Applied Marine Physics
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SURFACE WAVES IN AIR-SEA INTERACTIONS

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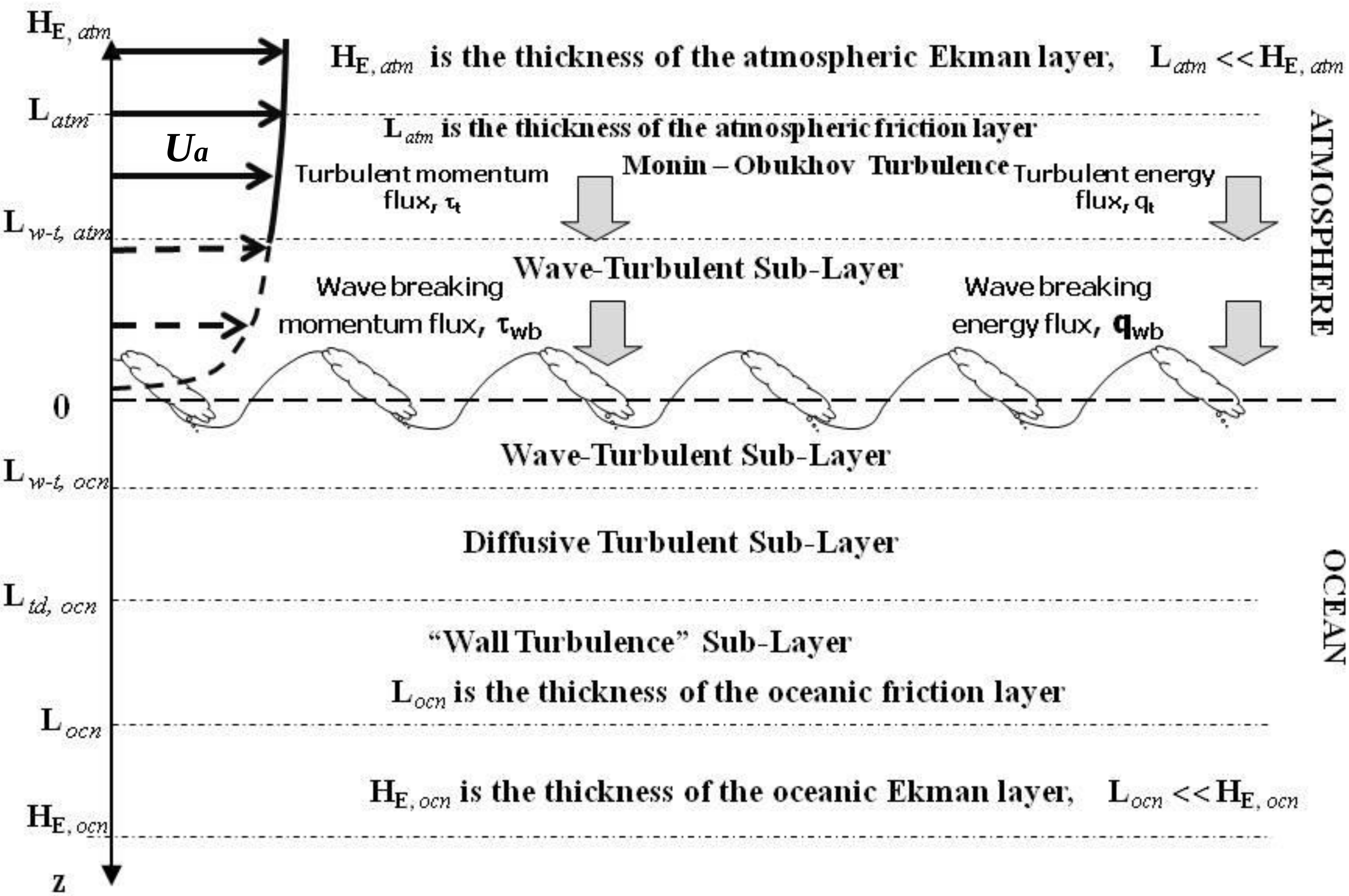


Diagram of Air – Sea Interaction

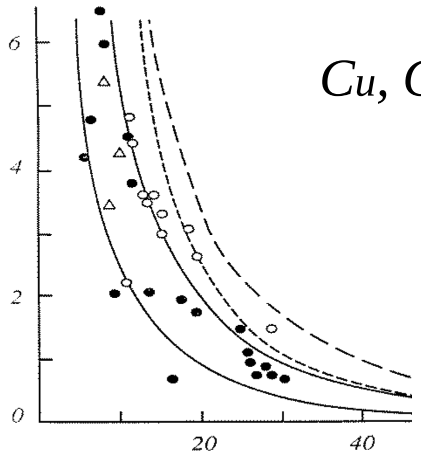
Monin – Obukhov Turbulent Layer
Turbulent Fluxes

