Changing basal conditions during the speed-up of Jakobshavn Isbræ, Greenland

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Motivation / Background

The Greenland ice sheet is receding more rapidly than previously predicted by ice sheet models. Outlet glaciers such as the Jakobshavn Isbræ have lost their floating tongues within years and a doubling in ice speed has been observed. It is inherently difficult to directly measure the conditions at the base of the glacier, which contribute to these rapid changes. Here, basal conditions for different years before and after the break-up of the tongue are inferred from surface velocity measurements to investigate the changes and to compare them with parameterizations of basal conditions commonly used in ice-sheet models.

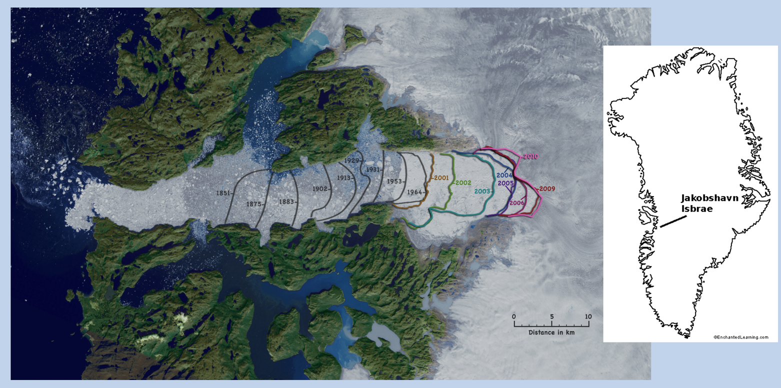


Figure : Jakobshavn Isbræ's grounding line retreat (http://earthobservatory.nasa.gov)

Methods

The Shallow Shelf Approximation with isothermal ice is used as a forward model, where input fields are: ice thickness H, surface elevation z_s, ice softness A, and a basal shear stress τ_b. The model output is surface velocity $\mathbf{u}$. The basal shear stress τ_b is parametrized through a power law:

$$\tau_b = \tau_c \frac{|\mathbf{u}|^{q-1}}{u_{\text{threshold}}^q} \mathbf{u},$$

where $\mathbf{u}$ is the basal sliding velocity. The purely plastic case is achieved by setting $q = 0$, whereas $q = 1$ leads to the common treatment of basal till as a linearly viscous material. We invert for τ_c, which has units of stress and is the basal yield stress if $q = 0$. Despite setting $q = 0.25$ for this study, we call τ_c the basal yield stress.
A Tikhonov cost functional with an added regularization term is used:

$$I(\tau_c, \alpha) = \alpha \mathbf{M}^2 + \mathbf{N}^2,$$

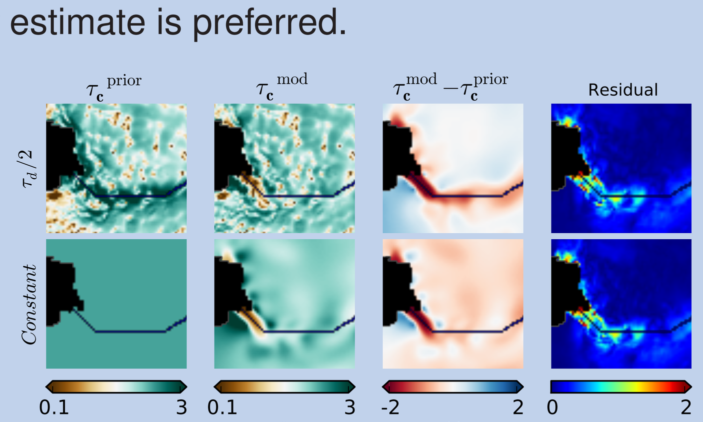
$$\mathbf{M}^2 = \frac{1}{\Omega} \int_\Omega ||\mathbf{u}(\tau_c) - \mathbf{u}^{\text{obs}}||^2 d\Omega$$

