

## 1. Abstract

We developed a ray-tracing algorithm for determining the tropospheric delay for Very Long Baseline Interferometry (VLBI), which is one of the main error sources. We tested different ray-tracing parameters to find the best parameterization for resolution, interpolation method and refractivity constants.

We validated our algorithm by using the calculated delays a priori in our analysis and estimated baseline length repeatabilities and Length-of-day values. The repeatabilities show a similar overall accuracy but improved results using ray-tracing for different stations. Also the Length-of-day comparison with GPS shows that the accuracy depends on the stations involved. Nonetheless we again find partly improvements when using ray-traced delays.

## 2. Tropospheric delay

Electromagnetic waves are bent and delayed while propagating through the atmosphere. This effect is a major error source for space geodetic techniques, such as Very Long Baseline Interferometry (VLBI) or Global Navigation Satellite Systems (GNSS).

The effect through the ionosphere can be measured using two frequencies, but must be modeled for the troposphere since it is non-dispersive.

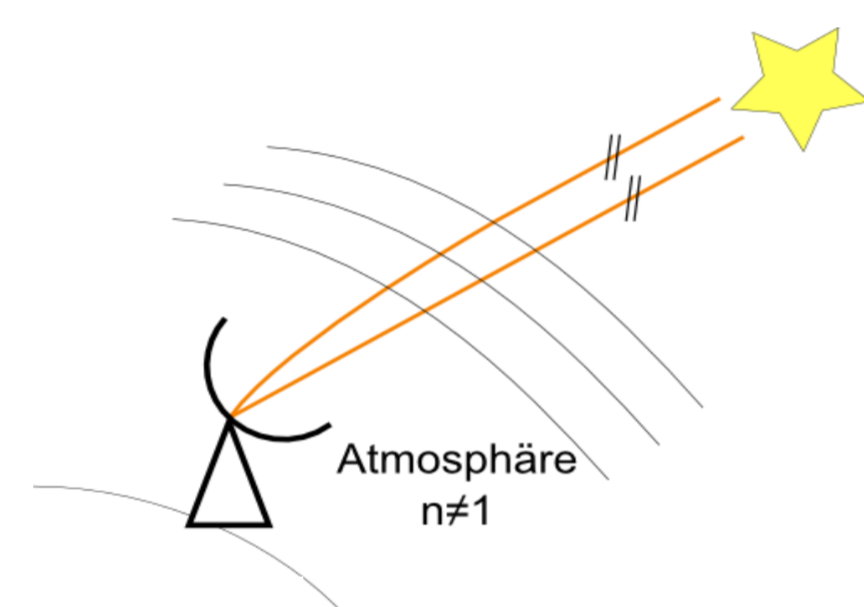


Fig. 1. Schematic chart of the path delay of electromagnetic waves through the atmosphere. Signals are bent and delayed.

## 3. Ray-tracing

Ray-tracing, primarily used in computer graphics, is a technique which tries to calculate the path through a medium. If the refractive index  $n$  is known at every point of the ray ( $\vec{r}$ ), the path length can be calculated using the Eikonal equation (in vector notation):

$$|\nabla L|^2 = n^2(\vec{r})$$

Where  $\nabla L$  stands for the components of the ray direction and  $L$  is the path length. The surfaces  $L(r) = \text{constant}$  are called geometrical wave surfaces of geometrical wavefronts.

The Eikonal equation yields a set of seven partial differential equations, six of them must be solved simultaneously.

The refractive index values can be derived from a NWM. We use operational data from the European Centre for Medium-Range Weather Forecasts (ECMWF) which give the meteorological parameters needed for the calculation of the refractive index.