C_0 - *wave phase velocity*

$Ua(z=10m)=U_{10}$ σ_η - *wave elevation RMS*

$\varsigma = C_0 / u_*$ - *wave age*

$\tau_{1,3} / \rho_{a0} = u_*^2 = C_u U_{10}^2$ $Q_T / \rho_{a0} c_{pa} = -C_T U_{10}(T_{a,10} - T_{ws})$ $p_{a0} Q_E / \rho_{a0} = -C_E U_{10}(E_{10} - E_{ws})$



C_u, C_T, C_E – coefficients of drag, heat and evaporation

$$0.5 < (C_u, C_T, C_E) \times 10^3 < 10$$

$$C_u(\varsigma) \approx 0.5[1 + 1.3 \times \tanh(\varsigma / \varsigma_*)]\varsigma^{-2}$$

$0 < \varsigma \le \varsigma_* \approx 32.87$ - *developed waves*

$$C_T(\varsigma) \approx [1 - 17\sqrt{C_u(\varsigma)}]C_u(\varsigma)$$

$$C_T \approx C_E$$

$$10 < \varsigma \le \varsigma_*$$

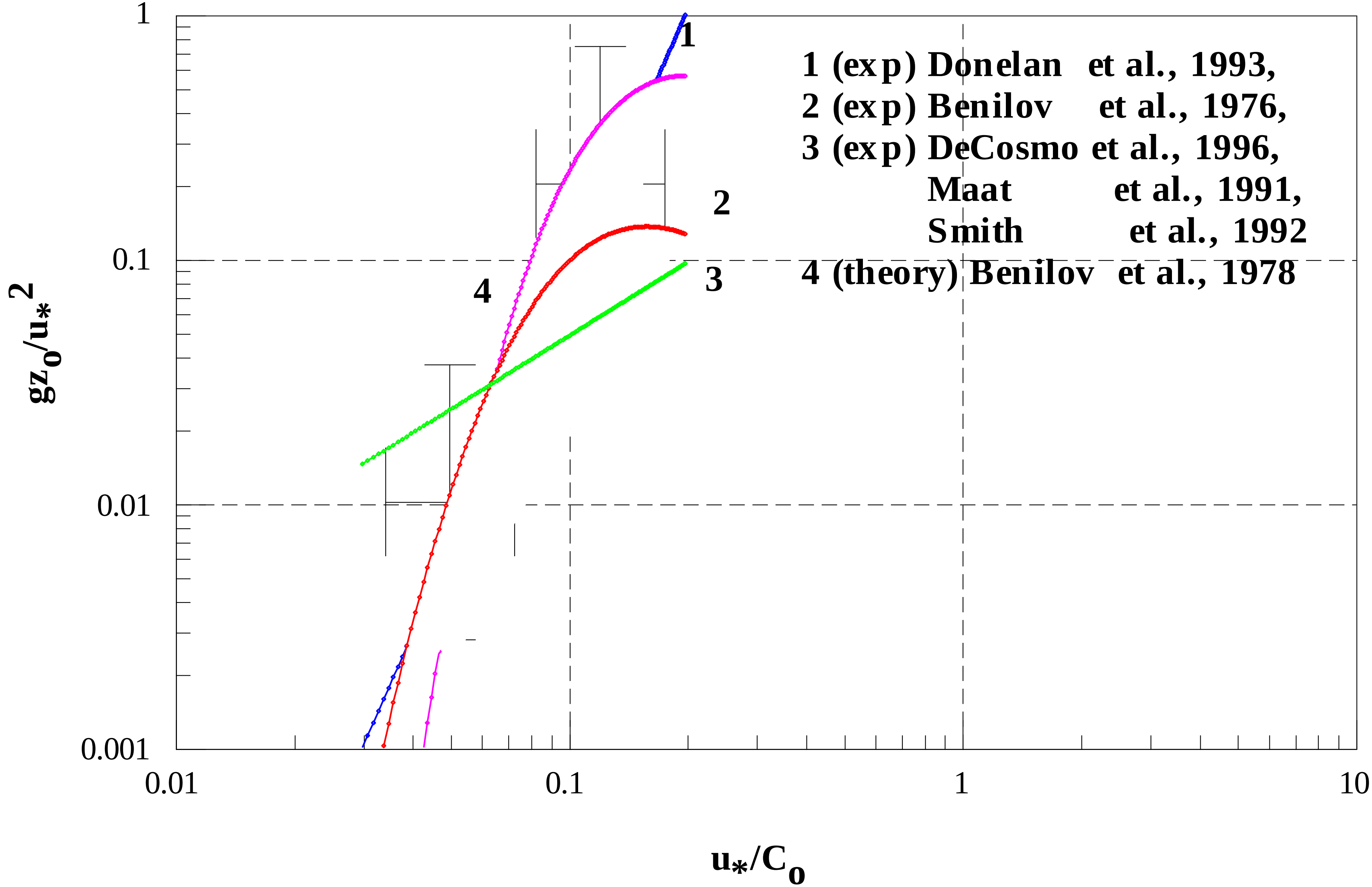
$$C_u(\varsigma_*) \approx 0.92 \times 10^{-3}$$

