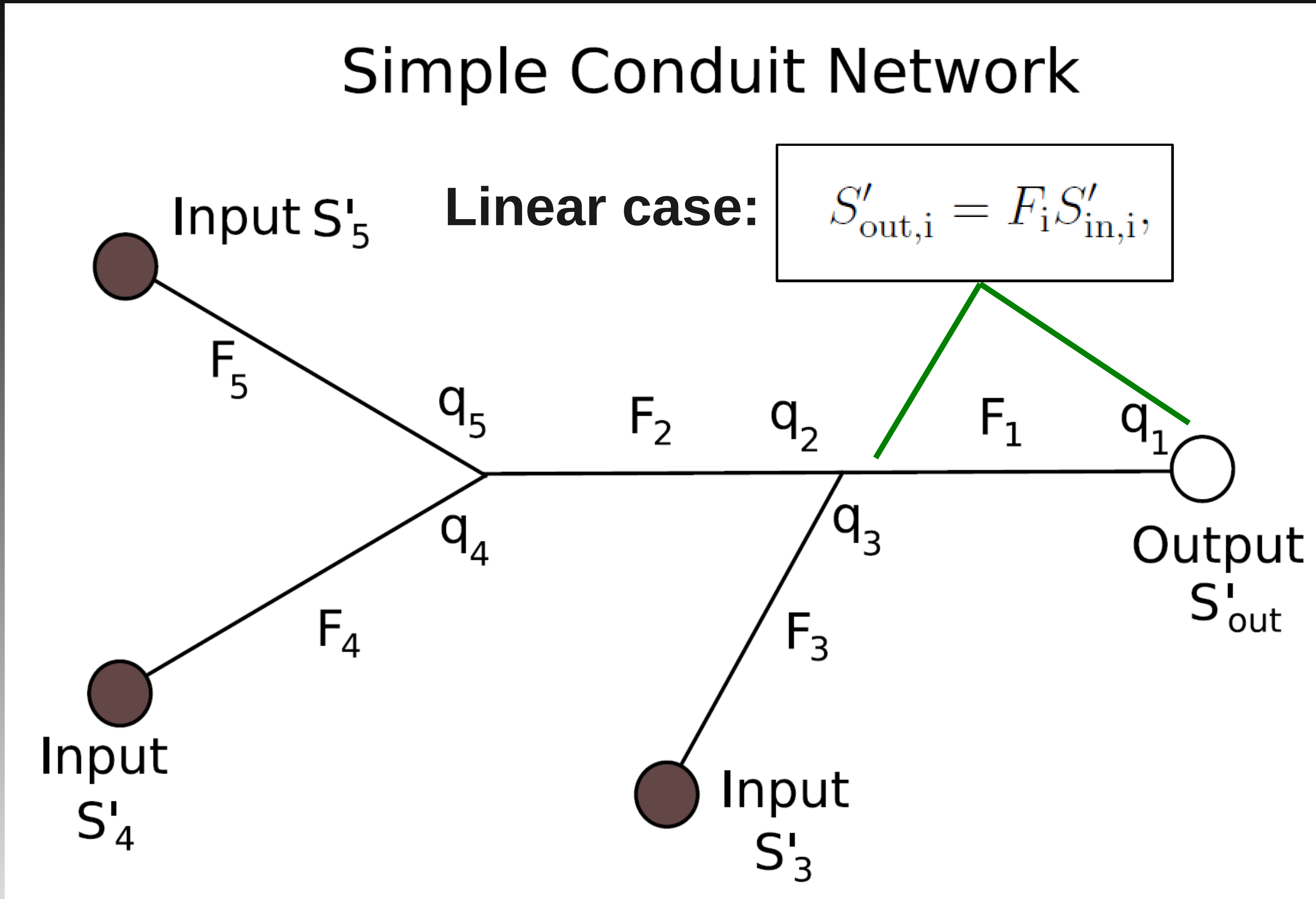
(Covington et al., 2012, J. Geophys. Res.)