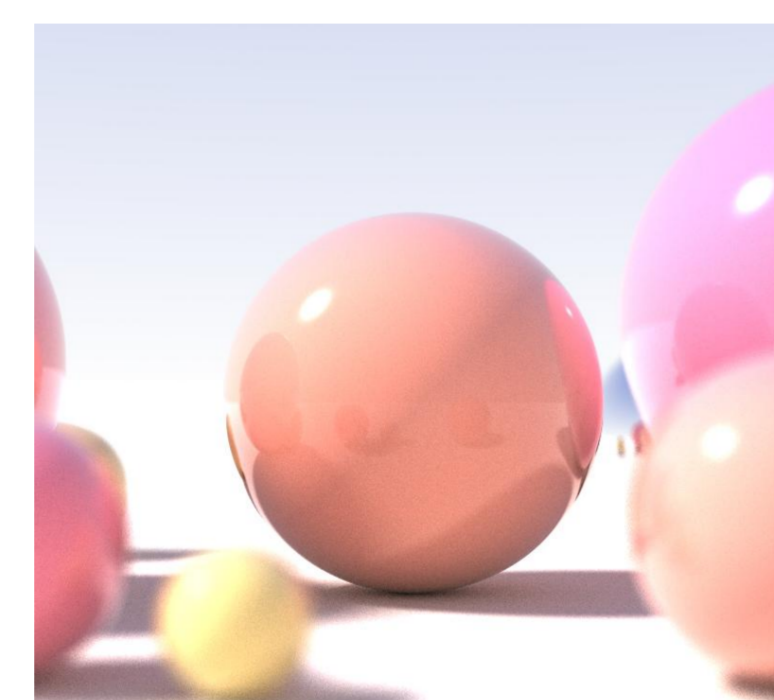


Fig. 2. Rendered computer graphic.

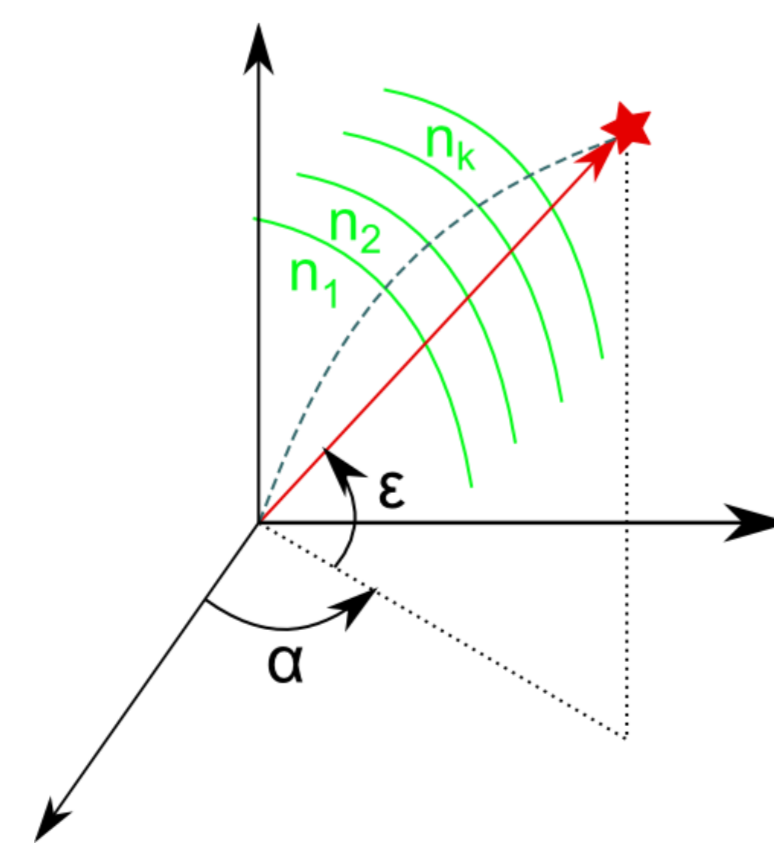


Fig. 3. Schematic chart of ray-tracing where the path of a signal from a source (red star) should be reconstructed on the basis of a known field of refractive index values.

## 4. Algorithm

Our algorithm is written in Matlab, which is on the one hand slow compared to a compiled programming language but eases the programming itself due to many built-in functions. Ray-tracing algorithms differ by many aspects:

Apart from the input dataset there are a number of variables for a ray-tracing program in general and a tropospheric ray-tracer in particular. These parameters may affect both the accuracy and the efficiency of the program, so they must be chosen carefully.

### • NWM resolution

Lower resolutions decrease computation time but could also worsen results. Therefore we calculated the differences of tropospheric delays using a resolution of  $0.5^\circ$  and  $1.0^\circ$  in latitude and longitude (Figure 4). Since they reach up to 1 decimeter we used  $0.5^\circ$ .

### • Interpolation and extrapolation

The original input data must be interpolated and vertically extrapolated to the upper limit of the troposphere. We achieved the best results using horizontal spline interpolation with 8 points around the meridian (orange dashed line in Figure 5).

