Development of an extended Kalman Filter on the Oise river's watershed : application to the hydrological model HYDRABV-SCS at Hirson

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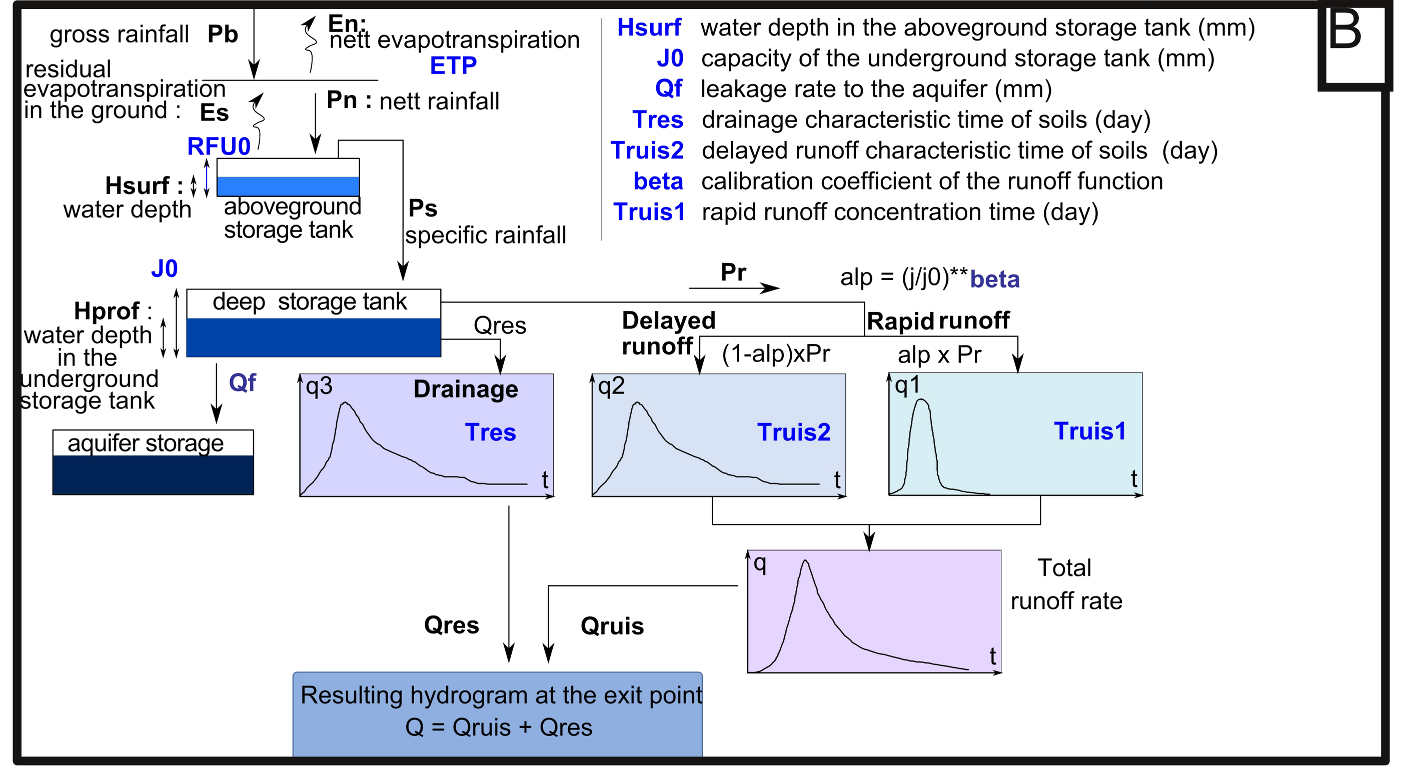
Study site and adjoint numerical model

Location of the watershed of the river Oise (France)



Block diagram of the numerical hydrological model HYDRABV-SCS ([1]) applied at Hirson