$$\mathbf{N}^2 = \frac{1}{\Omega} \int_\Omega c_{L^2}(\tau_c - \tau_c^{\text{prior}})^2 + K^2 c_{H^1} |\nabla(\tau_c - \tau_c^{\text{prior}})|^2 d\Omega$$

where $\mathbf{M}$ is the data-model misfit, $\mathbf{N}$ is the model norm (regularization term) and α is the regularization parameter. The model norm is composed of two parts: the familiar Euclidian L^2 norm and a Sobolov H^1 norm that measures the function's roughness. The factors c_{L^2} and c_{H^1} determine the relative weights of these two norms. K defines a typical length scale to rescale the H^1 norm, Ω as the misfit area.

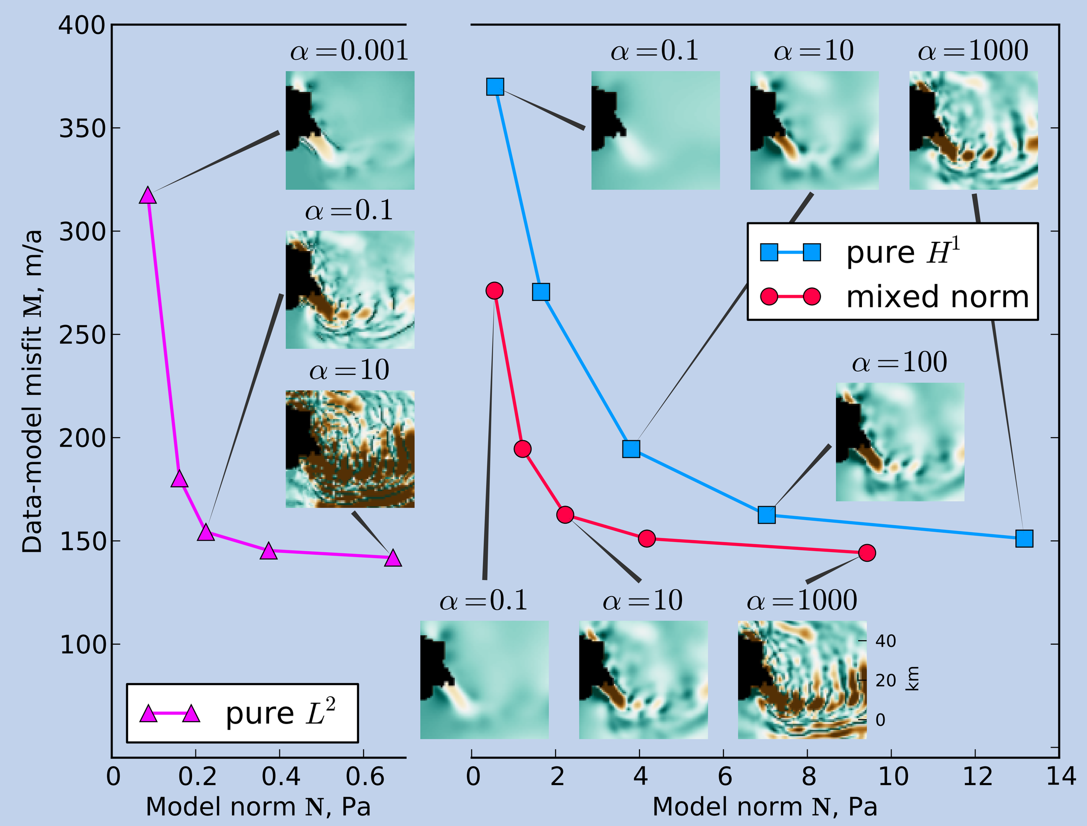
Parameter choices

A prior estimate of basal yield stress is necessary as a starting point for the minimization and for the model norm term. A prior estimate commonly used in glaciology is the driving stress field divided by two. However, small scale features remain unchanged because they do not affect the velocity field sufficiently. Therefore, a constant (or very smooth) prior estimate is preferred.



