

Lake eutrophication and environmental change: A viability framework for resilience, vulnerability and adaptive capacity



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Context & Problem:

Environmental change in a lake can be the result of a variety of phenomena and can happen under the form of extreme events and long-term changes, interacting with each other and natural variability. Yet, these changes can have lasting ecological and economic effects.

We propose a framework that describes these changes using the mathematics of viability theory and descriptive concepts such as resilience, vulnerability and adaptation.

A viability framework for the lake eutrophication case

Model: (all quantities dimensionless)

$$\begin{cases} P(t+1) = P(t) + \left[-b \cdot P(t) + L(t) + r \frac{P(t)^8}{m^8 + P(t)^8} \right] \cdot \Delta t \\ L(t) = L^* + w(t) \text{ where } w(t) \sim \mathcal{N}(0, \sigma) \\ L^*(t+1) = L^*(t) + u \cdot \Delta t \end{cases}$$

Where:

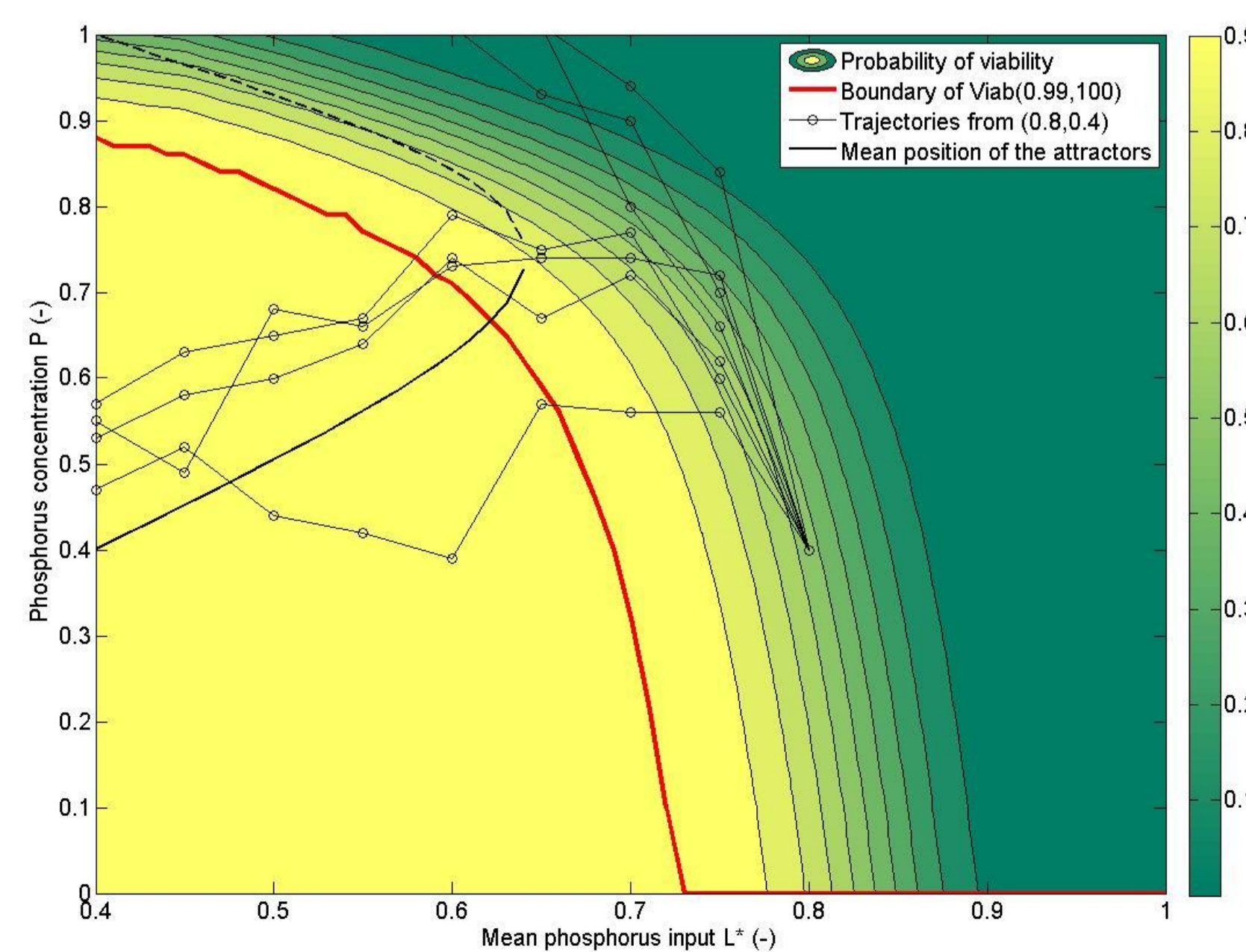
- a) (L^*, P) is the state of the system; P is the phosphorus concentration, L^* is the mean input, and L is the total input
- b) u is the control and represents the adaptive policies. Here we assume $|u| \leq 0.05$.
- c) Parameter values are $b = 5/6$, $r = m = 1$,

