A Montecarlo routine to simulate scattering in particulate media

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Introduction

Analytic solutions of radiative transfer equations have the advantage to be easy to handle and to give a clear physical picture of the scattering processes at work, however they relies on some assumptions and approximations that are not always satisfied. One of the most used solution in planetary studies is the one given in Hapke (1993). In this model one of the strongest approximations is relative to the multiple scattering process which assumes isotropic scattering in the derivation of the Chandrasekhar functions *H(x)*. In Hapke (2002) an improvement of the multiple scattering treatment is given, including anisotropic single particle phase functions ***p(g)*** expressed as Legendre polynomials, but the solutions depend on infinite series, and more and more terms are needed to describe complex *p(g)*. The necessity to treat **any given formulation of *p(g)*** encourages to develop a Montecarlo approach to the problem of scattering in particulate media. This is very important when the investigated media are represented by aggregates covered by regoliths (as for the case of planetary rings), that have a photometric behavior which is the result of the combination of single and multiple scattering processes and of other mechanisms as the opposition effect occurring in the powder which covers the surface.

Montecarlo routine

Montecarlo raytracing in particulate media has been described in several papers (Shkuratov and Grynko, 2005; Stankevic and Shkuratov 2004). Our aim in developing a Montecarlo raytracing model is to produce a simple, reasonably fast routine to simulate scattering in particulate media for planetary science applications in the **geometric optics limit**.

Our computer model, written in IDL language, includes two stages: the production of the particulate medium and the scattering simulation.

Production of the particulate medium

The medium is represented by tridimensional grid. Each cell of the grid represents a room for one "particle" (or aggregate or block). At this stage we assume that each single particle has spherical shape and same size. The number of positions randomly occupied by the particles is related to the filling factor ***Φ*** of the medium (which is a parameter of our model), and with the thickness of the medium it determines the optical depth ***τ***. Once the particles are placed in the grid (in the center of the cells) their position is moved inside the cell itself preventing overlapping with other particles. This makes the medium to be "quasi-random". However the fact that each cell can contain at most the center of only one particle makes the raytracing procedure fast.

In **fig.1** simulated slabs with different filling factors are shown.

Scattering simulation

Photons are shot from the upper side of the grid with a given direction which is individuated by the incidence angle (it is measured respect to the normal of the surface) and the azimuth angle). Rays enter the grid and are propagated in the same direction until they hit a particle. After the interaction the photon can be absorbed or scattered. **The scattering probability is proportional to the particle single scattering albedo w**. If w=0 the photon is absorbed. **Light is scattered following the single particle phase function *p(g)*. *p(g)*** can assume any given analytic formulation. During the scattering simulation photons that escape the medium from the upper and lower sides are counted as emitted photons while photons escaping from a lateral wall of the grid are re-entered from the opposite wall, simulating an infinite extension of the slab (horizontally stratified medium).