$$\frac{C_0(\varsigma_*)}{U_{10}} \approx 1$$

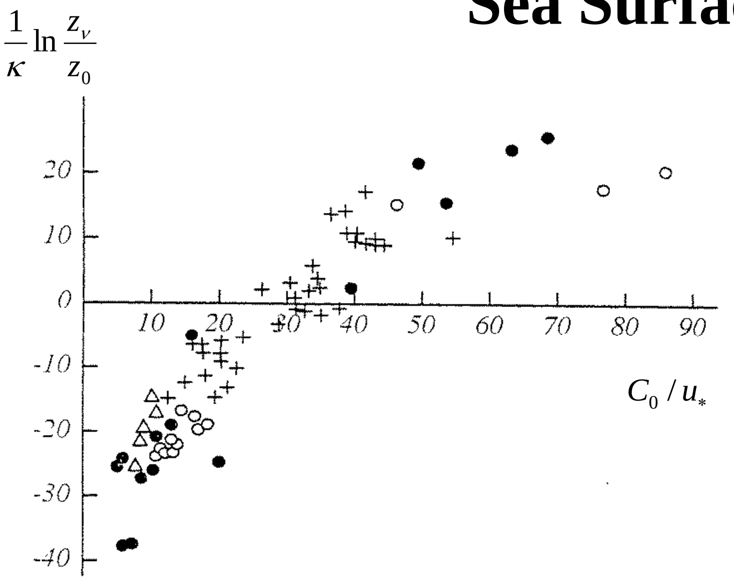
$$\frac{g\sigma_\eta}{U_{10}^2} \approx 5 * 10^{-2}$$

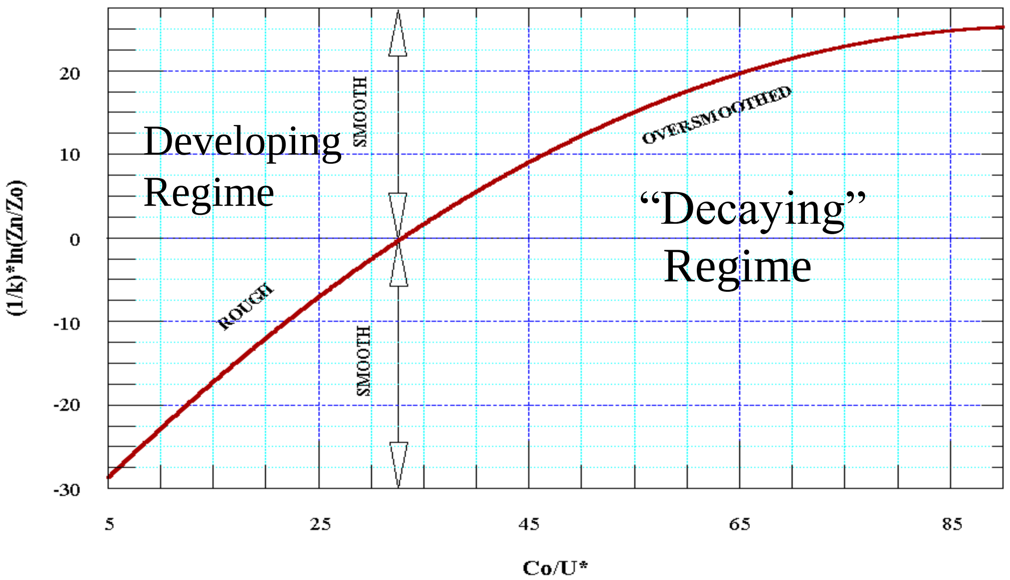
$$C_T(\varsigma_*) \approx 0.45 \times 10^{-3}$$

