Scale Dependence of Equivalent Permeability Tensor in Naturally Fractured Reservoirs

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Abstract

Fracture geometries in naturally fractured reservoirs are usually inferred from sparse subsurface observations from well logs, core samples and seismic surveys. Since the largest fractures tend to be the least frequent, they are often undersampled. Therefore, interpretations based on samples taken at any scale smaller than that of interest contain a bias, misrepresenting the true characteristics of the fracture system. This bias may lead to the incorrect characterisation of the reservoir, upscaling and gridding.

In this work, we investigate the variation of the equivalent permeability and its anisotropy in naturally fractured rocks as a function of sample size. We use a two dimensional fractured reservoir analogues mapped in field outcrops to compute equivalent permeability tensors for random samples which are taken inside the model at different length scales.

Introduction

Geological information about reservoirs is usually based upon data drawn from a wide range of length scales (e.g. well logs, core samples and seismic studies). Due to the storage and computational limits of reservoir simulators, these data must be integrated at the scale of the model grid cell (10–300 m). The process of upscaling enables this integration by some kind of averaging of the geological properties and flow characteristics of the reservoir to find equivalent properties at this coarse scale, such that the reservoir system on both scales acts similarly.

Numerous upscaling methods have been proposed and analysed (e.g. Qi and Hesketh, 2005). Each method assumes the existence of a representative elementary volume (REV). Finding this REV for permeability, k is not trivial in heterogeneous reservoirs where k often varies by orders of magnitude over very small distances.

Naturally fractured reservoirs (NFRs) tend to be highly heterogeneous and the fracture flow paths within them can aid or inhibit oil recovery in unforeseen ways (Matthäi *et al.*, 2007). A typical method of NFR property modelling is via permeability values derived from stochastically generated fracture geometries, with properties inferred from sparse observations such as well logs and cores (Min *et al.*, 2004). Since the largest fractures tend to be the most infrequent, they are rarely sampled (e.g. Odling, 1992). Therefore, interpretations based on samples taken at any scale smaller than the scale of interest contain an inherent bias which will affect the upscaled representation of the reservoir model.

In this work, we study the scale dependence of equivalent permeability in NFR analogues mapped in outcrops. Our aim is to investigate the existence of an REV in such reservoirs, and to subsequently determine the conditions under which permeability may be successfully upscaled.

Full Permeability Tensor

A Full tensorial representation of permeability is always required when: (1) the principal directions of anisotropy are not aligned with the simulation coordinate directions (2) non-orthogonal grids formulation (3) scale-up of permeability generates full permeability tensors although the fine-scale grids are isotropic and orthogonal (4) naturally fractured systems.

In a two-dimensional Cartesian coordinate system, $\mathbf{k}$ is:

$$\begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}$$

Since $\mathbf{k}$ is symmetric positive definite matrix (Bear, 1972)

$$k_{xy} = k_{yx}$$

Which the positive definition implies that:

$$k_{xx}k_{yy} > \left(k_{xy}\right)^2, k_{xx} > 0, k_{yy} > 0$$

Objectives

The objectives of this study are investigation of: (1) variation of the equivalent permeability of naturally fractured rocks as a function of sample size (2) existence of an REV in naturally fractured reservoirs (3) considering the full tensor of equivalent permeability.

Methodology

Geometry: we convert the traced fractures by boundary representation (BREP) via 3rd order NURBS curves in a 2D domain to represent the fractured porous medium.

Fracture permeability: we assume that fractures have smooth walls and will exhibit a parallel plate viscous (Poiseuille) laminar fluid flow within them. Then the permeability of each fracture segment, k_f, is (Witherspoon *et al.*, 1980):

$$k_\mathrm{f} = \frac{a^2}{12}$$

Single-phase steady-state flow: having established the fracture geometry and spatial permeability distribution in our NFR model, the geometry is discretised using a constrained conforming triangle-based mesh. We then use the Complex Systems Modelling Platform (CSMP++) software (e.g. Matthäi *et al.*, 2007) to determine pressure and velocity field over the entire domain by solving pressure equation:

$$\nabla \cdot (\mathbf{k} \cdot \nabla p) = 0$$

Boundary conditions: two flow problems have to be solved: (1) uniform pressure (Dirichlet) boundary conditions on the left (p_in) and on opposite edge of domain, and no-flow boundary conditions on the other edges (Figure 1 shows these conditions) (2) Dirichlet boundary conditions on the bottom and top edges and no-flow boundary conditions on the other edges.