### • Stop criterion

The eventually estimated output ray must point to the emitting source, therefore a feasible criterion is the difference between outgoing elevation angle and calculated (geometric) direction.

### • Refractivity constants

There exist different experimentally determined refractivity constants. For our analysis we took "Rüeger best average", which is suggested by the IAG Working Group 4.3.3. We calculated the potential difference when using different constants (Figure 7).

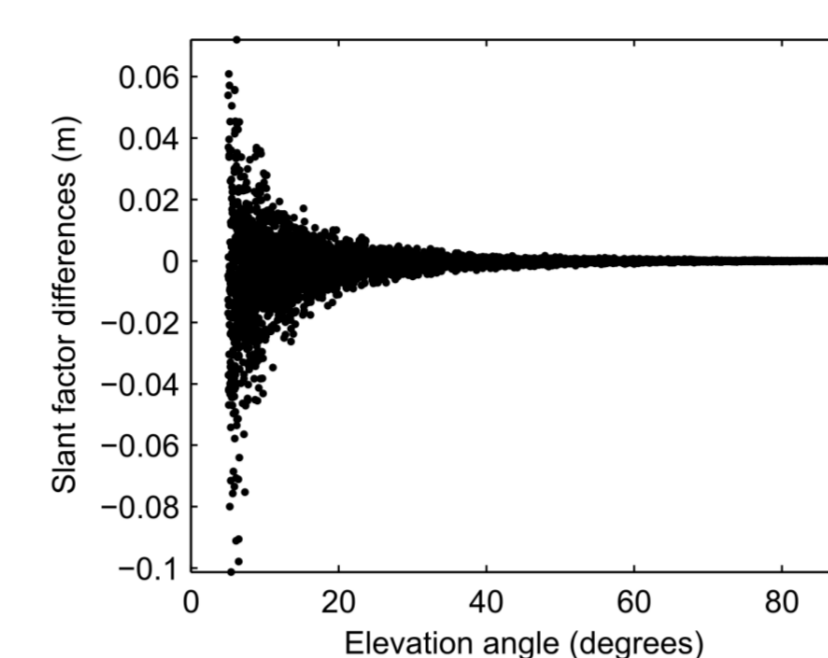


Fig. 4. Elevation dependent slant factor differences ( $1^\circ - 0.5^\circ$  resolution).

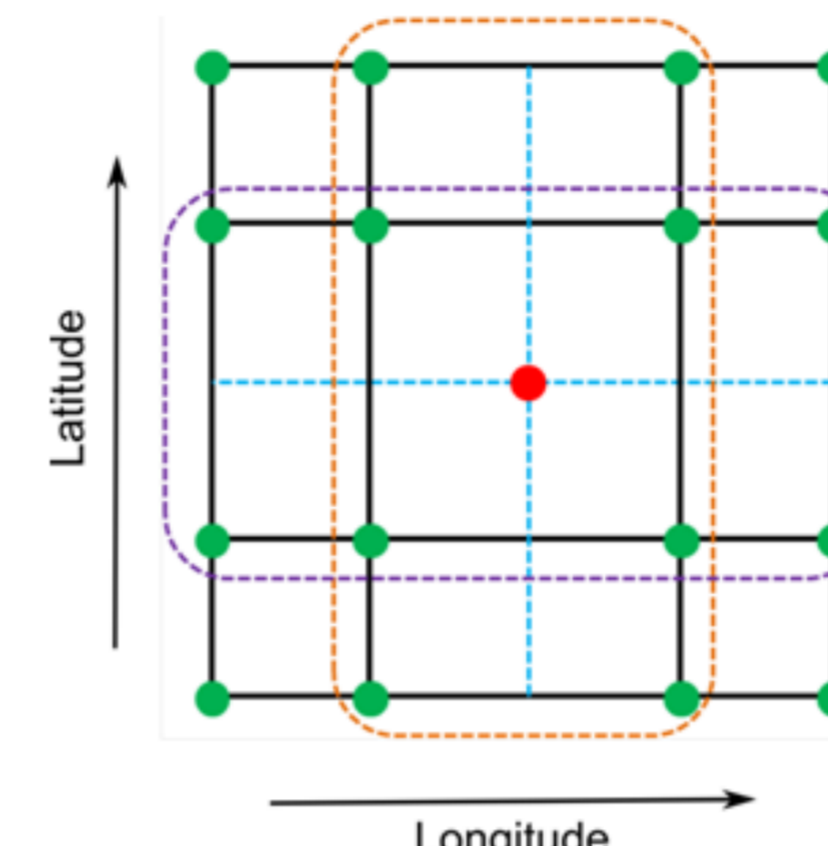


Fig. 5. Schematic chart of different interpolation methods.