Simple Conduit Network



The "transmission fraction," F_i, is related to process length scale

(Covington et al., 2012, J. Geophys. Res.)



(Covington et al., 2012, J. Geophys. Res.)

Response of Linear Conduit Networks

Individual conduit segment

$$S'_{\mathrm{out,i}} = F_{\mathrm{i}} S'_{\mathrm{in,i}},$$

F=transmission fraction (fraction of input that is transmitted)

Response of linear/linearized networks

$$S'_{\mathrm{out,tot}} = \sum_{i}^{inputs} S'_i R_i \prod_{j \in path} F_j = \sum_{i}^{inputs} S'_i R_i F_{\mathrm{path,i}},$$

Continuous representation using recharge distribution function, Φ_R

$$S'_{\mathrm{out}} = S'_{\mathrm{in}} \int_0^1 \phi_{\mathrm{R}} F_{\mathrm{path}} \; dF_{\mathrm{path}}.$$

(Covington et al., 2012, J. Geophys. Res.)

Response of Conduit Networks

$$S'_{\mathrm{out}} = S'_{\mathrm{in}} \int_0^1 \phi_{\mathrm{R}} F_{\mathrm{path}} \, dF_{\mathrm{path}}.$$

Physical Interpretation:
factors that control signal amplitude

1. Input signal amplitude

2. The capability of individual flow paths to transmit or dampen the signal

3. The distribution of flow among paths with different transmission factors

Response of Conduit Networks

$$S'_{\text{out}} = S'_{\text{in}} \int_0^1 \phi_{\text{R}} F_{\text{path}} \, dF_{\text{path}}.$$

Physical Interpretation:
factors that control signal amplitude

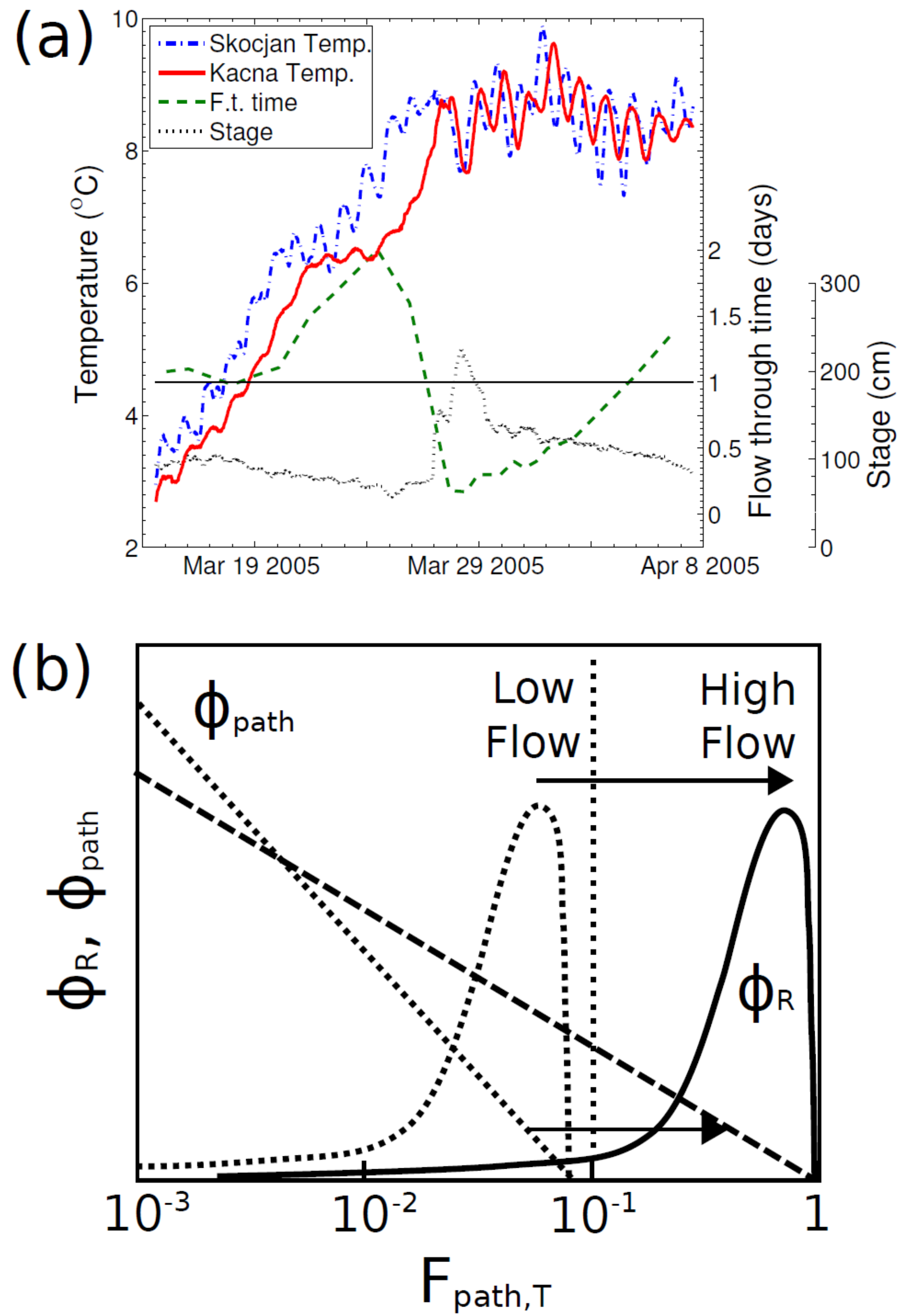
1. Input signal amplitude
2. The capability of individual flow paths to transmit or dampen the signal
3. The distribution of flow among paths with different transmission factors