The goal of viability is to keep the system within constraints that represent its desirable properties. Here we have:

- 1) an ecological constraint: the lake is oligotrophic for $P \leq P_{max} = 1$;
- 2) an economic constraint: farming is profitable for $L^* \geq L^*_{min} = 0.4$;

Stochastic viability kernel: the set of states such that there is a given minimal probability β respecting the constraints for T time steps.

$$Viab(\beta, T) = \{x(0), \exists u(\cdot), P(\forall t \in [0, T], x(t) \in K) \geq \beta\}$$



Computations done through dynamic programming (also gives the optimal control strategies)

Resilience and vulnerability to extreme events

Extreme event:

An extreme rainfall event can carry an important quantity of phosphorus from the soil into the lake, causing an abrupt increase in P .

Resilience:

The concept refers to the ability for a system to retain or recover its properties and functions after a perturbation.

We consider that the properties are recovered when they are safe from more ordinary events, i.e. inside the stochastic viability kernel, here $Viab(0.99, 100)$.

Dynamic programming allows for the computation of the probability of entering $Viab(0.99, 100)$ within a time horizon T : this is the **probability of resilience**.

Vulnerability: (IPCC definition)

The concept refers to the degree to which a system is susceptible to, and unable to cope with, adverse effects of climate change, including climate variability and extremes.

Vulnerability is a statistic on a cost distribution found by taking into account all possible trajectories for an optimal strategy:

I. Recovery time

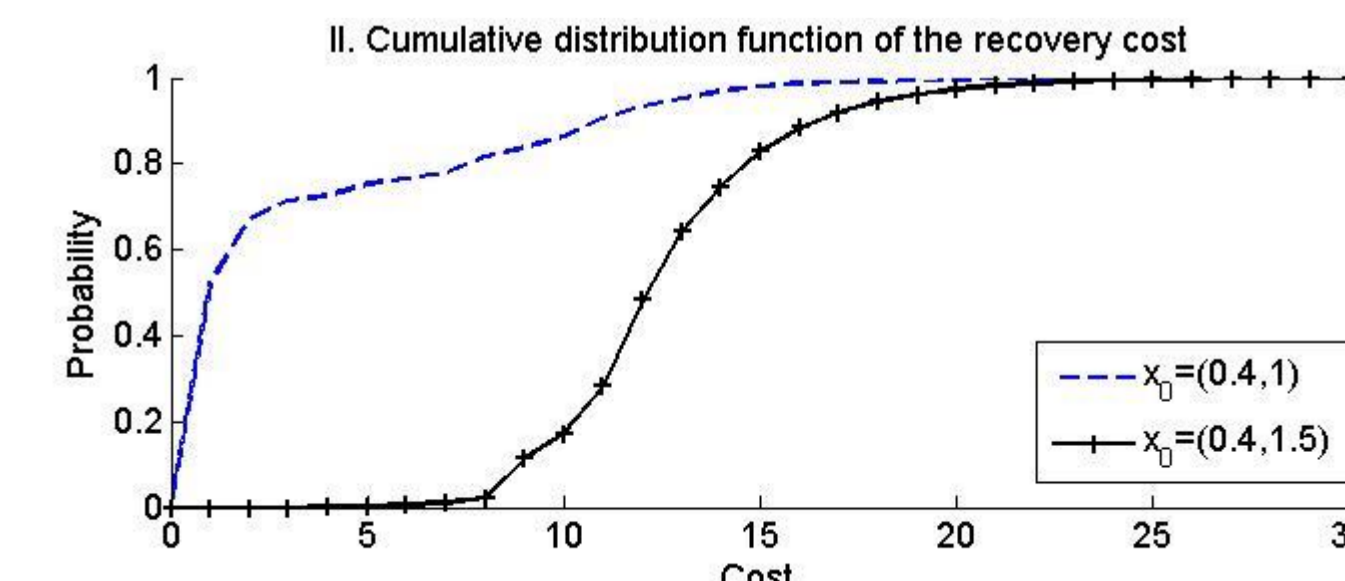
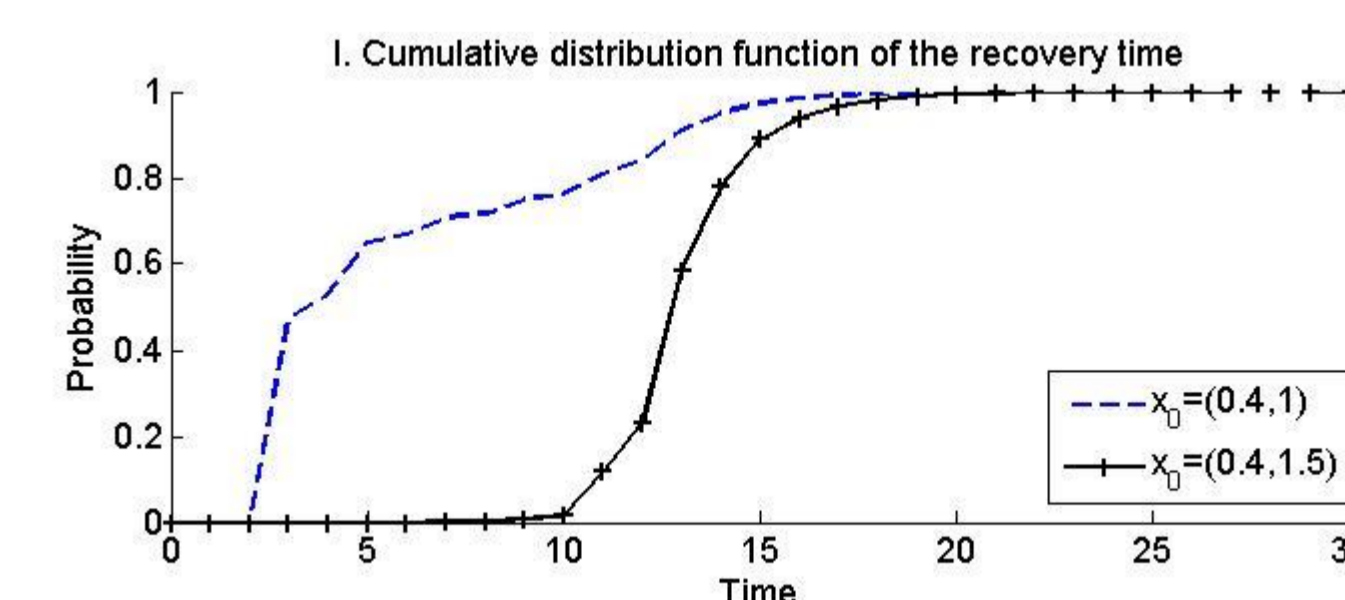