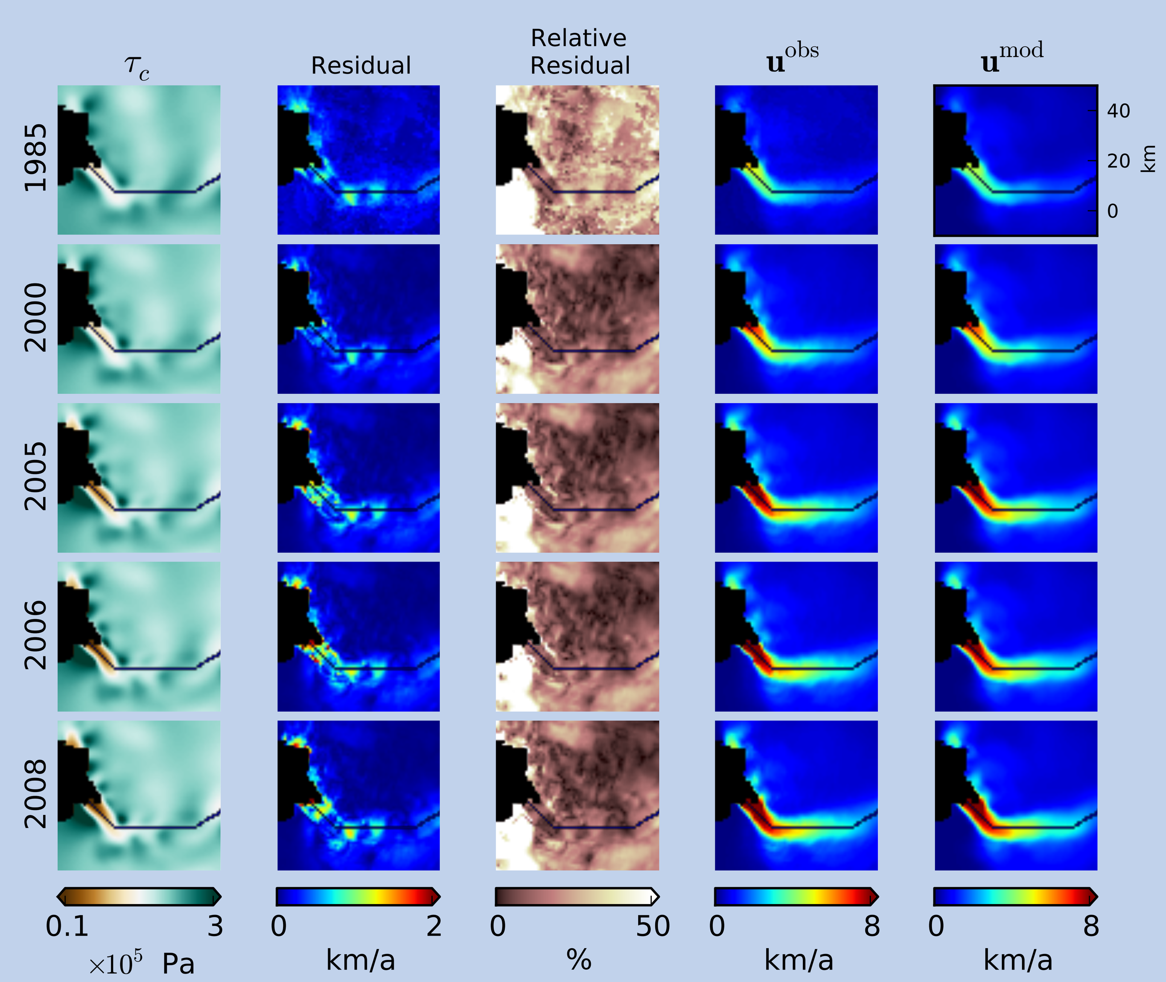
Parameter choices (cont.)

