Global modelling of surface and internal tides

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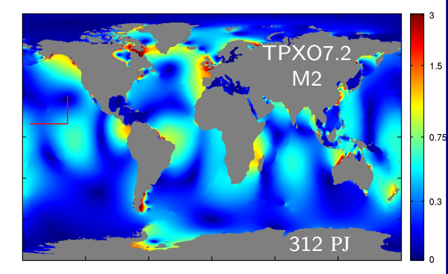
NATURAL ENVIRONMENT RESEARCH COUNCIL

Surface and Internal Tides

The long and short



Surface (barotropic) lunar M2 tide (m), generated from the TPXO, data-assimilative solution of Egbert et al.

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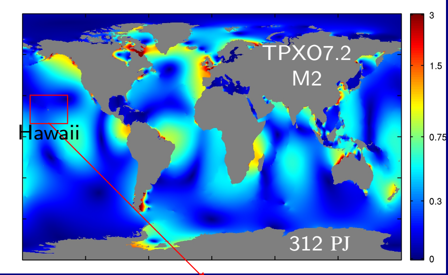

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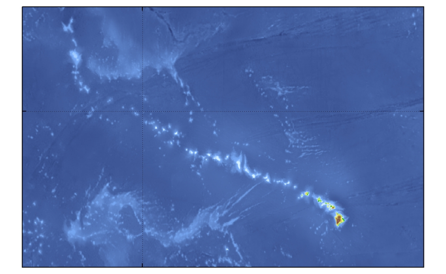
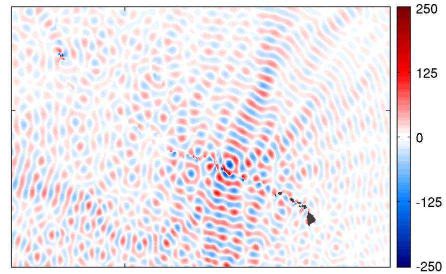
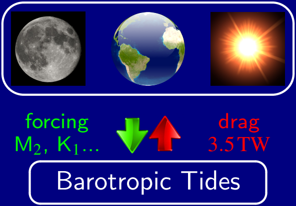


Surface tide ⇄ Internal tide (N/m^2)



Coupled system



Coupled system

forcing
M_2, K_1...

drag
3.5 TW

Barotropic Tides

- Long waves
- 1-layer SWE
- Easy to solve

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forcing / drag

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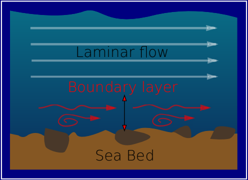

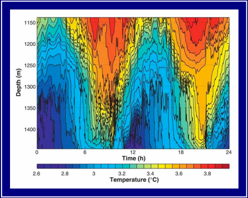

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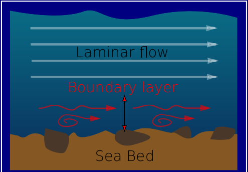
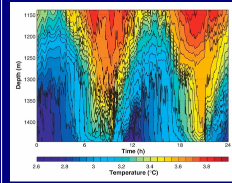
Internal Tides

Parametrized BBL drag

$$d_{BL} = \rho c_d \left| u \right| u$$





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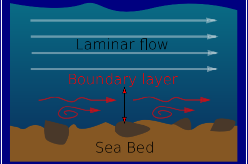
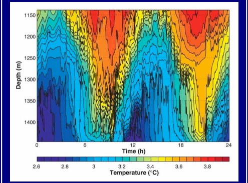


Parametrize/Model(!)

IT drag

- short waves!



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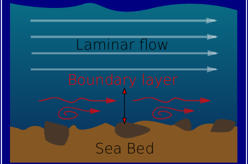

Mixing on continental shelves

Vertical mixing in deep ocean

Parametrize/Model(!)

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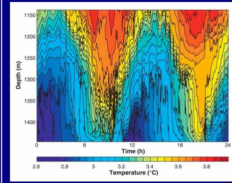
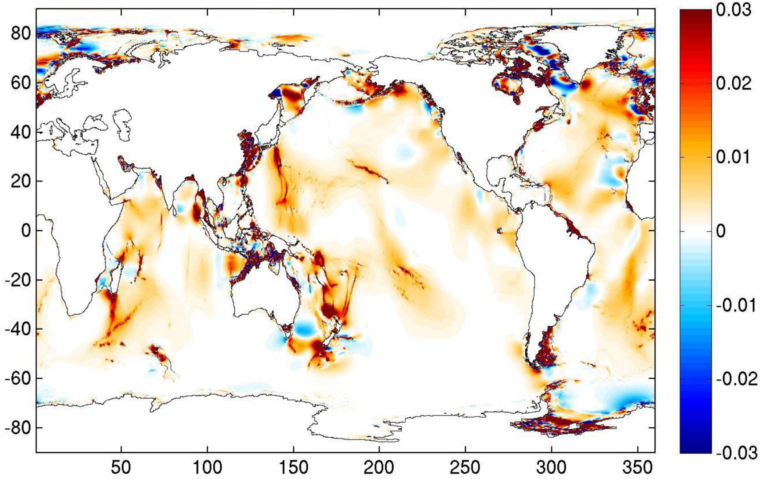
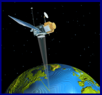