Sea Surface Roughness - Charnock Constant



Sea Surface Aerodynamic Features



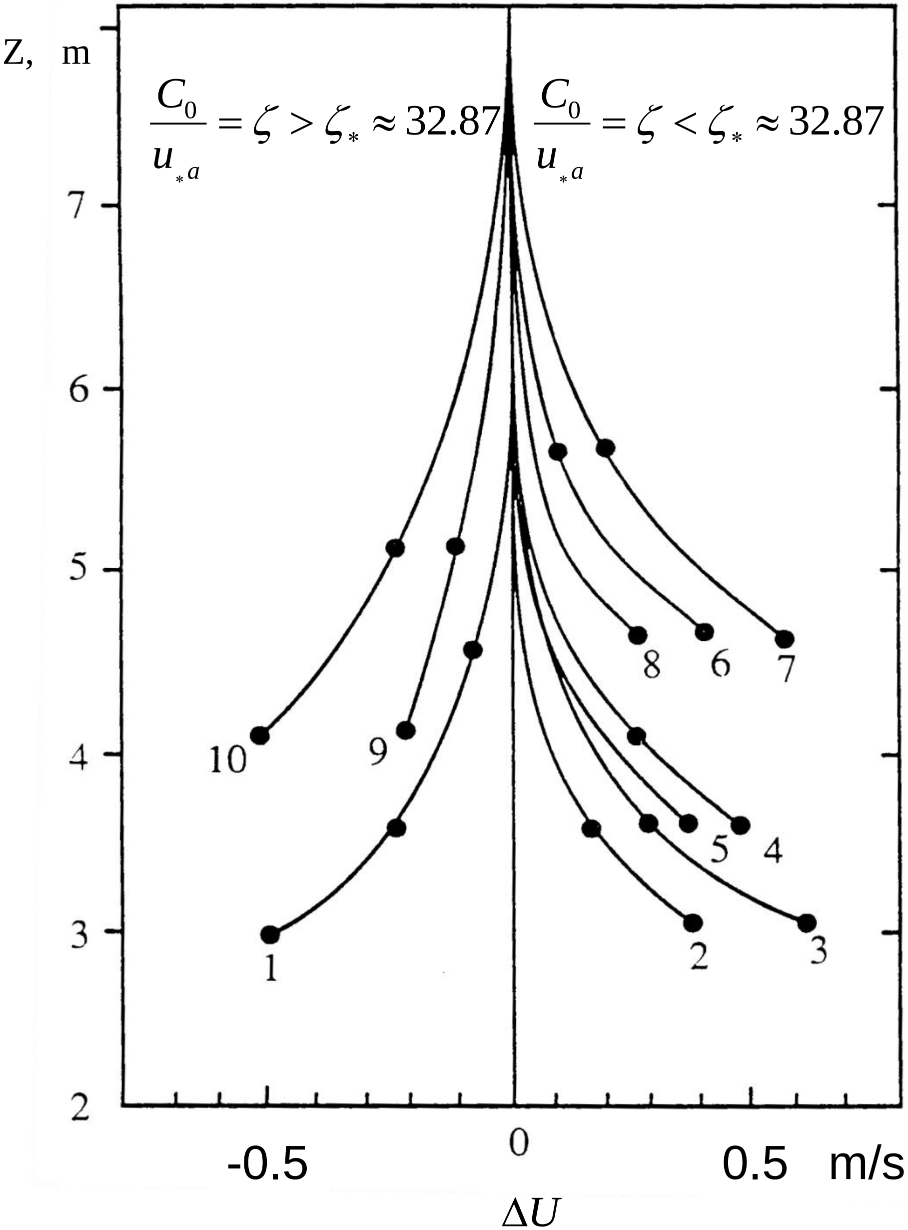


$$\overline{U}_a = \frac{u_{*a}}{\kappa}\ln\frac{z}{z_0} \qquad z_{\nu a} = \frac{\nu_a}{9u_{*a}} \qquad \overline{U}_{as} = \frac{u_{*a}}{\kappa}\ln\frac{z}{z_\nu} \qquad \overline{U}_a = \frac{u_{*as}}{\kappa}\ln\frac{z}{z_\nu} \qquad \frac{C_0}{u_{*a}} = \zeta$$