The value of constant ice softness was chosen as $A = 2.5 \times 10^{-24} \text{Pa}^{-3}\text{s}^{-1}$ because this value gives a low data-model misfit for all years. The H^1 model norm biases towards smooth solutions, below we show basal yield stress solutions for different model norms and different regularization parameters α. The 'L-curve' criterion was used to choose $\alpha = 10$ as the regularization parameter for the H^1 model norm, this should prevent over- or underfitting of the data.



Results

All inversions reproduce the overall pattern of observed surface velocities, which shows that, in general, our data and model choices are capable of reproducing the observations by only adjusting basal yield stress. In the lower 5 km of the glacier a clear trend from higher to lower basal yield stress values is visible.

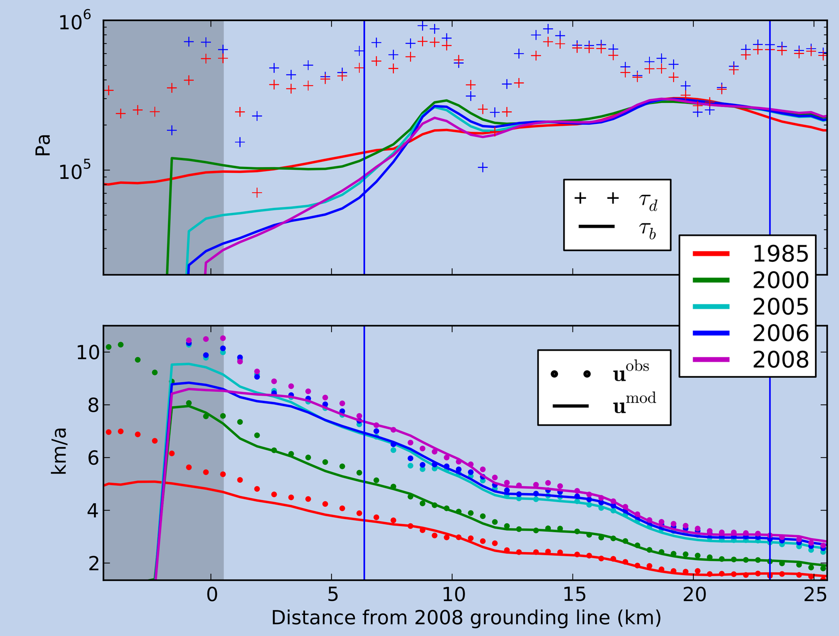


Acknowledgement/Data Sources

We acknowledge the use of data and/or data products from CReSIS generated with support from NSF grant ANT-0424589 and NASA grant NNX10AT68G. As well as Joughin, I., B. Smith, I. Howat, and T. Scambos. 2010. MEaSUREs Greenland Ice Velocity Map from InSAR Data. Boulder, Colorado, USA: NSIDC

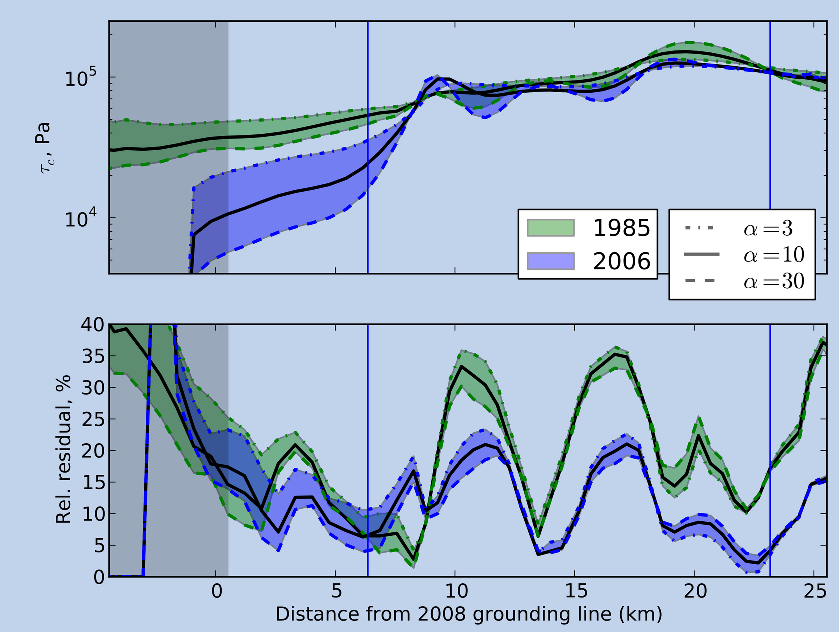
Results (cont.)