M_2 DISSIPATION (W/m^2)

FROM THE SATELLITE ALTIMETRY





TPXO8: $2.45\,\mathrm{TW} = 1.65\,\mathrm{TW}$(shallow) $+\ 0.8\,\mathrm{TW}$(deep)

Numerical Strategies

Aim

- A purely dynamical model (not data-constrained).
  - Multiscale: long (barotropic) and short (baroclinic) waves.
  - Computationally efficient and flexible (parametric studies).

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Global IT resolving models

- Multi-layer SW: Arbic et al. 2004; Simmons et al. 2004
- OGCMs: Arbic et al. 2010, 2012; Müller et al. 2012
  - Issues: resolution $> 1/12^\circ$ to get ITs; scalar approximations for SAL.

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An Alternative Approach

- Exploit near-linearity of both surface and internal tides.
- Modal decomposition in z for arbitrary topography: 3D $\rightarrow$ 2D.
- Frequency domain: long runs of time-stepping $\rightarrow$ matrix inversions.

Boussinesq equations and B.C.

$$\frac{\partial \mathbf{u}}{\partial t} + f\mathbf{e_z} \times \mathbf{u} = -\frac{\nabla p}{\rho_0} + \mathbf{F}, \ \frac{\partial p}{\partial z} = -\rho g, \ \nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} = 0, \ \frac{\partial \rho}{\partial t} - \frac{\rho_0 N^2}{g} w = 0,$$

$$w = \mathbf{u} \cdot \nabla h \quad \text{at} \quad z = -H(\mathbf{x}); \quad w = \frac{\partial \zeta}{\partial t}, \ p = \rho_0 g \zeta \quad \text{at} \quad z = 0.$$

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Vertical modes

$$\left(\frac{Z'}{N^2}\right)' + \frac{Z}{c_n^2} = 0, \quad Z'(-H) = 0, \quad N^2(0) Z(0) + gZ'(0) = 0$$

- Single barotropic mode: $c_0 \approx \sqrt{gH}$
- Infinite set of internal modes: $c_n \approx \bar{N}H/\pi n, \ n \geq 1$
- Modal expansion: $\psi = \sum_{n=0}^{\infty} \psi_m(\mathbf{x}) Z_m(z, H(\mathbf{x}))$.

Modal Shallow Water Equations

(cf. Griffiths & Grimshaw, JPO 2007)

Barotropic mode:

$$\frac{\partial \boldsymbol{u_0}}{\partial t} + \boldsymbol{f} \times \boldsymbol{u_0} = -\nabla(P_0 - g\zeta_{eq} - g\zeta_{sal}) - \frac{\boldsymbol{D}_{BL} + \boldsymbol{D}_{IT}}{\rho H},$$

$$\frac{1}{g}\frac{\partial P_0}{\partial t} + \nabla \cdot (H\boldsymbol{u_0}) = 0, \quad \boldsymbol{D}_{IT} = -\rho \nabla H \sum_n P_n.$$

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Internal modes:

$$\frac{\partial \boldsymbol{u_m}}{\partial t} + \boldsymbol{f} \times \boldsymbol{u_m} = -\nabla P_m - \nabla H \sum_n I_{mn} P_n,$$

$$\frac{\partial P_m}{\partial t} + \nabla \cdot (c_m^2 \boldsymbol{u_m}) = -(c_m^2)' u_0 \cdot \nabla H - \nabla H \cdot \sum_n I_{mn} c_n^2 u_n.$$

Here, c_m and $I_{mn}(c_{m,}c_n)$ are horizontally varying.