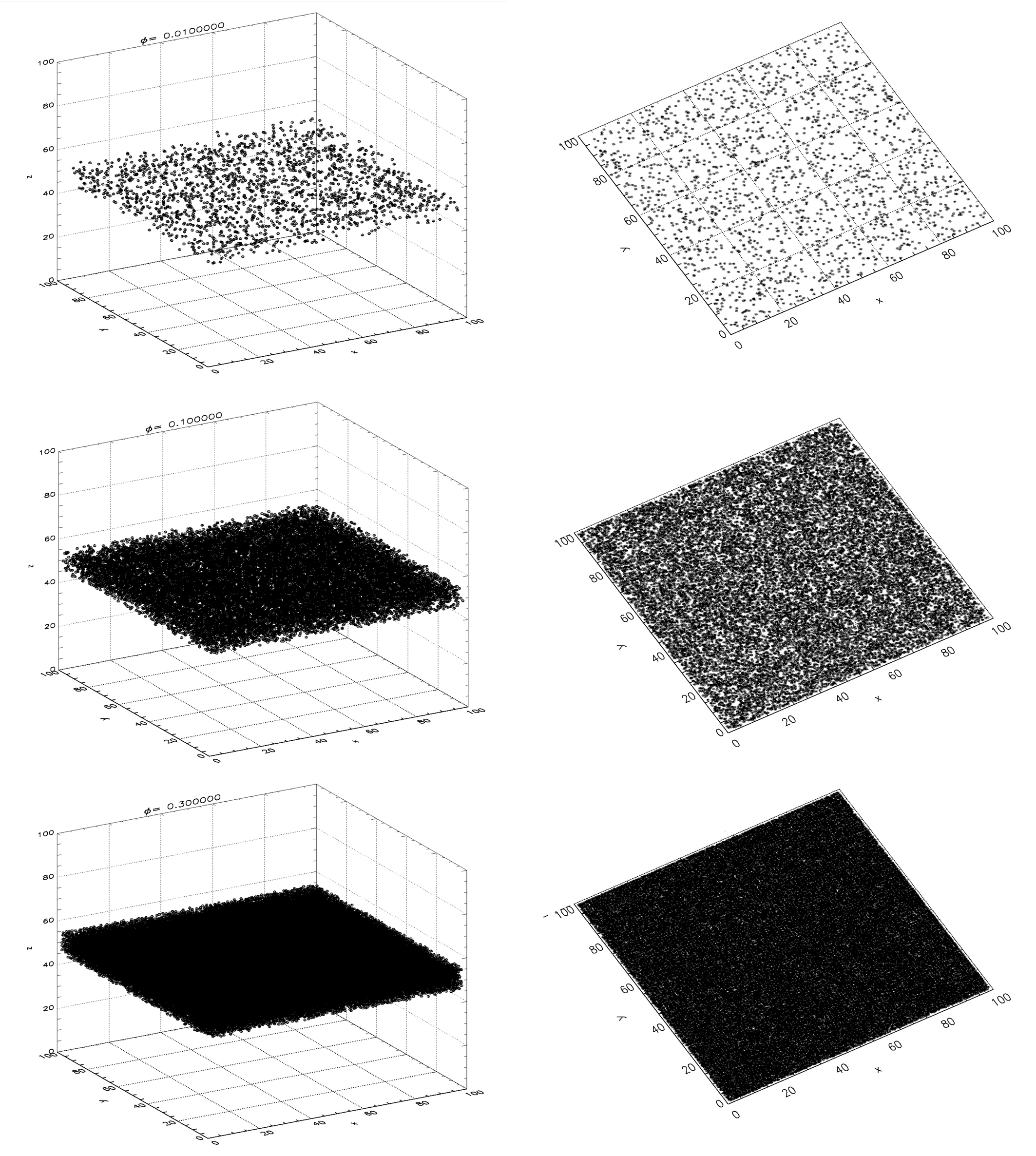

Fig. 1. Portions of simulated slabs with different filling factors (from top to bottom: 0.01, 0.1 and 0.3). On the right the projection on the XY-plane is shown.

Integral quantities

In order to test the correctness of our simulations we have computed two integral quantities which depend on the total amount of scattered photon: the directional-hemispherical reflectance r_{dh} of a semi-infinite medium and the transmission factor of a slab ***T***.

Directional-hemispherical reflectance r_{dh}

The directional-hemispherical reflectance is the ratio between the total power emitted by the medium in the upper hemisphere and the total amount of incident power, when the source is collimated. According to Hapke (2008) this quantity depends on the filling factor ***Φ*** of the medium and can be expressed as:

$$\begin{aligned} r_{dh} &= 1-\gamma H(\mu_0) \\ \mu_0 &= \cos(i) \end{aligned} \qquad \begin{aligned} \gamma &= \sqrt{1-w} \\ H(\mu_0) &= \frac{1+2\mu_0/K}{1+2\gamma\mu_0/K} \end{aligned}$$

where ***i*** is the incidence angle and ***H(μ₀)*** is an approximation of the Chandrasekhar function corrected for the porosity by mean of the term ***K***:

$$K = -\ln(1-1.1209\phi^{2/3})/1.1209\phi^{2/3}$$

Simulations with ***i=0°*** have been performed for various ***w***'s and the derived r_{dh} has been compared with its analytic value (fig.2). The agreement is very good.