To compare the results for the different years in more detail the basal shear stress, τ_b, is shown and compared to the driving stress along the centerline depicted in the previous figure. As seen in the spatial distribution of basal yield stress, the values in the first 5 km are clearly lowered compared to higher upstream, and they generally decrease over time.

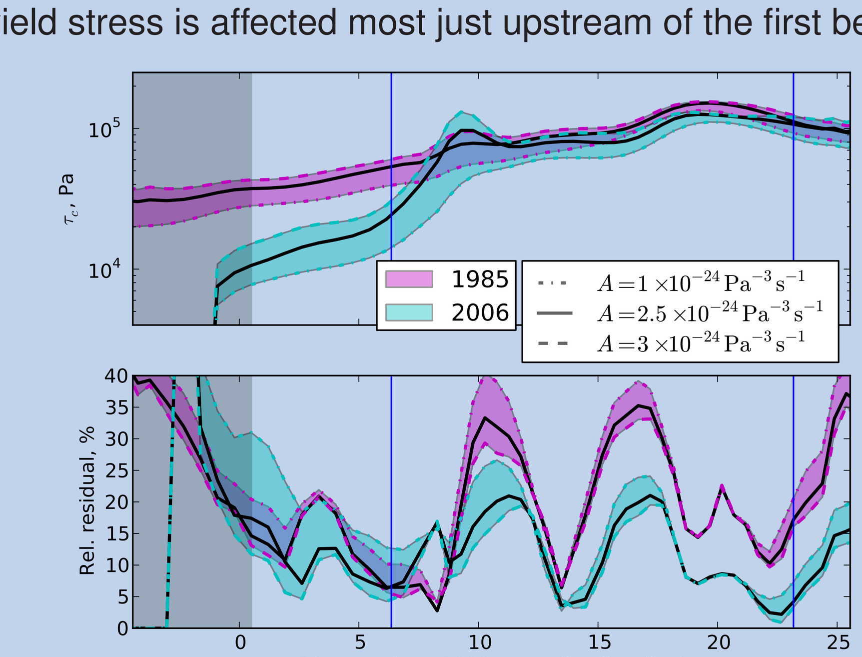


Robustness of results

The solution to our inverse problem is not unique, many of the parameters are not well constrained and a range of parameter choices would be equally acceptable. To evaluate the robustness of our results a range of parameters is explored for the years 1985 and 2006. Robustness of basal yield stress for regularization parameter values is shown below. The choice of regularization parameter mostly affects the first 5 km where a smaller data-model misfit in velocities is expensive (in the model norm sense) because the narrow trough makes abrupt changes in τ_c necessary.



The solutions for softer ice lead to generally higher τ_c values, because of the more localized stress balance. The 2006 basal yield stress solution exhibits a higher sensitivity to changes in ice softness and the basal yield stress is affected most just upstream of the first bend.