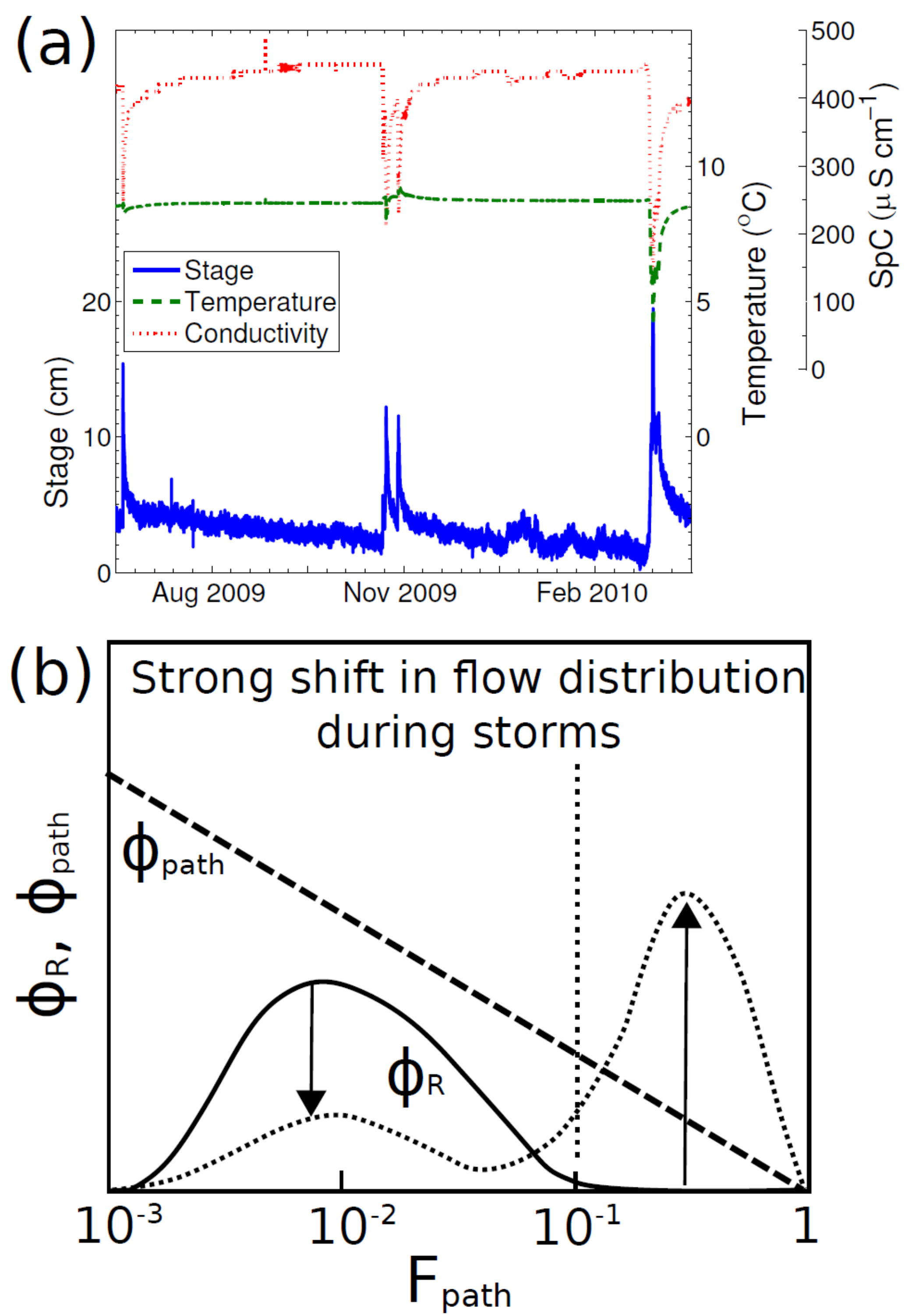
Response of Conduit Networks

$$S'_{\mathrm{out}} = S'_{\mathrm{in}} \int_0^1 \phi_{\mathrm{R}} F_{\mathrm{path}} \; dF_{\mathrm{path}} \, .$$

Physical Interpretation:
factors that control signal amplitude

1. Input signal amplitude
2. The capability of individual flow paths to transmit or dampen the signal
3. The distribution of recharge among paths with different transmission factors


(a)
Skocjan Temp.
Kacna Temp.
F.t. time
Stage
Temperature (°C)
Flow through time (days)
Stage (cm)
Mar 19 2005
Mar 29 2005
Apr 8 2005
(b)
φpath
Low Flow
High Flow
φR
φR, φpath
Fpath,T
10-3
10-2
10-1
1


(a)
Stage
Temperature
Conductivity
Stage (cm)
Temperature (°C)
SpC (μS cm⁻¹)
0
10
20
5
500
400
300
200
100
Aug 2009
Nov 2009
Feb 2010
(b)
Strong shift in flow distribution during storms
ϕ_{path}
ϕ_R
ϕ_R, ϕ_{path}
10^{-3}
10^{-2}
10^{-1}
1
F_{path}

Conclusions

- Process length scales provide a quantitative tool for understanding signal transport along single flow paths.

- For conduit networks, one can consider the transmission fraction, F_{path}, of individual network segments or entire input-output paths. In the case of linear networks, results are easily extended from the segment to the network scale.

- In linear conduit networks, the recharge distribution function, Φ_R, as a function of transmission fraction, F_{path}, provides a general framework for understanding network response. Φ_R subsumes previous explanations of the presence or lack of spring variability, such as diffuse vs. conduit flow systems, or the nature of recharge.