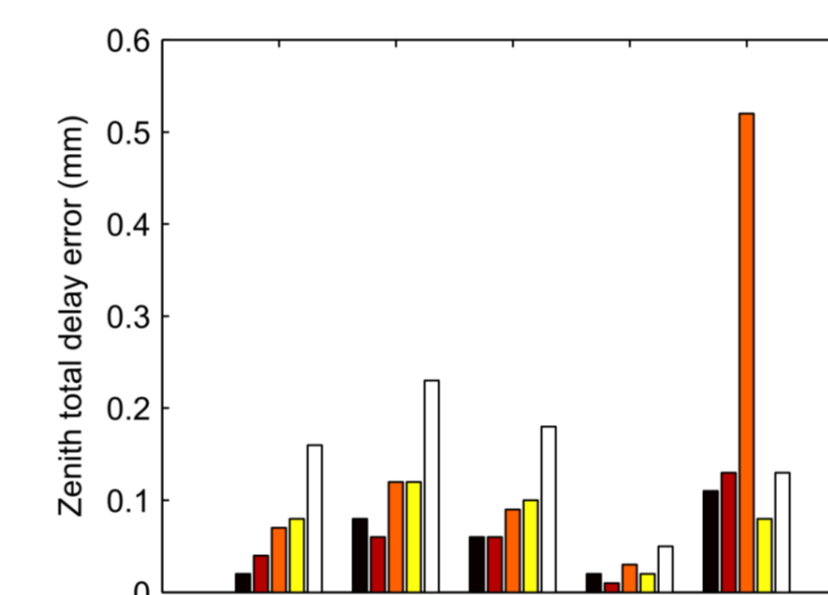


Fig. 6. Schematic chart of different interpolation methods: Spline 16 points (black), spline 8 points along meridian (red), spline 8 points along parallel (orange), mean 4 points (yellow), mean 16 points (white).

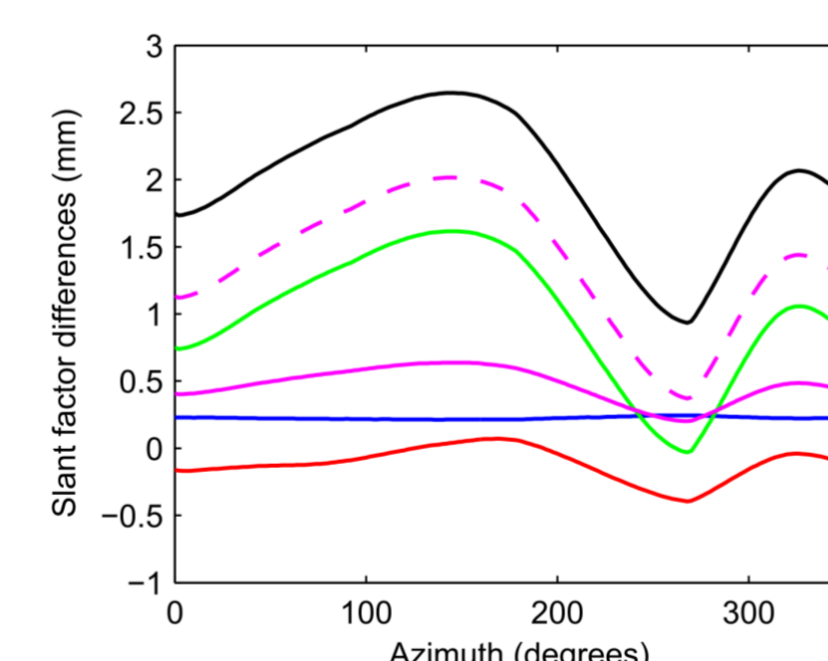


Fig. 7. Azimuth dependent slant factor differences w.r.t. Rüeger best average: green: IUGG, magenta-solid: Bevis et al., red: Smith & Weintraub, magenta-dashed: Essen, black: Essen & Froome at station Tsukuba, 18.8.2008, 0h.