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$$\begin{pmatrix} Hsurf(k) \\ Hprof(k) \\ Hprof(k-1) \\ qout(k-1)-qini \\ \vdots \\ qout(k-npk)-qini \end{pmatrix} = M_k * \begin{pmatrix} Hsurf(k-1) \\ Hprof(k-1) \\ Hprof(k-2) \\ qout(k-2)-qini \\ \vdots \\ qout(k-npk-1)-qini \end{pmatrix} + B_k$$

(rows numbered 1, 2, 3, 4, …, npk+3)

$$X_k = M_k * X_{k-1} + B_k$$

Development of the adjoint model of the numerical hydrological model HYDRABV-SCS ([1]) applied at Hirson

Construction of the Kalman Filter

Assimilated Diagnostic variable : the flow rate q_{tot} observed at Hirson

$$\frac{q_{tot}(n) - q_{base}}{s_m} = \begin{bmatrix} 0 & 0 & \frac{1}{T_{res}} & dt*htrf(npk) & \cdots & dt*htrf(1) \end{bmatrix} * X_k + y_0$$

$$y_k = H_k * X_k + y_0$$

Equation linking the observation vector y_k to the state vector x_k via an observation operator H_k

with
- y_k : observation vector at k,
- x_k : state vector at k,
- s_m : superficy of the watershed (m²)
- q_{tot} : flow rate observed at Hirson (m³/s)
- dt : time step of the simulation (s)

Main steps for data assimilation and forecast for an **EXTENDED** Kalman Filter (Fig D)

1. **INITIALISATION** : state of the system x_0^f and matrix of error covariance P_0^f

For next time steps:

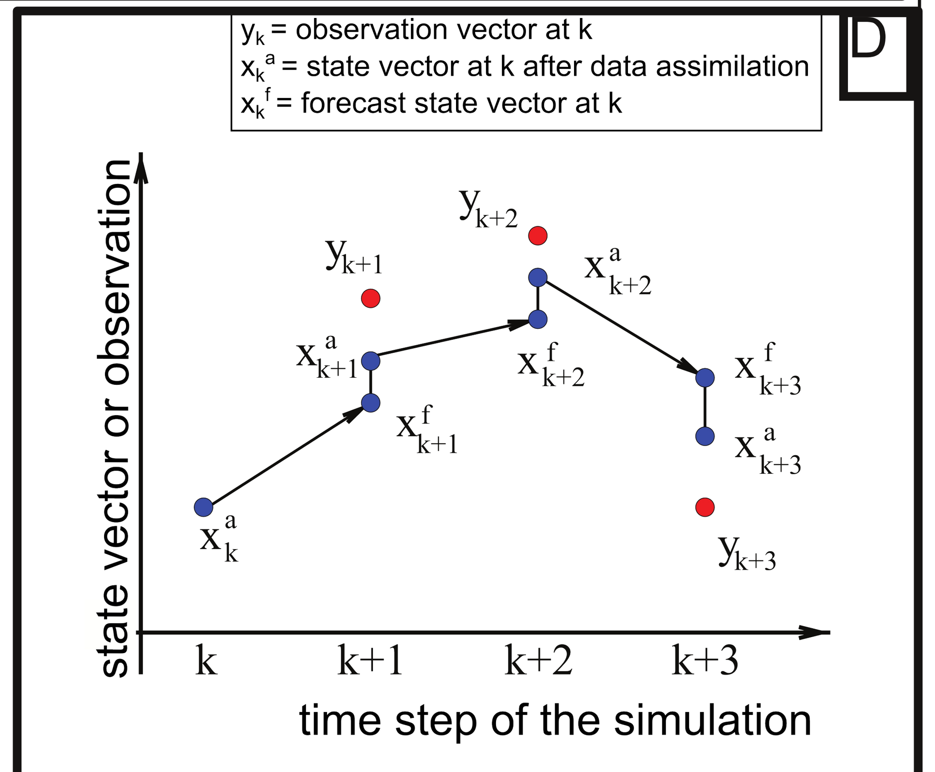
2-a **ANALYSIS**: calculation of the gain matrix K_k^*, of the analysis estimated **state vector** x_k^a and of the matrix of error covariance P_k^a

$$K_k^* = P_k^f * H_k^T * (H_k * P_k^f * H_k^T + R_k)^{-1}$$
$$x_k^a = x_k^f + K_k (y_k - H_k * [x_k^f])$$
$$P_k^a = (I - K_k^* * H_k) * P_k^f$$