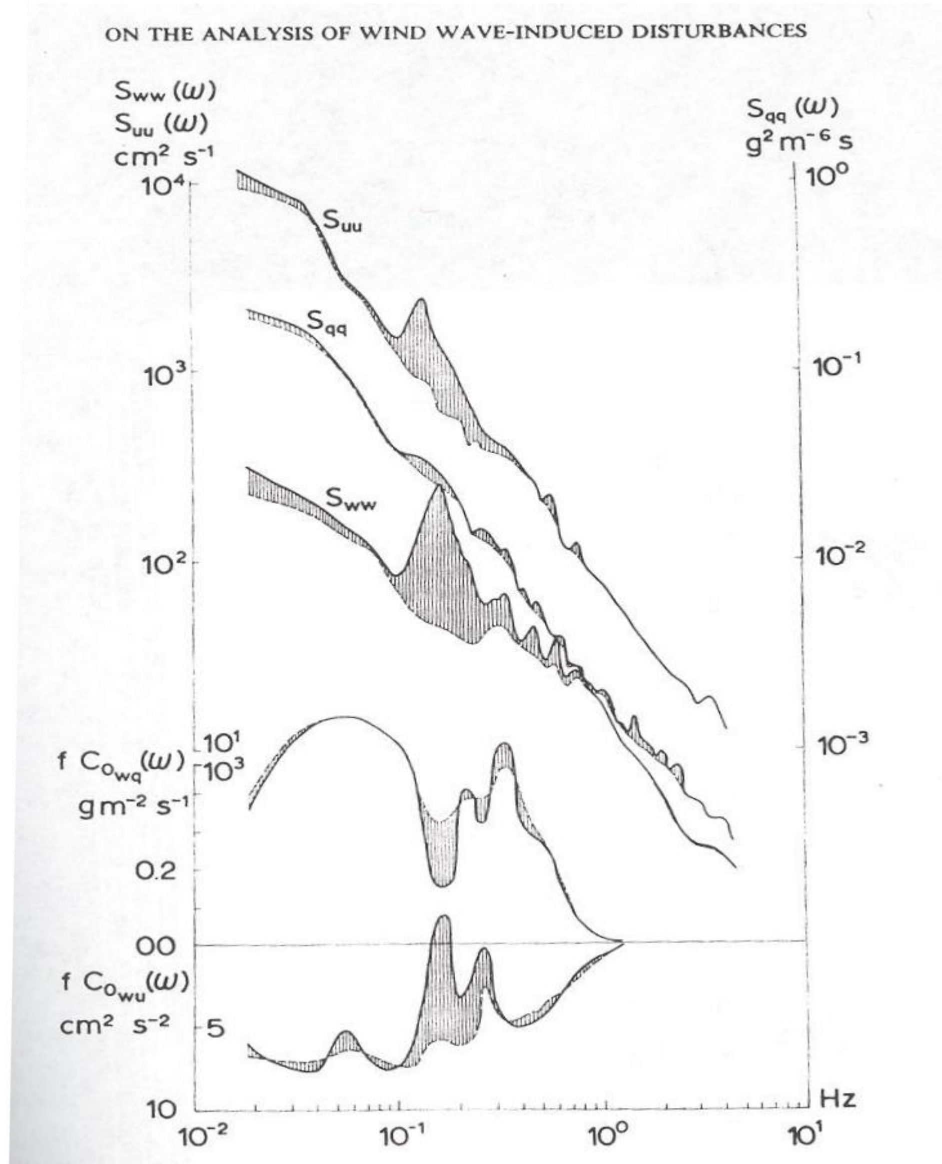
$$5 \le \zeta \le 90, \qquad \frac{1}{\kappa}\ln\frac{z_\nu}{z_0} = -35 + 1.29 * \zeta - 6.9 * 10^{-3} * \zeta^2, \qquad \frac{z_0}{z_\nu} = 1, \qquad \zeta = \zeta_* \approx 32.87$$

$$5 \le \varsigma \le \varsigma_* \qquad m = \frac{gz_0}{u_*^2} \approx 355 \times \varsigma^{-3.64} \qquad 10^{-3} < m < 1$$

Atmospheric Wave – Turbulent Sub-Layer



Wind Velocity Deviation from the Log- Profile



Wave Manifestation in the Turbulent Spectra

Momentum and Energy of Wave Motion

The wave momentum $\boldsymbol{M}_w = \rho_w \iint \boldsymbol{C}\Phi(\mathbf{k}) k\, dk_1 dk_2 \approx \rho_{\boldsymbol{w}} \frac{\beta}{3g} \boldsymbol{C_0} C_0^2$