## References

- Nafisi, V. et al, Ray-traced tropospheric delays in VLBI analysis, Radio Sci., 47
- Böhm, J. et al., Asymmetric troposphere delays from numerical weather models for UT1 determination from VLBI Intensive sessions on the baseline Wettzell-Tsukuba, J.Geod.,84(5)
- IVS webpage (<http://ivsc.gsfc.nasa.gov/>)
- IGS webpage (<http://igs.cbl.jpl.nasa.gov/>)

## Acknowledgement

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## 5. Baseline length repeatabilities

We use ray-traced delays in the VLBI analysis using the Vienna VLBI Software VieVS. We analyze a continuous VLBI campaign in August 2008 (CONT08) and calculate baseline length repeatabilities which are standard deviations of baseline lengths.

Those are used to validate ray-tracing as method for determining the tropospheric delay. Zenith delays and tropospheric gradients are estimated in the analysis.

We compare them to a standard mapping function approach which is also derived from a numerical weather model. We hardly find differences in the fitting curve, although the accuracy depends much on the stations. For example ray-tracing yields very good results for station Concepcion in Chile, whereas the results for Tsukuba in Japan deteriorate.



Fig. 8. Stations taking part in CONT08, a continuous VLBI campaign in August 2008.

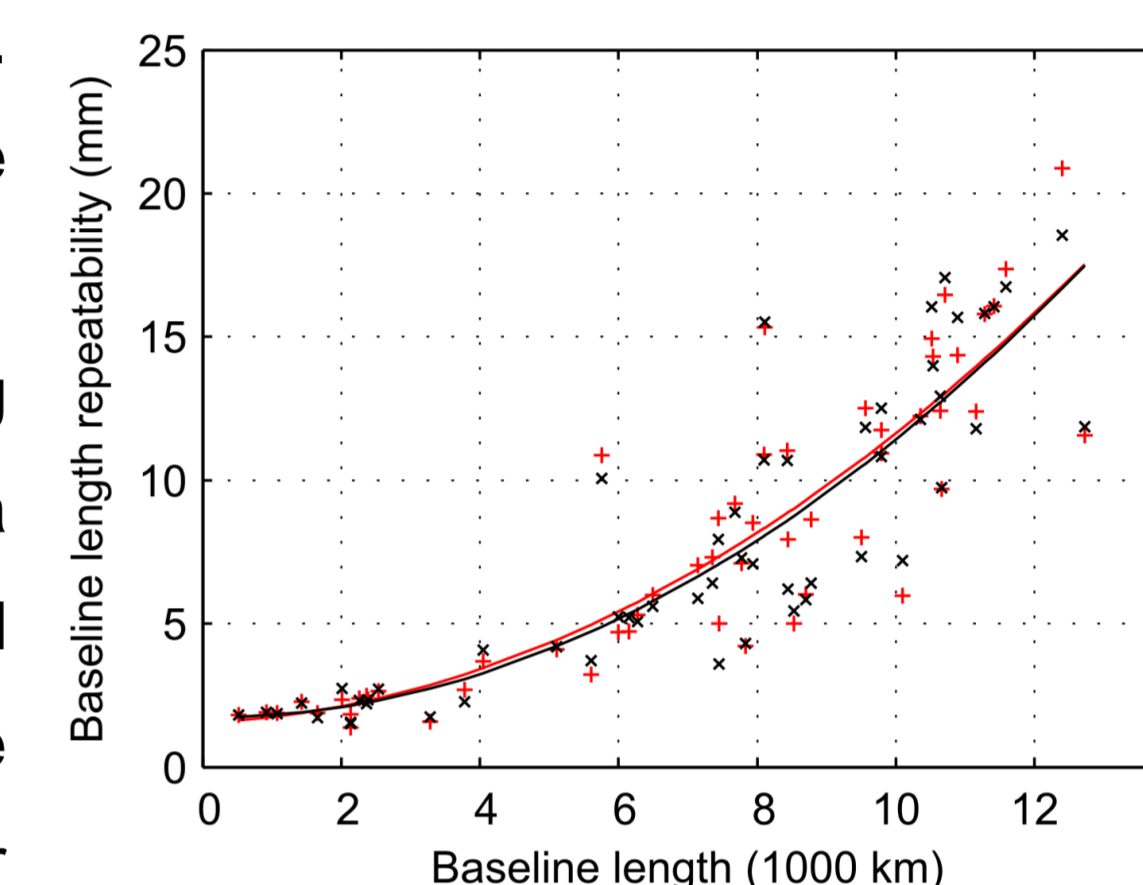


Fig. 9. Baseline length repeatabilities for CONT08 using two models for the a priori tropospheric delay: Red: Ray-tracing, black: standard mapping function (Vienna Mapping Function 1) approach, also based on numerical weather models.