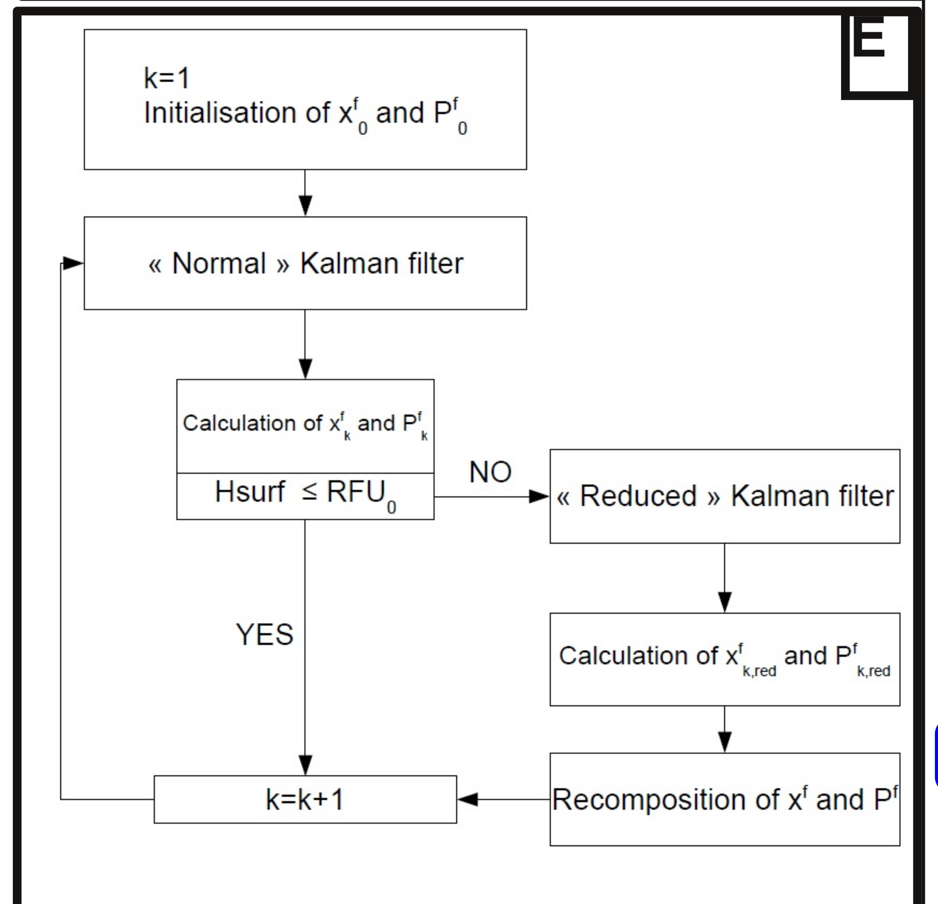
2-b **FORECAST**: calculation of the forecast estimated x_k^f and of the forecast matrix of error covariance P_{k+1}^f

$$x_{k+1}^f = M_{k+1}[x_k^a] + B_k * u_k$$
$$P_{k+1}^f = M_{k+1} * P_{k+1}^a * M_{k+1}^T + Q_k$$

with
- M_{k+1}: tranfert matrix from k to k+1,
- Q_k : the input noise covariance matrix,
- u_k : the forcing term
- H_k : a linear observation operator
- R_k : the observations' errors' covariance matrix
- RFU_0: capacity of the aboveground storage tank



Block diagram of data assimilation with an "extended" Kalman Filter ([2])



Block diagram of data assimilation with an EXTENDED and **"DISCONTINUOUS"** KALMAN FILTER

Results for the seasons 2007/2008 and 2009/2010

INITIAL STATE
(Figures F and G)
- under-estimation of flood peaks
- bad simulation of secondary flood peaks

AFTER ASSIMILATION
(Figures F and G)
- good evaluation of each flood peak

BUT

- jumps in results because of the discontinuous character of the Kalman Filter and deep influence of numerical parameters introduced for the recomposition of the numerical error covariance matrix (Figure G);
- runoff flow rates with and without data assimilation very different, whereas drainage flow rates are the same ; data assimilation provides negative non-physical values for runoff flow rates (Figure H and I);
- the "weight" of runoff processes in the original model and its contribution to the total flow rate should be higher and more equitable towards drainage processes;
- this could lead to the re-evaluation of runoff thresholds and respective weights for the 36 preceding hours.