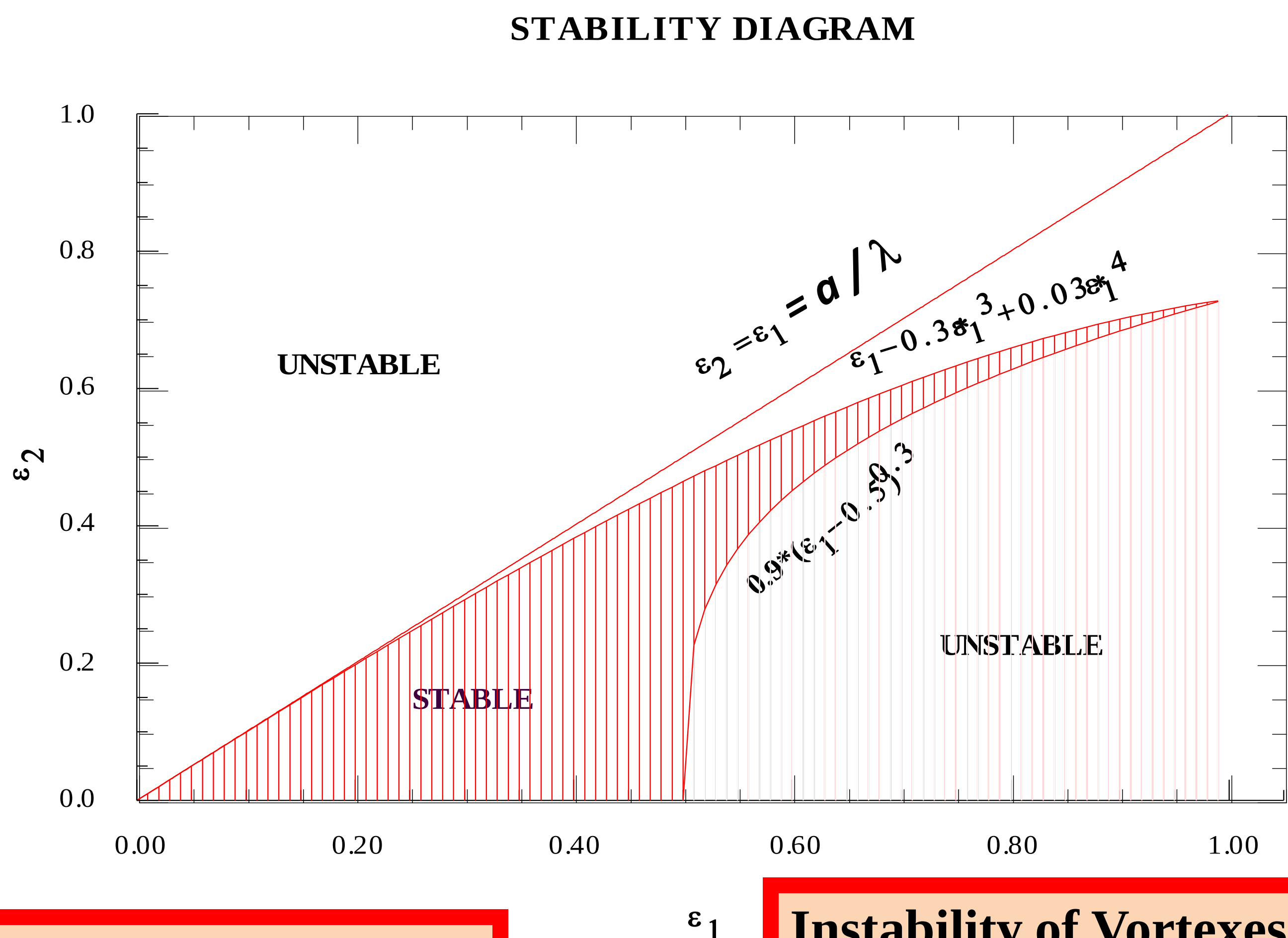
The wave energy $E_w = \rho_w g \iint \Phi(\boldsymbol{k}) dk_1 dk_2 \approx \frac{3}{4} \boldsymbol{C}_0 \cdot \boldsymbol{M}_w$

The budget of wave momentum and energy with the wave breaking
(HUWF - horizontally uniform wave field)

$$\partial_t M_w + \tau_{wb} = \tau_w \qquad \partial_t E_w + q_{wb} = q_w$$

$$\frac{\tau_w}{\rho_a u_{*a}^2} \le 1 \qquad \tau_{\mathrm{wb}} \le 10^{-4} \rho_w \beta C_0^2 \qquad q_{wb} \le 10^{-4} \rho_w \beta C_0^3$$