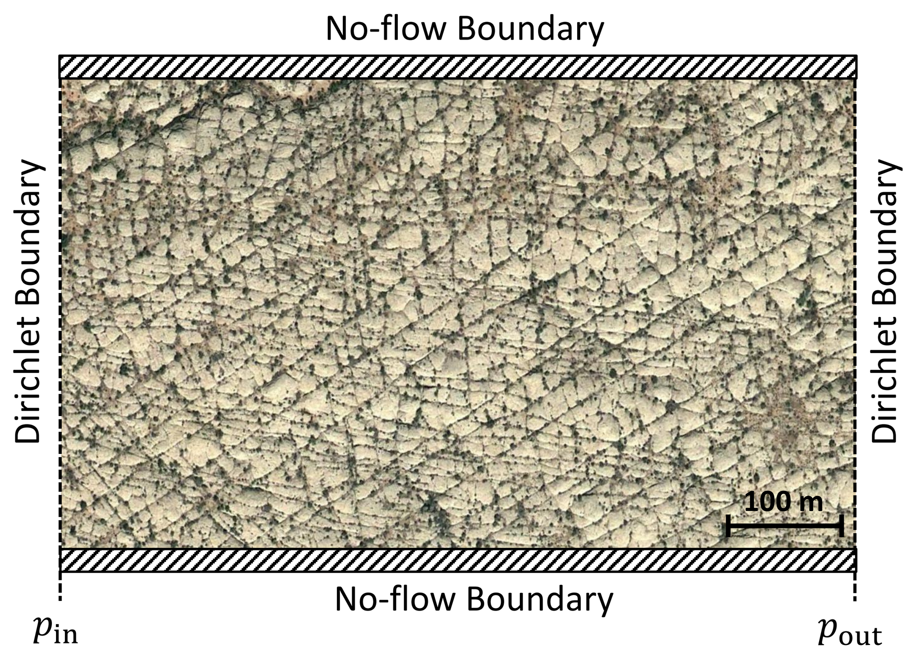

Figure 1. Typical fractured outcrop map from Arches national park (680 m × 400 m)

Assumptions: we use $\Delta p = 10^8$ Pa ($\Delta p = p_\mathrm{in} - p_\mathrm{out}$) in this study. Also we use a constant permeability for rock matrix surrounding the fractures, k_m, in order to focus our investigation on the contribution of the fractures to k_eq.

Sampling procedure: we randomly sample square shaped box (l_i) from different regions inside the domain at different sizes.

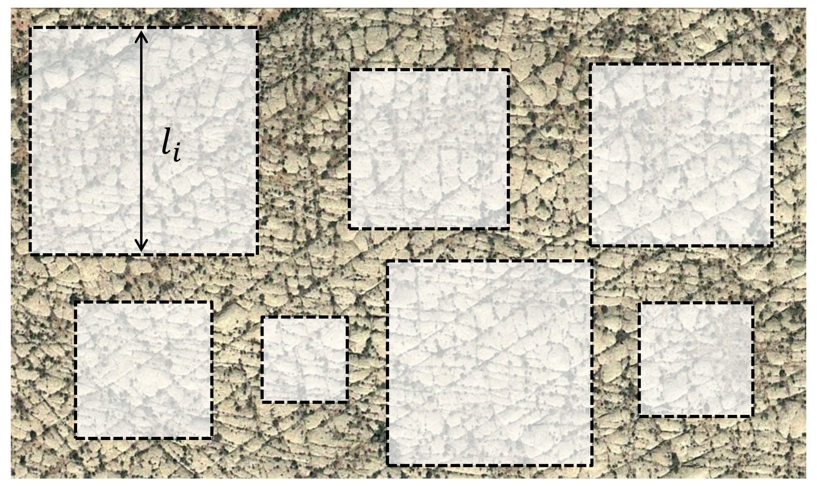

Figure 2. Random sampling at different sizes

Calculation of equivalent permeability tensor: by incorporating global flow effects into hydraulic property calculations on the local scale (sample l_i), the full equivalent permeability tensor is computed by making use of volume-averaged fluid velocities and pressure gradients over the sample (Durlofsky, 2005) in the form of Darcy's law:

$$\langle \mathbf{u} \rangle = -\mathbf{k}^* \cdot \langle \nabla p \rangle$$

The matrix form of this equation would be:

$$\begin{bmatrix} \langle \nabla p_x \rangle^1 & \langle \nabla p_y \rangle^1 & 0 & 0 \\ 0 & 0 & \langle \nabla p_x \rangle^1 & \langle \nabla p_y \rangle^1 \\ \langle \nabla p_x \rangle^2 & \langle \nabla p_y \rangle^2 & 0 & 0 \\ 0 & 0 & \langle \nabla p_x \rangle^2 & \langle \nabla p_y \rangle^2 \end{bmatrix} \begin{Bmatrix} k_{xx}^* \\ k_{xy}^* \\ k_{yx}^* \\ k_{yy}^* \end{Bmatrix} = - \begin{Bmatrix} \langle u_x \rangle^1 \\ \langle u_y \rangle^1 \\ \langle u_x \rangle^2 \\ \langle u_y \rangle^2 \end{Bmatrix}$$

where the superscripts 1 and 2 denote the flow problems to be solved.

Model Verification

To verify the accuracy of the computed permeability tensor using the methodology described above, we consider a conceptual fractured medium consisting of a rock matrix block embedded with three fractures which are not-aligned with the flow direction (Durlofsky, 1991). Figure 3 shows the model and its corresponding parameters. Analytical and numerical simulation results for different fracture orientations are presented in Table 1. Results demonstrate the applicability of the proposed method for the calculation of equivalent permeability tensor for such systems.