## 6. Length-of-day

In addition we analyze 1h-Intensive sessions which are used for the near real-time estimation of UT1-UTC. As external validation we convert UT1-UTC to Length-of-day (LOD) using the formula:

$$LOD = -\frac{\delta(UT1-UTC)}{\delta t}$$

Global Navigation Satellite Systems (GNSS) also derive Length-of-day values. Comparing VLBI-derived LOD to GPS-derived LOD allows an external validation. We use final earth rotation parameter (ERP) files from the International GNSS Service (IGS).

Table 1 shows RMS values of the difference of GPS-derived and VLBI-derived LOD values. Since the results depend on the baseline, Table 1 is separated for different baselines – the corresponding Intensive sessions are shown in Figure 10.

Model	RMS LOD w.r.t. IGS-LOD [ $\mu$ s]			
	INT1	INT2/3	Ts-Wf	all
p0 Saast.	30.8	27.1	24.6	29.8
ECMWF	30.4	26.1	25.1	29.3
Ray-tracing	30.8	25.3	23.5	29.5

Table 1. RMS values for the difference of VLBI-derived and IGS-derived Length-of-day values. Rows indicate different models, columns different baselines (see Figure 10).

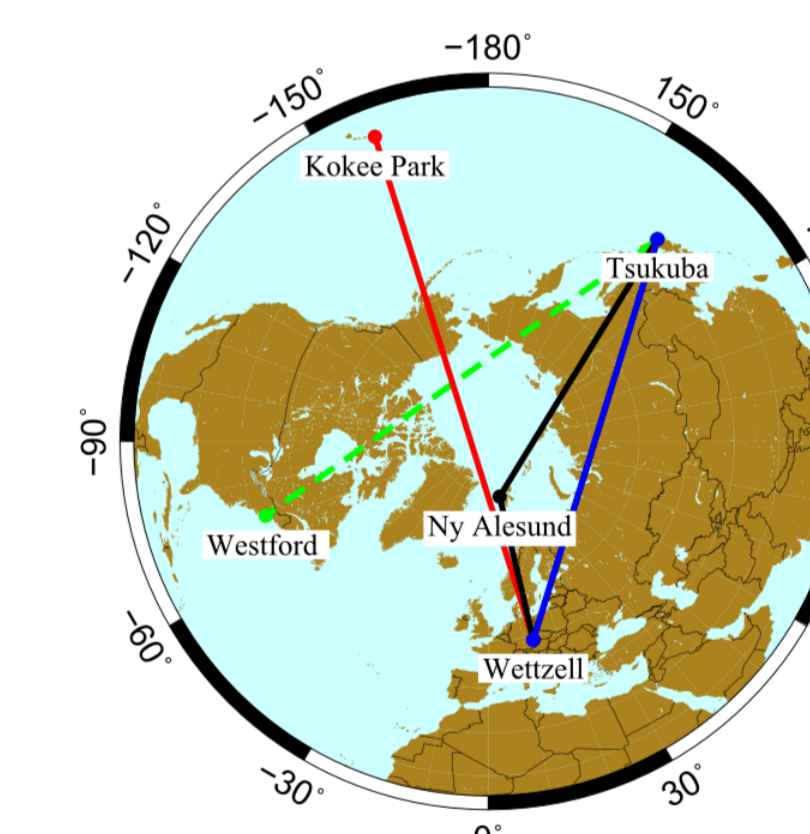


Fig. 10. Baselines of IVS Intensive sessions which are carried out on a regular basis (red, black, blue) and a temporary baseline (green-dashed). Red: Kokee Park – Wettzell (INT1), blue: Tsukuba-Wettzell (INT2), black: Tsukuba-Wettzell-NyAlesund (INT3), Green: Tsukuba-Westford (Westford replaced Wettzell).

Also the Length-of-day validation shows that the accuracy depends on the stations taking part in the observation: Ray-tracing improves observations from INT2 and INT3 baselines but has no benefit for INT1 session. Due to the large number of INT1 sessions, the overall accuracy shows no improvement (column "all" in Table 1).