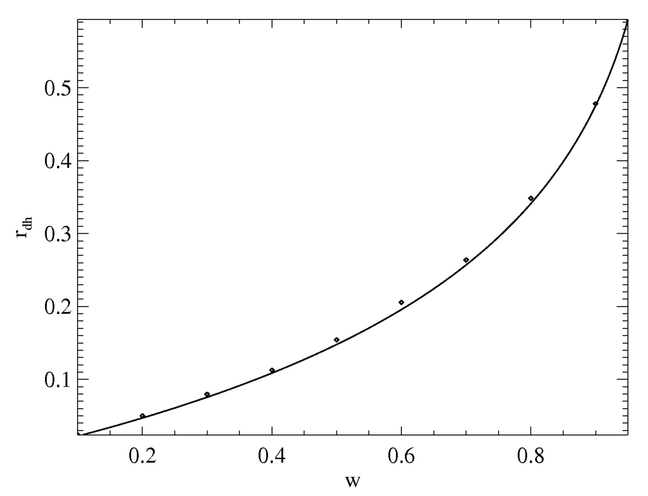

Fig. 2. Simulated directional-hemispherical reflectance (diamonds) and its analytic expression given in Hapke (2008) (solid line) against ***w***.

Transmission factor of a slab *T*

From Hapke (2008) the normal transmission of a slab of thickness ***z*** is:

$$T(z) = Ke^{-KEz} \qquad E = \frac{\phi}{v}\sigma Q_E$$

where ***E*** is the extinction coefficient of the medium, ***v*** the particle volume, ***σ*** the particle cross section and ***Q_E =1*** the extinction efficiency in the geometric optics limit.

We have computed ***T(z)*** from our simulations with ***Φ=0.0001, 0.001, 0.01, 0.1*** for a medium with ***z=100, w=0*** and particle radius ***r=0.5***. Then we have compared these values with the analytic prediction given above (table 1). The agreement is remarkably good.

Φ	T (analytic)	T (simulated)
10^{-4}	0.987	0.986
10^{-3}	0.870	0.868
10^{-2}	0.234	0.230
10^{-1}	7×10^{-8}	~0

Table 1. Comparison between analytic and simulated transmission factors of a slab of absorbing spherical particles for various filling factors.

Effect of porosity

In Hapke (2008) the effect of porosity is discussed and the equations of bidirectional reflectance of a semi-infinite medium from Hapke (1993) have been modified taking into account for a variable filling factor. We then used our Montecarlo routine to test the two equation of bidirectional reflectance given respectively in Hapke (1993) and Hapke (2008). We simulated a semi-infinite medium with ***Φ=0.3, w=0.8, p(g)=1*** and light incident with an angle i=30° respect to the normal at the surface. In fig. 3 the following quantity (for simplicity we refer to that as "reflectance")

$$r(i,e,g)\frac{\mu}{\mu_0} \qquad \begin{aligned} \mu &= \cos(e) \\ \mu_0 &= \cos(i) \end{aligned}$$

is plotted for the simulation and the two Hapke models, against the emission angle ***e*** in the plane containing the incident direction and the normal at the surface.