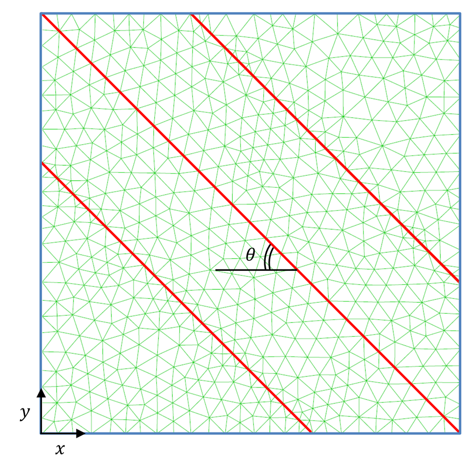

Figure 3. Porous medium with fractures not-aligned with coordinate axes.

$k_\mathrm{m} = 2 \times 10^{-15} [\mathrm{m}^2]$

$k_\mathrm{f} = 1 \times 10^{-12} [\mathrm{m}^2]$

Permeability tensor $\times 10^{-15}$ [m^2]		$\theta = 0$	$\theta = -45$	$\theta = -90$
k_{xx}	Analytical	98.581	50.397	2.219
	Numerical	98.581	51.688	2.000
k_{xy}	Analytical	0.0	-48.183	0.0
	Numerical	0.0	-49.674	0.0
k_{yy}	Analytical	2.219	50.397	98.581
	Numerical	2.000	51.659	98.581

Table 1. Analytical vs. numerical results of permeability tensor calculation with different fracture orientation.

Results

To investigate equivalent permeability tensors in NFRs, we use the above methodology on an outcrop from Arches national park. Figure 4 shows log-normally distributed apertures which are assigned to individual fractures in the model. The result of the numerical simulation for this model is shown in Figure 5.

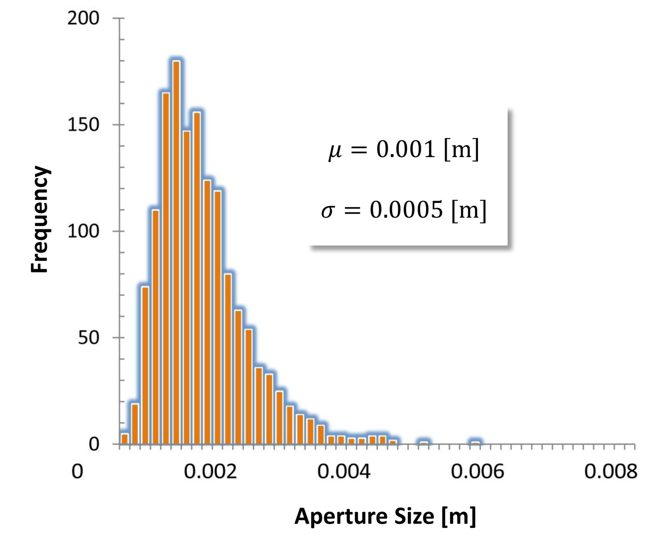

Figure 4. Log-normally distributed apertures

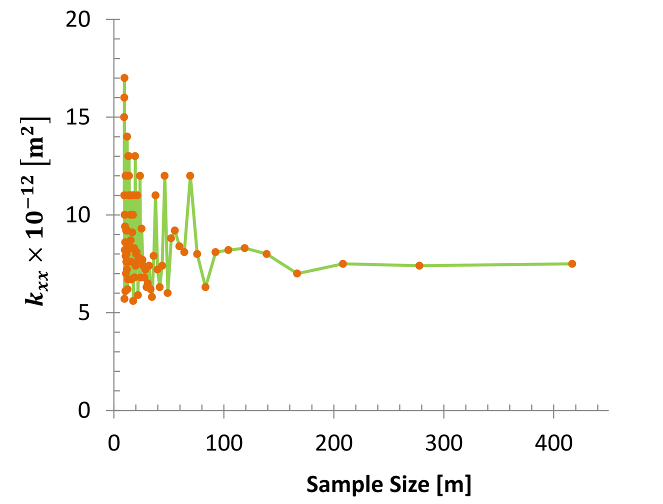

Figure 5. Variation of tensor component of permeability vs. sample size

Conclusions

The main conclusions from this work are as follows:

- There is a decreasing trend in the equivalent permeability with increasing sample size, up to a scale where it did not change significantly with further increase in sample size.
- These findings demonstrate that there is a strong dependence of equivalent permeability upon scale for the NFRs as we have studied.
- Results also suggest the existence of an REV which should be used to determine the appropriate scale to upscale the permeability for these NFR analogues to use in numerical reservoir simulators.
- Fracture geometries in naturally fractured reservoirs should be sampled at the scales of interest; if not, the equivalent permeability of such reservoirs might be significantly unrealistic.

Future Works

- Sensitivity analysis of different outcrops.
- Using geo-mechanical models to create realistic aperture values for the fractures.

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