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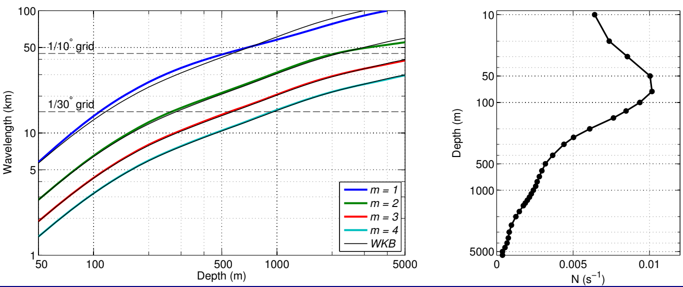

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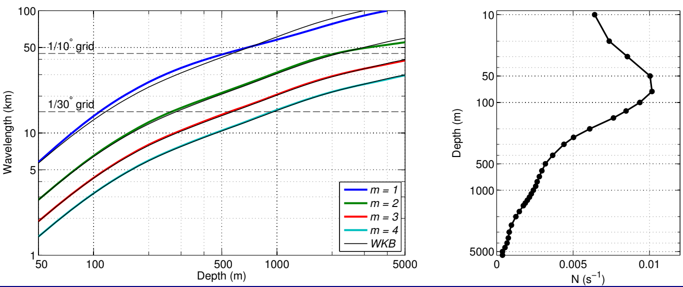

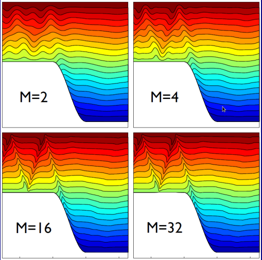

- ITs are notoriously sensitive to topographic slopes via
$$\varepsilon = \frac{|\nabla H|}{\alpha}, \quad \alpha = \left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2}$$
- But most of the energy flux is in low modes for large-scale topo.
- We truncate to 2-4 modes.

Spectral approach

$$\boldsymbol{u_m} = \mathrm{Re}\Big(\sum_j \hat{\boldsymbol{u}}_m^{(j)} \exp\big(-\mathrm{i}\omega_\mathrm{j}\mathrm{t}\big)\Big) \quad \text{and} \quad P_m = \mathrm{Re}\Big(\sum_j \hat{P}_m^{(j)} \exp\big(-\mathrm{i}\omega_\mathrm{j}\mathrm{t}\big)\Big)$$

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Barotropic mode:

- Solve iteratively and average at each step.
- Projection for non-linear terms (e.g. $\hat{\boldsymbol{D}}_{BL}$).

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Internal modes:

- Calculate $N(\mathbf{x}, z)$ from WOA09 and then $c_m(\mathbf{x}, H(\mathbf{x}))$.
- Split topo into resolved and unresolved
  - large-scale topo $\rightarrow$ model explicitly (most conversion into low modes);
  - small-scale rough topo $\rightarrow$ parameterized IT wave drag.
- Add sponge layers (nonlinear wave breaking in shallow regions).

FD Scheme and Convergence

Discretization

- 2^{nd}-order energy-conserving FD (*Arakawa C-grid*).
- Lat-lon grid with a rotated pole.
- Resolution up to $1/30^{\circ}$.

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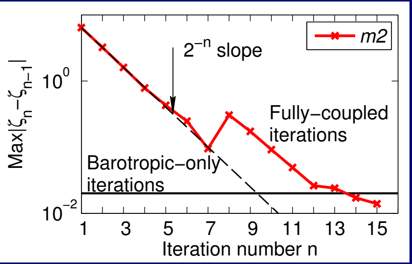
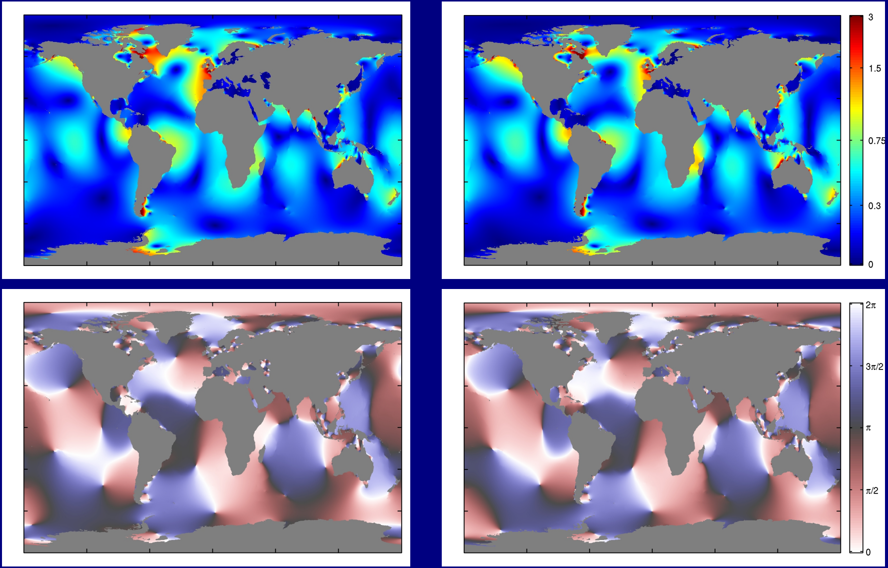
Converging solution M_2