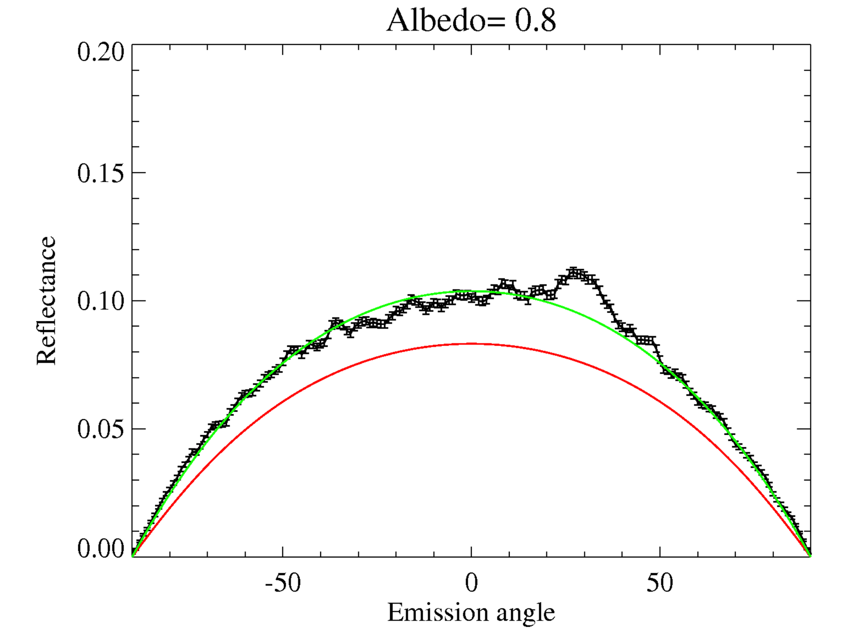

Fig. 3. Simulated reflectance (black line) and reflectance from Hapke (1993) (red line) and Hapke (2008) (green line line) against ***e*** (the minus sign for the emission angles represents directions that are on the opposite semi-plane respect to the one of the incident direction). Error bars represent estimated Poissonian fluctuations.

We find an almost perfect agreement with the Hapke (2008) result (opposition effect is not included but it naturally appears in the simulation). Ripples on the simulation plot are due to the limited number of traced rays. Angular resolution is 2°.

Planetary rings photometry

The commonly used bidirectional reflectance (BDR) equation of planetary rings often relies on the approximation of single scattering (Cuzzi and Estrada, 1998), which neglects multiple scattering between the blocks (aggregates) which form the ring and corrects only for the optical depth. The typical expression of BDR is:

$$r(i,e,g) = \frac{\mu_0}{\mu_0+\mu}\frac{w}{4\pi}p(g)(1-e^{-\tau\frac{\mu+\mu_0}{\mu\mu_0}})$$

where ***τ*** is the optical depth.

We ran simulations with our code to test this hipotesys. In particular we find that single scattering approximation is valid only for low optical depth (filling factor) and low ***w*** (fig. 4).

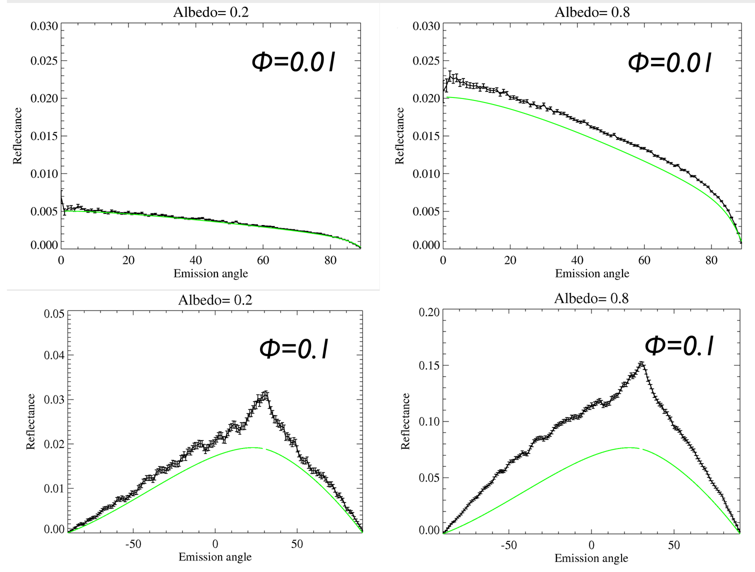

Fig 4. Reflectance against emission angle. On the top *i*=0°, on the bottom i=30°. Rings thickness is 10 m and particle radius is 0.5 m. Filling factor and albedo are indicated in the plot. Single particle phase function *p(g)* is a Lomell-Seeliger. The black line represents the results of the simulations and the green line is from the equation above (opposition effect is not modeled).