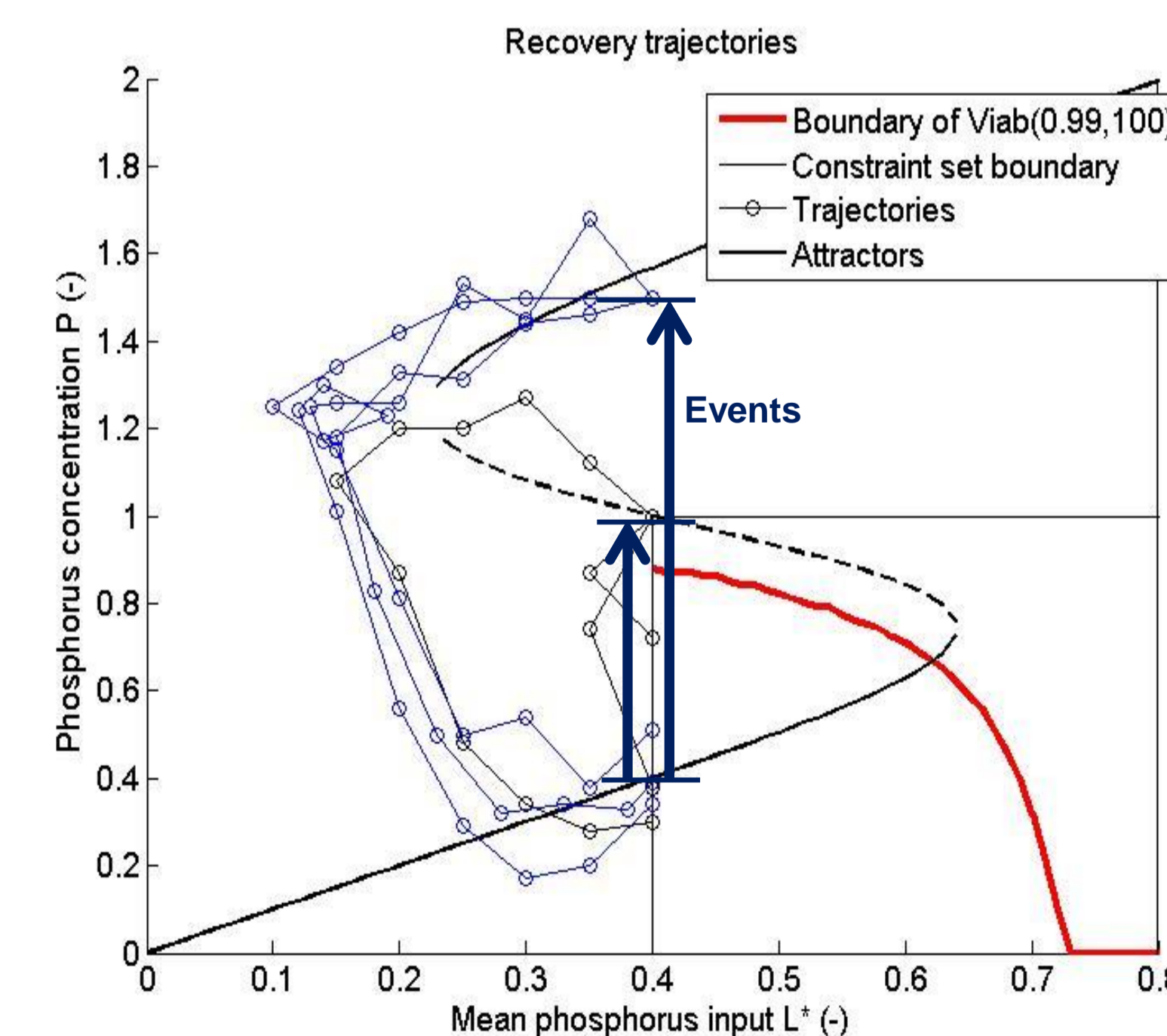
(\approx a decreasing function of resilience)