Oceanic Wave –Turbulent Sub-Layer

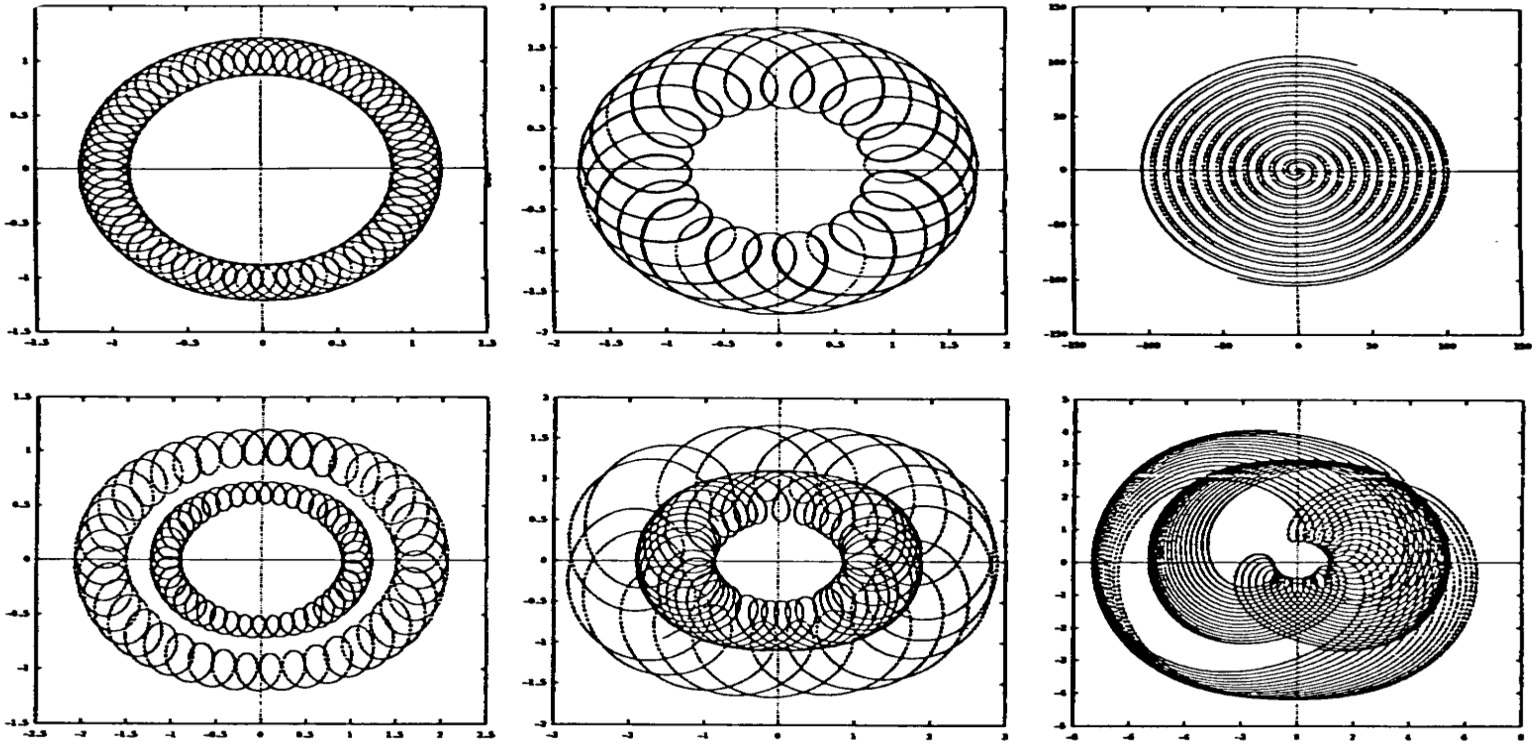


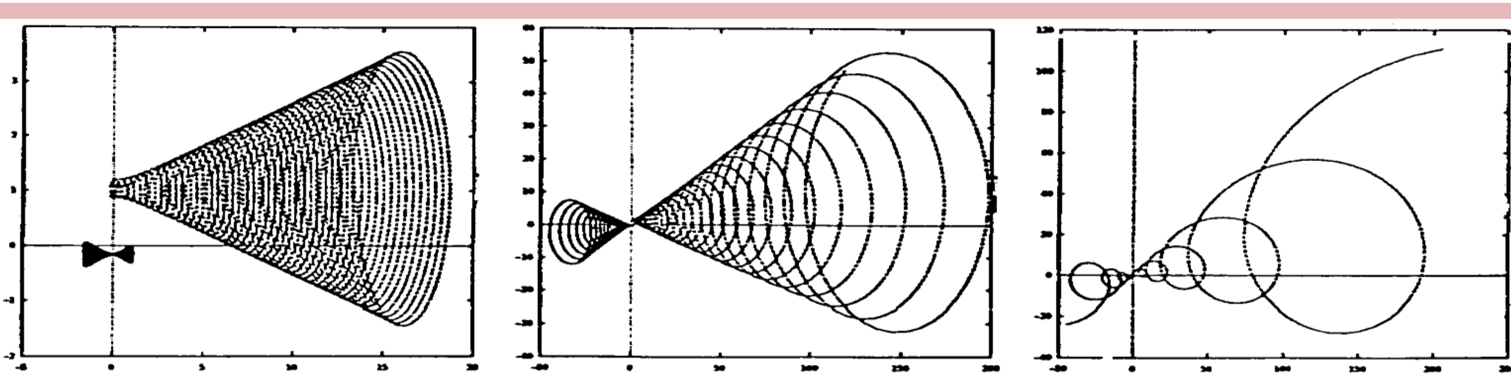
The Turbulence Generated by the Potential Surface Waves

Instability of Vortexes in the Field of the Potential Surface Waves

$$\varepsilon_2 = \varepsilon_1 = a/\lambda$$

The examples of phase portraits on the (ω1, ω2)-plane





$$\varepsilon_2 = \varepsilon_1 = a/\lambda$$

Turbulence of the Wave - Turbulent Sub-Layer

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, $|\partial_z k_T| << \sigma |\partial_z k_v|$ $\quad \nu_T (\partial_z \overline{U})^2 << \varepsilon$ $\quad 0 \le z \le L_{w-v}$

$$k_v = \frac{\beta_P C_0^2}{2}\left(1+\sqrt{\frac{z}{L_{*v}}}\right)\exp\left(-\sqrt{\frac{z}{L_{*v}}}\right)$$

$$L_{*v} = C_0^2 / 24g = \sigma_\eta / 12\sqrt{\beta_P}$$

$$k_T(0) \approx 1.3(\gamma_1 \beta_P)^{2/3} c_0^2 = 2.7\times 10^{-4} c_0^2$$

$$k_T = k_T(0)\frac{3\left(1+\sqrt{\frac{z}{L_{*v}}}\right)\exp\left(-\sqrt{\frac{z}{L_{*v}}}\right)}{2+\left(1+\sqrt{\frac{z}{L_{*v}}}\right)^3\exp\left(-3\sqrt{\frac{z}{L_{*v}}}\right)}$$