- K_1: Numerical issues with coastally trapped IT

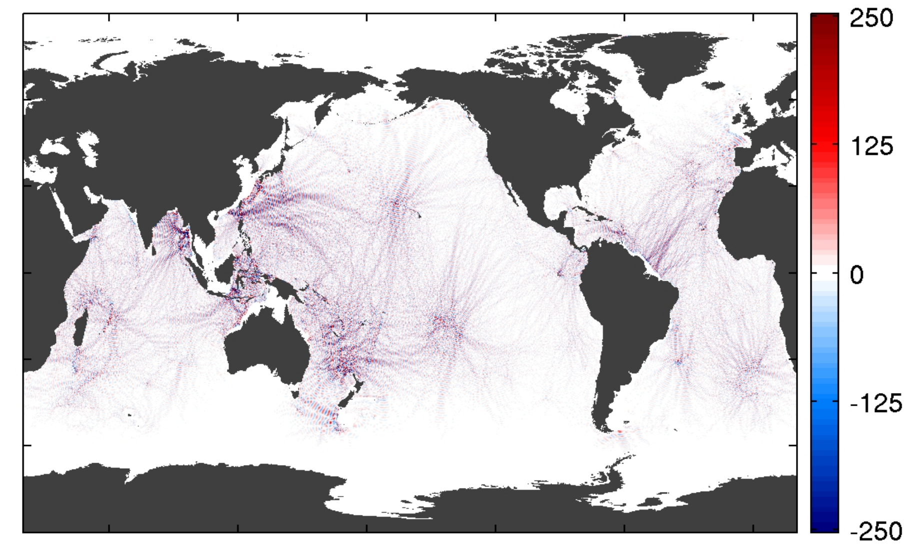
Results: M_2 Amplitude and Phase

Model | Data (TPXO7.2)