$$\text{So } C(t, x) = \begin{cases} 0 & \text{if } x \in K \\ 1 & \text{otherwise} \end{cases} \text{ and } v = \sum_T C(t)$$

II. Recovery cost, the distance from the desirable properties:

1. Economic cost $C_1(t)$: distance to $L=0.4$
2. Ecological cost $C_2(t)$: distance to $P=1$

Here $k = 0.2$.



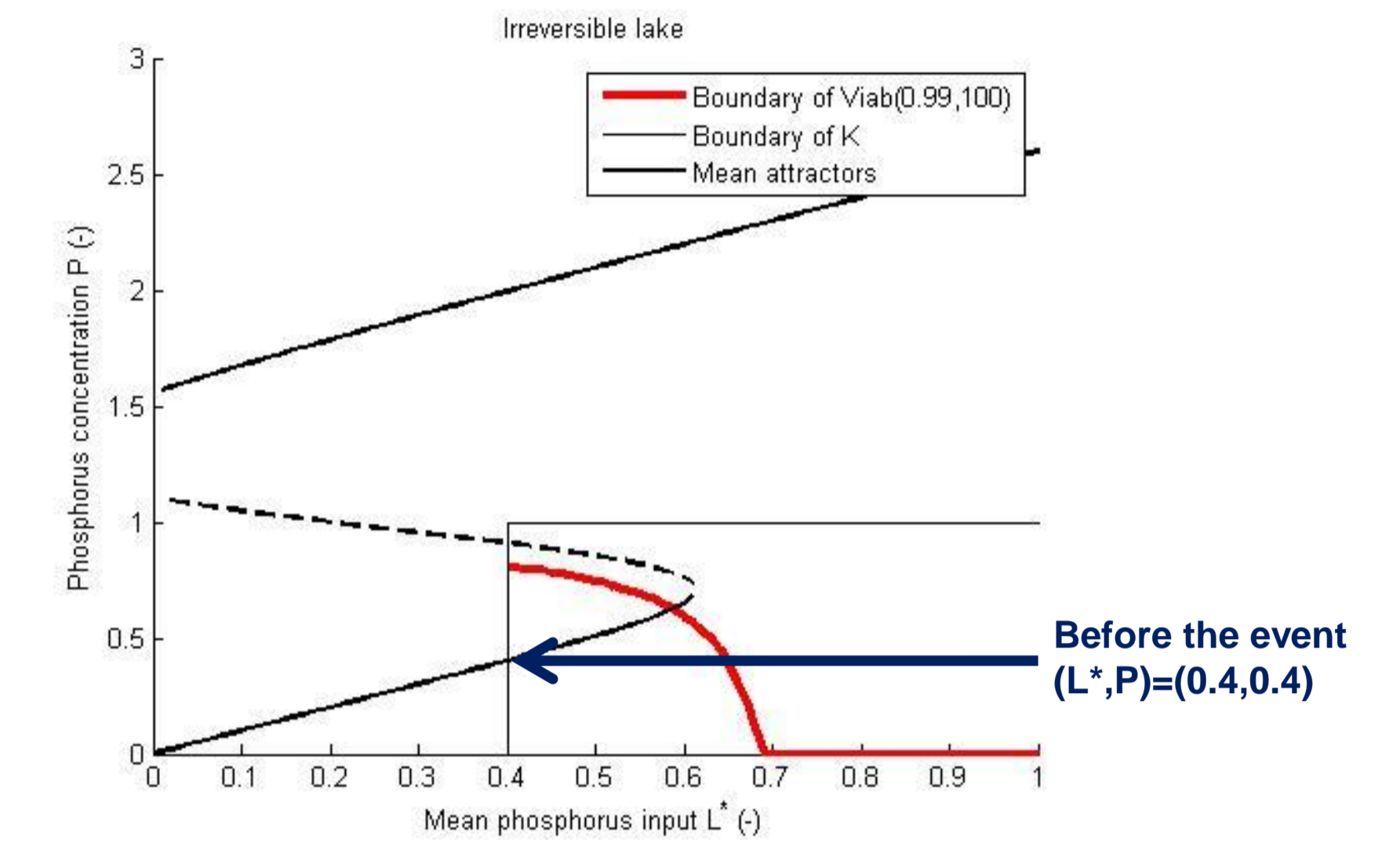
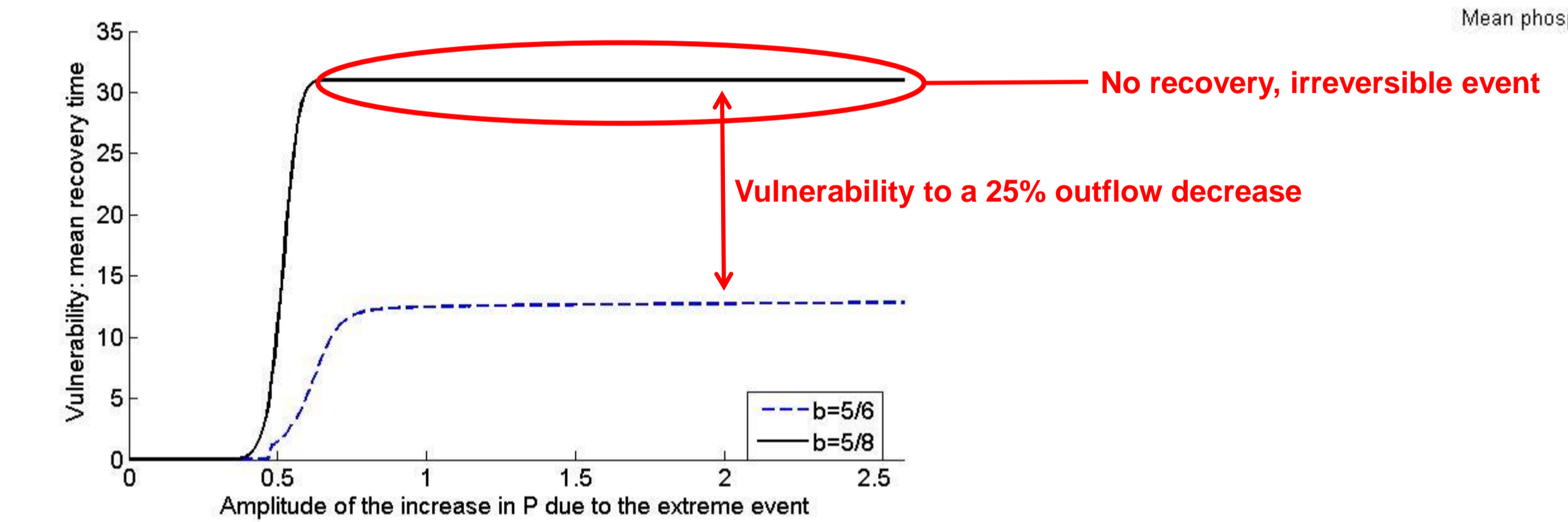
Extension to environmental changes (change in model parameters)