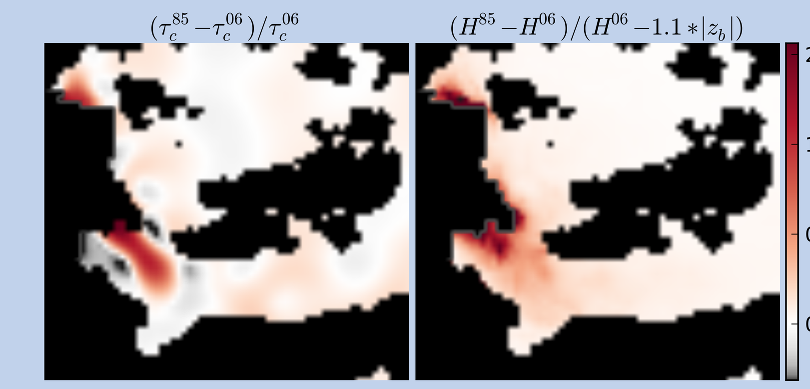


Discussion

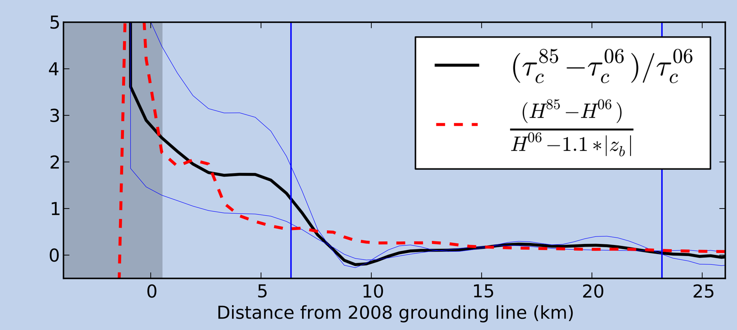
A common way to parameterize the basal yield stress in time dependent ice-sheet model runs is through a Mohr-Coulomb model:

$$\tau_c = \tan(\phi)\,(\rho g H - p_w),$$

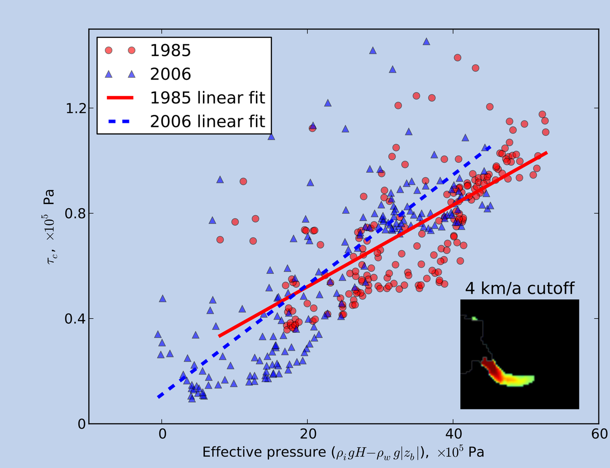
where $(\rho g H - p_w)$ is the effective pressure, p_w is the pore water pressure, and ϕ is a 'till friction angle'. The inferred relative change in τ_c is much more localized to the trough than the predicted relative change in height above floatation. But the broad pattern is similar, confirming that the relative change in height above floatation accounts for most of the relative changes in τ_c.



The relative changes in inferred τ_c are shown along the centerline for different regularization parameters; $\alpha = 3$ (lower envelope) and $\alpha = 30$ (upper envelope). The relative change in height above floatation has a different qualitative shape, but falls within the envelope of regularization parameters. The choice of regularization parameter gives rise to large uncertainties, especially in the terminus area.



To investigate if using a constant-in-time value for the till friction angle ϕ is reasonable, we plot the inferred value of τ_c against the predicted effective pressure for each grid point. In areas with a constant till friction angle we would expect a linear relationship with a slope of $\tan(\phi)$. The overall thinning from 1985 to 2006 should lead to a decrease in effective pressure and a simultaneous decrease in τ_c. When we limit the analyzed points to the areas of fast flow, a linear relationship emerges. The slope of this linear fit indicates that $\tan(\phi) \approx 0.02$ and thus $\phi \approx 2°$, which is a very low value of till friction angle compared to the measured values between 19° and 26°.



Summary

- Implemented Tikhonov inversion methods for basal yield stress in the Parallel Ice Sheet Model (PISM).
- Justified choices of ice softness and model norm with data-model misfit metric and chose the regularization parameter with the L-curve method.
- Changes close to terminus are a robust feature when testing different parameter choices.
- The observed changes are in broad agreement with a Mohr-Coulomb parameterization of basal yield stress, although the derived value of till friction angle is not physically realistic.

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