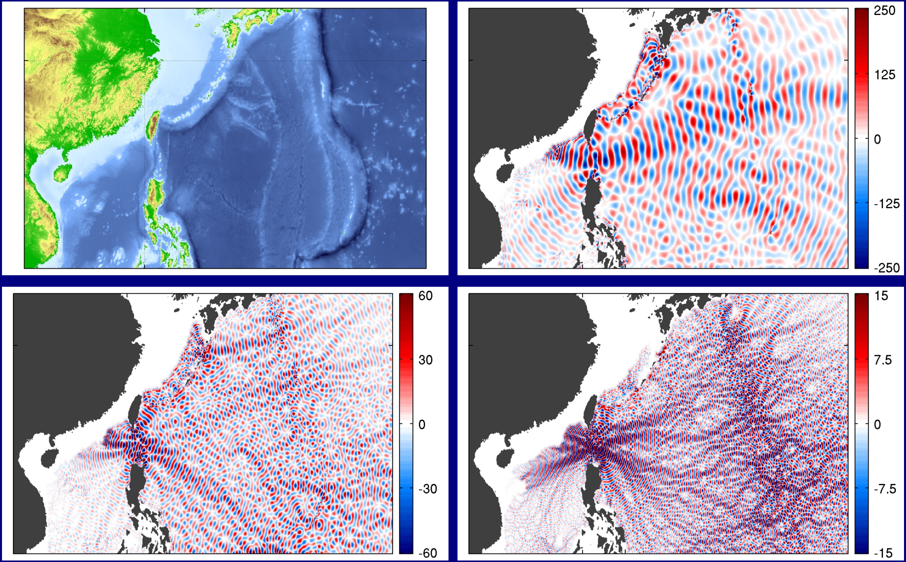
M_2 Global Internal Tides

bottom pressure, N/m^2

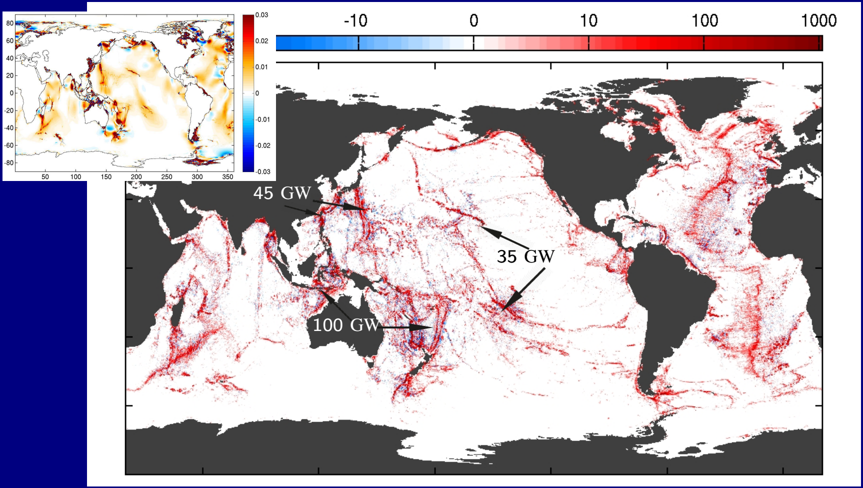


Luzon Straight: modes 1, 2 and 3

bottom pressure, N/m^2



M_2: Baroclinic conversion (mW/m^2)



- M+P: 1.1 TW = (0.3+0.3) TW (open) + (0.1+0.4) TW (coastal)
- TPXO: 1 TW = 0.75 TW (open) + 0.25 TW (coastal)

SUMMARY

- A purely dynamical model (no data assimilation).
  - Surface and internal tides are modelled with quasi-linear SWEs.
  - Works in frequency domain. Fast: 12h/constituent on $1/25^{\circ}$ grid.
  - Showed results for M2. Other constituents (K_1) also modelled.
- Good for
  - high-res present day maps for the internal tides;
  - paleotides (e.g. in the last glacial maximum);
  - simulate the tidal evolution of the Earth-Moon system?
- Not so good for
  - present-day global maps of tides (data-constrained models are better);
  - modelling of shelf tides (nonlinear and non-tidal interactions).

SUMMARY 2

LIMITATIONS & FUTURE WORK

- **IT model**: $\uparrow$ resolution will give more resolved IT conversion and $\downarrow$ parametrized IT drag
  - subdomain decomposition + parallelization
  - more baroclinic modes, mode coupling....
- **FD scheme**: grid points are clustered at high lats, no local refinement.
  - Move to FE? (cf. Lyard et al., 2006)
- **Sparse solver**: direct sparse solvers (PARDISO, UMFPACK) are used but iterative methods are desirable.
- **Nonlinearities**: advection can be added to the iterative scheme.

Convergence with resolution for M_2

Res	DBL, TW	DIT, TW	D, TW	KE, PJ	PE, PJ	RMS
1/10	1.695	1.308	2.998	236.8	204.3	0.093
2/25	1.689	1.305	2.986	237.5	204.8	0.096
1/15	1.616	1.350	2.958	231.3	198.8	0.092
1/20	1.829	1.177	2.997	250.0	215.7	0.100
1/25	1.958	1.068	3.026	262.8	227.3	0.105

Res	M1, TW	M2, TW	M3, TW	DITM, TW	DITP, TW
1/10	0.146	0.017	0.000	0.163	1.145
2/25	0.175	0.039	0.000	0.214	1.092
1/15	0.179	0.059	0.008	0.246	1.105
1/20	0.215	0.092	0.027	0.333	0.843
1/25	0.253	0.110	0.044	0.407	0.661

Discretization

For each tidal constituent write $\hat{\boldsymbol{a}} = \left(\hat{U}^{(j)}, \hat{V}^{(j)}, \hat{\zeta}^{(j)}\right)$, giving

$$(\mathbf{L} - \mathrm{i}\omega_j \mathbf{I})\hat{\boldsymbol{a}} \quad = \quad \hat{\mathbf{b}} + \hat{\mathbf{s}}(\hat{\boldsymbol{a}}) - \hat{\mathbf{d}}(\hat{\boldsymbol{a}})$$

- $\mathbf{L} = \begin{pmatrix} 0 & -f & \frac{g}{re\cos\theta}\frac{\partial}{\partial\varphi} \\ f & 0 & \frac{g}{re}\frac{\partial}{\partial\theta} \\ \frac{1}{re\cos\theta}\frac{\partial}{\partial\varphi}H & \frac{1}{re\cos\theta}\frac{\partial}{\partial\theta}\cos\theta H & 0 \end{pmatrix}$ – the linear dynamics;
- $\hat{\mathbf{b}}$ – forcing; $\hat{\mathbf{s}}$ – self-attraction; $\hat{\mathbf{d}}$ – nonlinear friction.