Example: reduction of the outflow by 25%

Assuming that the phosphorus sink term $-b \cdot P$ is solely due to outflow, the lake becomes irreversible: the oligotrophic property ($P < 1$) cannot be recovered after it is lost. The value of b decreases to $5/8$.

Then **vulnerability to this change** is the difference in vulnerability before and after the change. Here for vulnerability as the time spent outside of K , this is also a **resilience loss**.

Recovery time before and after change (horizon $T=30$)

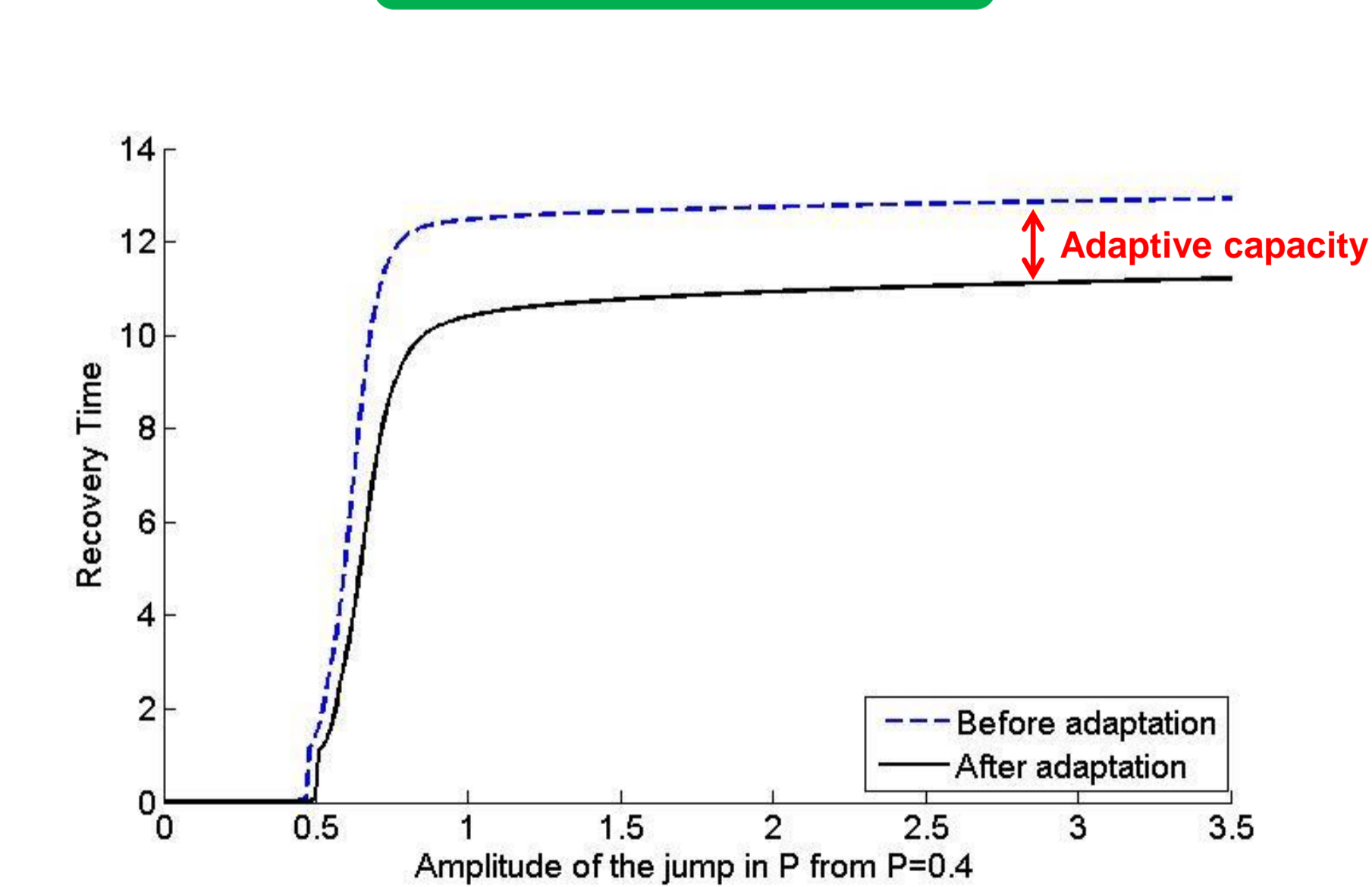


Adaptive capacity

Adaptive capacity can be defined as the vulnerability reduction due to the introduction of new controls

Example: new technological developments or management practices lower the minimum economically acceptable phosphorus input to $L^* = 0.35$.

Before change, $b=5/6$



After change, $b=5/8$