$$\varepsilon(0) \approx 4\gamma_1 \beta g c_0 = 1.2\times 10^{-5} g c_0$$

$$\ell(0) = \sigma_\zeta / 15\sqrt{\beta_P} = c_0^2 / 30g$$

$$\ell = \ell(0)(1+\sqrt{z/L_{*v}})$$

$$\varepsilon = \varepsilon(0)\frac{\sqrt{27\left(1+\sqrt{\frac{z}{L_{*v}}}\right)\exp\left(-\frac{3}{2}\sqrt{\frac{z}{L_{*v}}}\right)}}{\left[2+\left(1+\sqrt{\frac{z}{L_{*v}}}\right)^3\exp\left(-3\sqrt{\frac{z}{L_{*v}}}\right)\right]^{\frac{3}{2}}}$$

$$\nu_T = \nu_T(0)\frac{\sqrt{3}\left(1+\sqrt{\frac{z}{L_{*v}}}\right)^{\frac{3}{2}}\exp\left(-\frac{1}{2}\sqrt{\frac{z}{L_{*v}}}\right)}{\sqrt{2+\left(1+\sqrt{\frac{z}{L_{*v}}}\right)^3\exp\left(-3\sqrt{\frac{z}{L_{*v}}}\right)}}$$

$$\nu_T(0) = 0.038(\gamma_1 \beta_P)^{1/3}\frac{c_0^3}{g}$$

Diffusive Turbulent Sublayer, k-ε turbulence

$$L_{w-v} \le z \le L_w \qquad \left|\partial_z k_T\right| << \sigma \left|\partial_z k_v\right| \qquad \nu_T \left(\partial_z \overline{U}\right)^2 << \varepsilon \qquad 0 \le z_1 \le L_w - L_{w-v}$$

$$k_{T1} = \frac{k_{T1}(0)}{\left(1 + z_1 / L_{*1}\right)^{n_k}} \qquad n_k = \frac{4}{\mu - 7}, \quad n_\varepsilon = \frac{\mu - 1}{\mu - 7}, \quad n_{\nu T} = \frac{9 - \mu}{\mu - 7}$$

$$\varepsilon_1 = \frac{\varepsilon_1(0)}{\left(1 + z_1 / L_{*1}\right)^{n_\varepsilon}} \qquad 7 < \mu = \sqrt{1 + 24 c_2 \,\mathrm{Pr}_\varepsilon / \mathrm{Pr}_k} \le 9 \qquad 2 < c_2 \,\mathrm{Pr}_\varepsilon / \mathrm{Pr}_k \le \frac{10}{3}$$

$$\nu_{T1} = \frac{\nu_{T1}(0)}{\left(1 + z_1 / L_{*1}\right)^{n_{\nu T}}}$$

$$\ell = \ell(0)\left(1 + \frac{z_1}{L_{*1}}\right)$$

$$\nu_{T1}(z_1, \mu = 9) = \nu_{T1}(0)$$

$$n_k = 2, \quad n_\varepsilon = 4 \qquad k_{T1} \sim z^{-2}$$

$$c_2 \,\mathrm{Pr}_\varepsilon / \mathrm{Pr}_k = \frac{10}{3} \qquad \varepsilon_1 \sim z^{-4}$$

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"Wall Turbulence" Sublayer, k-ε turbulence

$$L_w \le z \le L \quad \left|\Pr_k^{-1} d_{z_2} \nu_{T2} d_{z_2} (k_{T2} + \sigma k_\nu)\right| << \nu_{T2} \left(d_{z_2} \overline{U}\right)^2 \quad 0 \le z_2 \le L - L_w$$

$$\overline{U}(L_w) - \overline{U}(z_2) = \frac{u_{*w}}{\kappa} ln\left(\frac{z_2 + L_{*2}}{L_{*2}}\right)$$

$$k_{T2} = \frac{u_{*w}^2}{\sqrt{c_\nu}} = const$$

$$\varepsilon = \frac{\varepsilon_1(L_w) L_{*2}}{z_2 + L_{*2}}$$

$$\nu_{T2} = \kappa(L_{*2} + z_2) u_{*w}$$

$$\ell = \kappa \sqrt[4]{c_\nu}\,(L_{*2} + z_2)$$

$$c_2 \Pr_\varepsilon / \Pr_k = \frac{10}{3}$$

$$\Pr_k = \frac{3 c_2 \kappa^2}{10\sqrt{c_\nu}\,(c_2 - c_1)} \approx \frac{16}{25}$$

$$\Pr_\varepsilon = \frac{\kappa^2}{\sqrt{c_\nu}\,(c_2 - c_1)} \approx \frac{10}{9}$$

Consistent with $\varepsilon \sim z^{-4}$ of the diffusive sublayer

CONCLUSION

* The turbulent boundary layers around the air-sea interface and the interface itself are the unified dynamical system

* The final results provide the complete set of turbulent characteristics important in the air-sea interactions.

* The turbulent characteristics are well consistent with observations

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THANK YOU