Solve iteratively, averaging at each time step $\hat{\boldsymbol{a}}_{n+1} \to 0.5\left(\hat{\boldsymbol{a}}_{n+1} + \hat{\boldsymbol{a}}_n\right)$

$$\left(\mathbf{L} - \mathrm{i}\omega_j \mathbf{I} - \hat{\mathbf{S}}_n + \hat{\mathbf{D}}_n\right)\hat{\boldsymbol{a}}_{n+1} \quad = \quad \hat{\mathbf{b}} + \left[\hat{\mathbf{s}}(\hat{\boldsymbol{a}}_n) - \hat{\mathbf{S}}_n\hat{\boldsymbol{a}}_n\right] - \left[\hat{\mathbf{d}}(\hat{\boldsymbol{a}}_n) - \hat{\mathbf{D}}_n\hat{\boldsymbol{a}}_n\right]$$

$\hat{\mathbf{S}}$ and $\hat{\mathbf{D}}$ – sparse approximations to self-attraction and nonlinear drag.

Topographic smoothing

Sensitivity problem:

- IT generation is notoriously sensitive to topographic slopes via
  - criticality parameter $\varepsilon = \frac{|\nabla H|}{\alpha}, \quad \alpha = \left(\frac{\omega^2 - f^2}{N^2 - \omega^2}\right)^{1/2}$.

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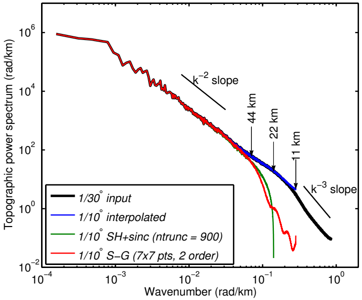
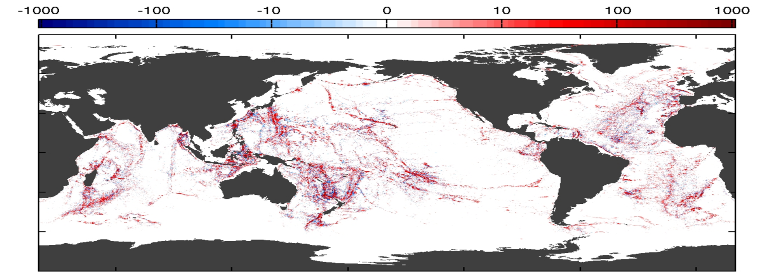
Splitting topography

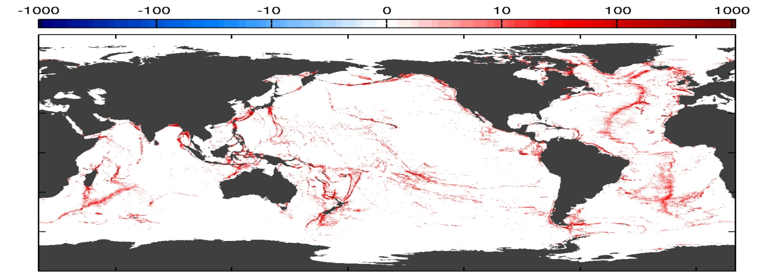
- Split topo into resolved and unresolved
  - Large-scale topo → model explicitly (most conversion into low modes)
  - Small-scale rough topo → parameterized IT wave drag
- "Global" or "local" filters:
  - SH truncation or Savitzky-Golay filter.



M_2: Baroclinic conversion (mW/m^2)

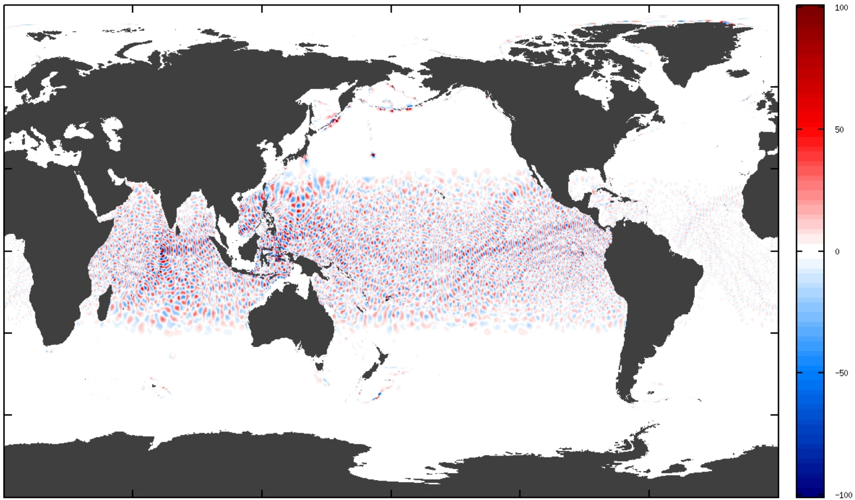
Dynamical and parameterized





K_1: IT BOTTOM PRESSURE (N/m^2)