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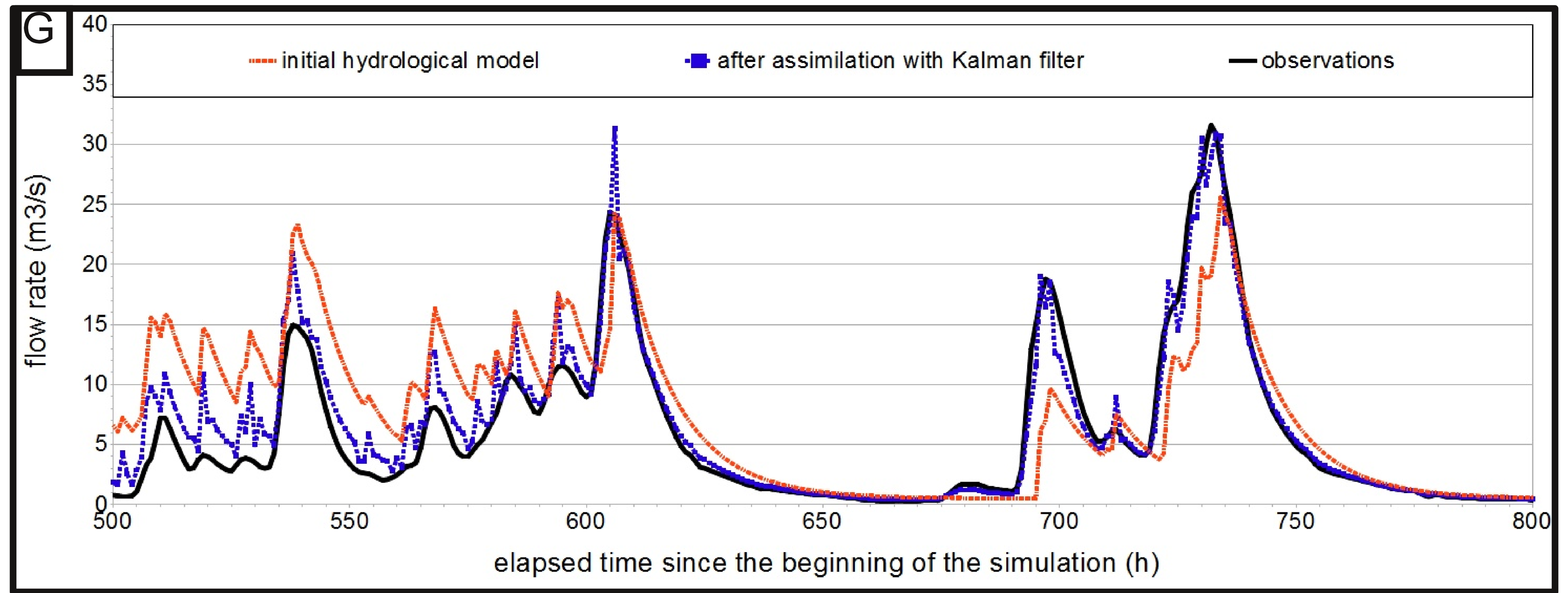
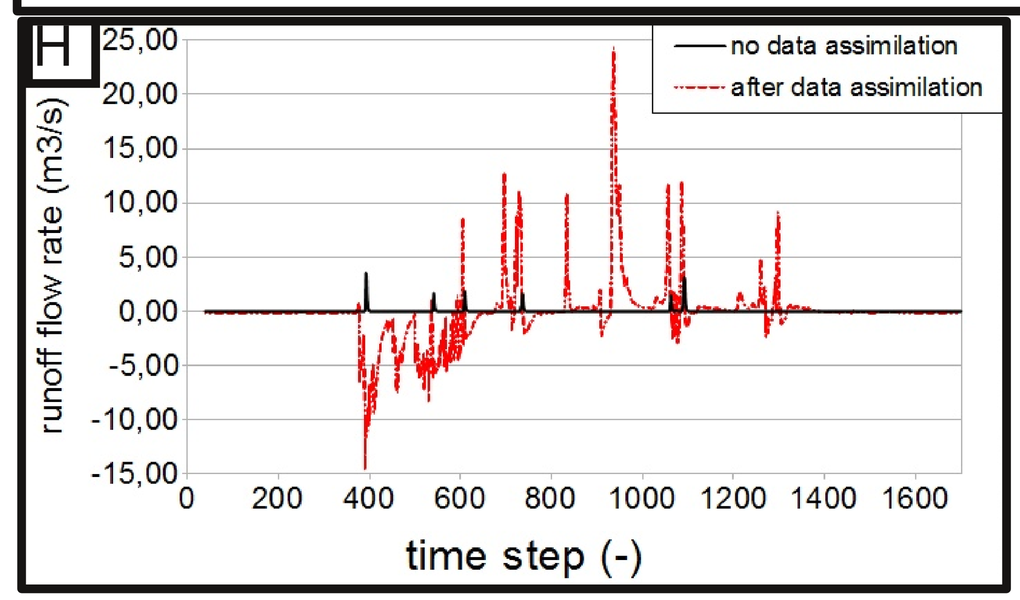
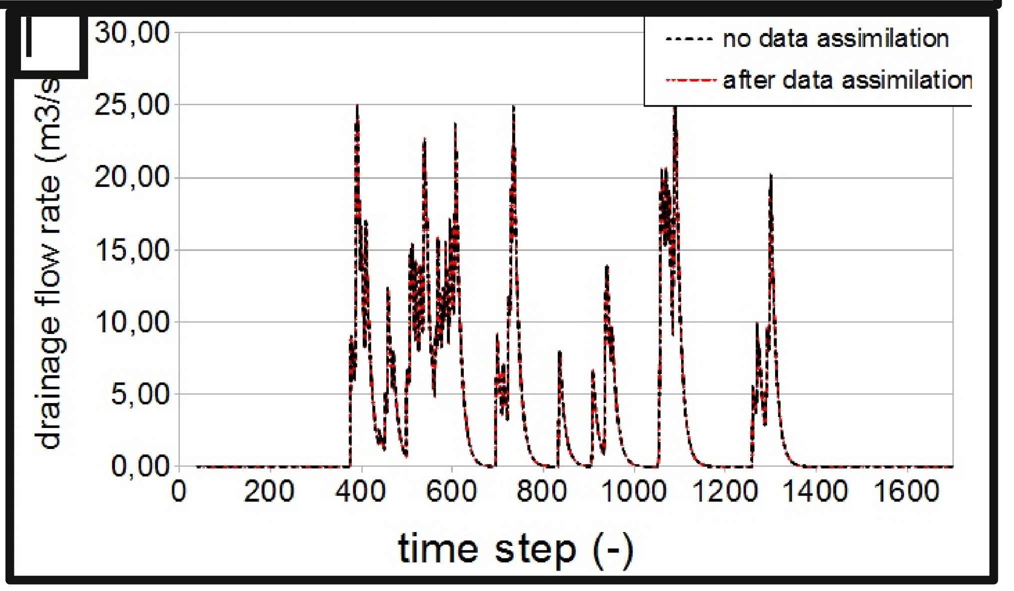
season	Nash Coefficient of the initial hydrological model	Nash Coefficient after data assimilation
2007/2008	0.89	0.997
2009/2010	0.41	0.92

Results for the 2009/2010 season







Perspectives

- the evaluation of the benefits of data assimilation for a "forecast" operating mode and also for other observation points
- the developement of an "Ensemble" Kalman filter less heavy to implement and test

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