Comparison with Saturn's rings data from the VIMS spectrometer onboard Cassini

In *EGU#7039 poster titled "Multi-wavelength studies of Saturn's rings to constrain ring particle properties and ring structure: the VIMS perspective, by Filacchione et al.* Saturn's rings spectrogram from VIMS data have been shown and their spectral properties investigated. One important spectral quantity that has been computed is the 2 μm band depth (BD) of water ice, which is diagnostic for the grain size of the regolith covering the icy chunks which form the rings. In fig.5 (left) the BD is plotted against the phase angle for different regions of the rings, showing a positive trend. We produced simulations for the B ring (R=94300 km, τ=1.7) at the same geometric conditions of the VIMS observations, confirming the same trend for the BD (fig.5, right), which is induced by a possible dependence of the multiple scattering on phase angle.

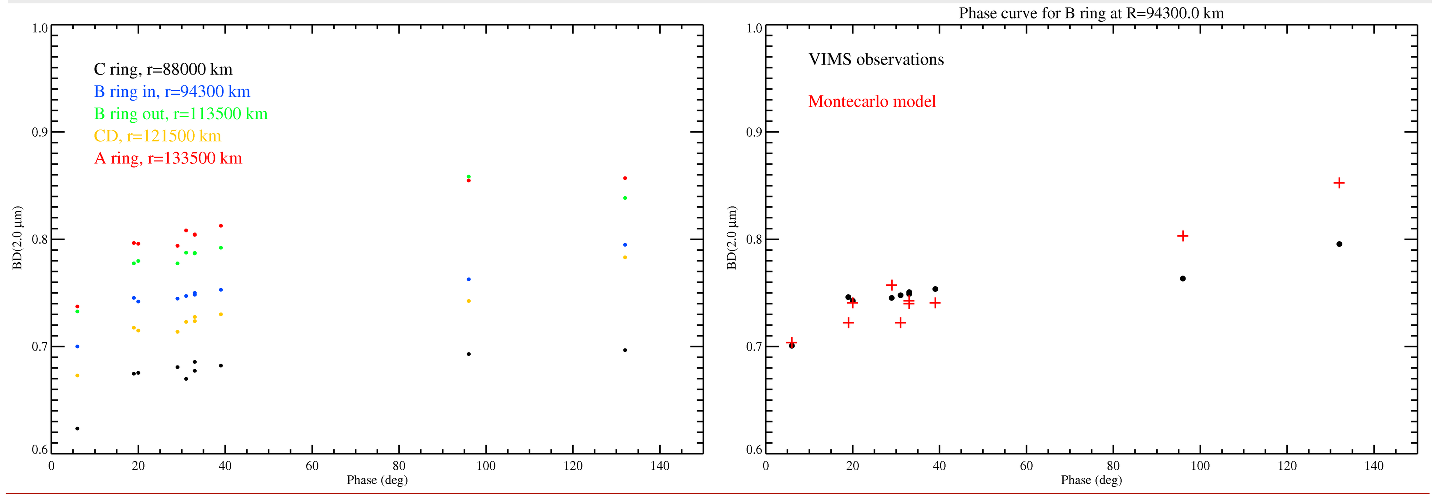

Fig 5. Left: water ice BD at 2 μm for the main rings of Saturn. Right: comparison of the B ring BD and prediction from the Montecarlo simulations.

Future work

The Montecarlo routine we have developed correctly reproduces multiple scattering between particles with known ***p(g)*** (isotropic or Lommel-Seeliger) and ***w***. The next step of our work is to simulate photometric properties of real planetary rings, whose particles are coated by regolith and must be characterized by a complex phase function which accounts for the scattering processes that take place in the regolith itself (multiple scattering, opposition effect, ...).

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