MODES 1 AND 2



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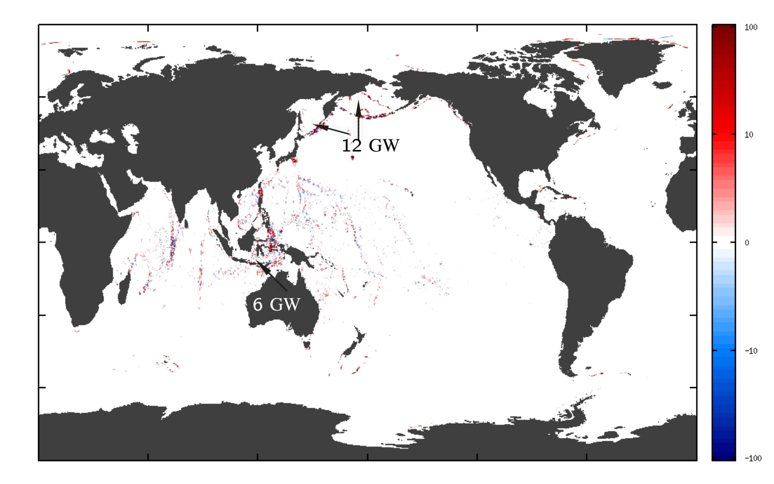
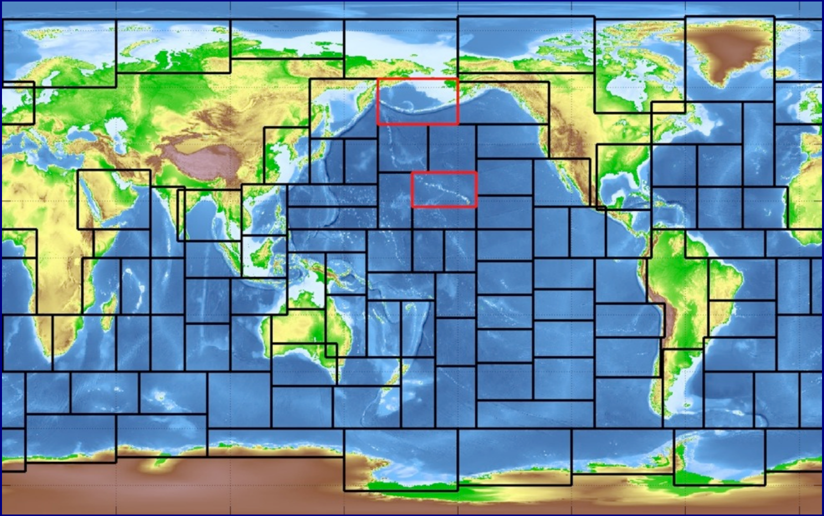


- I.T. MODEL: 0.04 TW = 0.02 TW (open ocean) + 0.02 TW (coastal)
- TPXO: 0.1TW = 0.05 TW (open ocean) + 0.05 TW (coastal)