Open questions

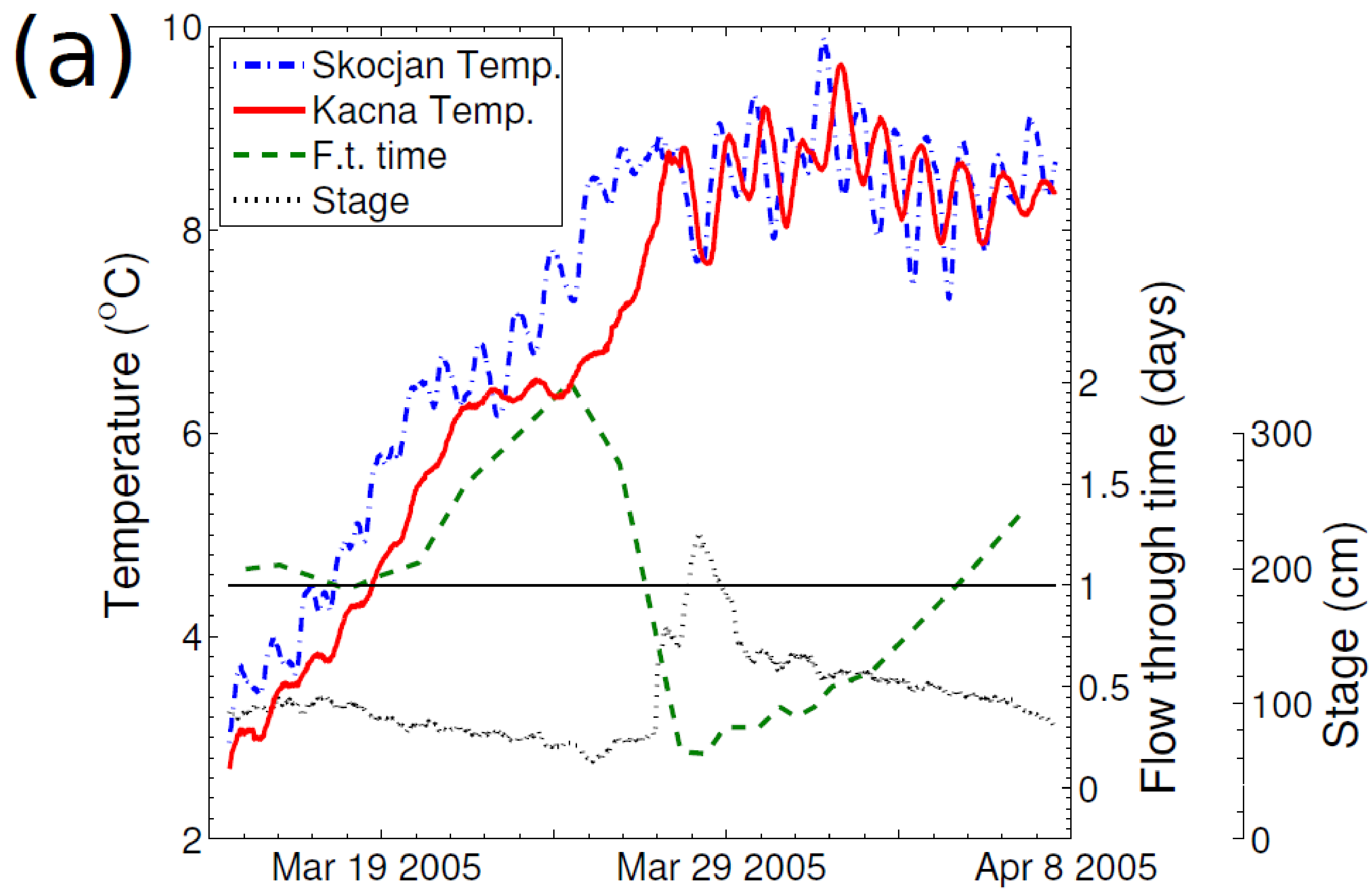
- What does Φ_R really look like in karst aquifers? Are there strong correlations with hydrological, geological, or speleogenetic factors?

- To what extent does linear network theory apply to real systems? Do non-linearities lead to qualitatively different types of behavior?

A Simple Rule of Thumb

$$\lambda_{\mathrm{T,early}} \approx \bar{V}t.$$

A pulse is damped if its duration is shorter than the flow through time.



(Covington et al., 2012, J. Geophys. Res.)