Subdomain Decomposition Approach



- Solve in parallel on 100 cores

Other generation mechanism

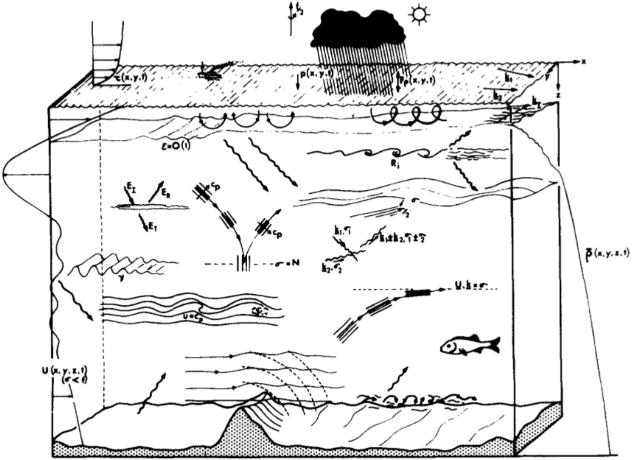
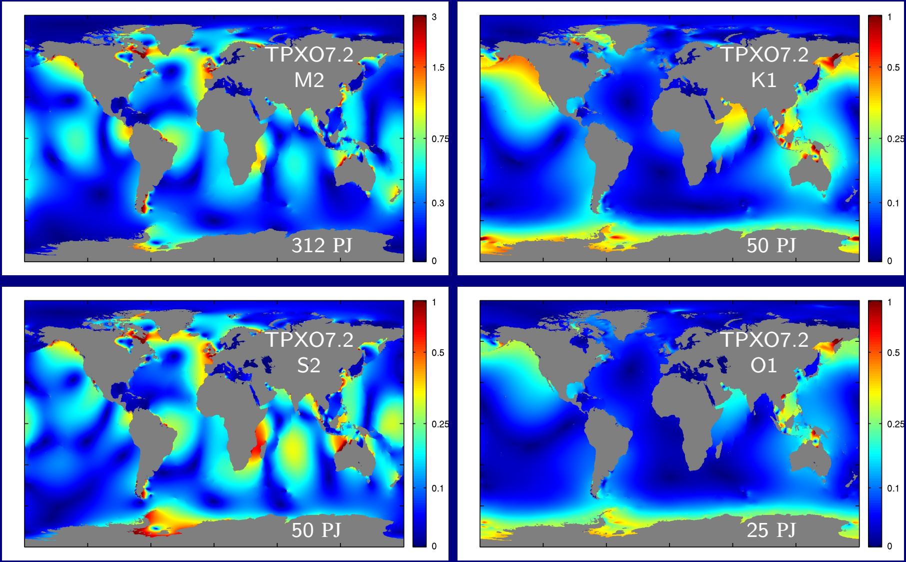


Fig. 7. A solar view of physical processes affecting internal waves. "As a challenge, the reader is left to decipher the symbols himself" (Thorpe, 1975)

- Spectra of internal waves show a strong peak at M_2 frequency

OBSERVED TIDES



Tidal amplitudes (m), generated from the TPXO7.2

Linear single-layer SWE for is the depth perturbation $\zeta = \zeta_o - \zeta_e$ and volume transprt $\boldsymbol{U} = (H + \zeta)\boldsymbol{u}$:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{f} \times \boldsymbol{U} = -gH\nabla(\zeta - \zeta_{eq} - \zeta_{sal}) - \rho_0^{-1}(\boldsymbol{D}_{BL} + \boldsymbol{D}_{IT}),$$

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \boldsymbol{U} = 0.$$

ζ_{eq} is the equilibrium tide representing astronomical forcing;
ζ_{sal} is the self-attraction and loading potential;
ρ_0 – averaged density; f – the Coriolis parameter.

Simple Barotropic Tide Model

Our numerical approach

Linear single-layer SWE for is the depth perturbation $\zeta = \zeta_o - \zeta_e$ and volume transprt $\boldsymbol{U} = (H+\zeta)\boldsymbol{u}$:

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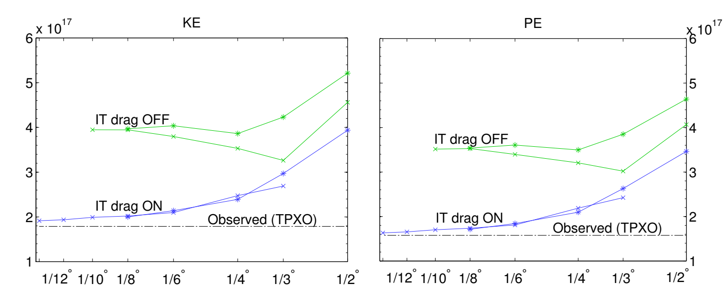
Parameterized IT drag

$$\boldsymbol{d}_{IT} = \frac{\rho_0 N_B^2}{3\omega}(\boldsymbol{u} \cdot \nabla H)\nabla H \times \begin{cases} 1, & |f| < \omega, \\ 0, & |f| \geq \omega, \end{cases}$$

where N_B is the buoyancy frequency at the bottom.

Resolution Convergence

M2 energy (J)



Dissipation

	D_{BL}(TW)	D_{IT}(TW)	D(TW)
M2 model	~ 1.35	~ 1.4	~ 2.75
M2 obs	< 1.65	> 0.8	~ 2.45
K1 model	~ 0.2	~ 0.14	~ 0.34
K1 obs	< 0.3	> 0.04	~ 0.34