Assessing predictability of a hydrological stochastic-dynamical system

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Outline

Preliminary notes

Theoretical basis and analytical illustration

Application of the proposed approach to assess predictability of moisture content of frozen soil

- *Dynamic-stochastic modeling hydrothermal regime of soil column*
- *Is the process of frozen moisture dynamics stable? Sensitivity to initial conditions*
- *Assessing predictability and its sensitivity to changes of soil properties and climatic conditions*

Conclusions

Motivation

To develop a conceptual approach to answer the questions:

What are the predictability limits of different water cycle components and what metrics can be used to quantify it?

How to identify predictable and unpredictable patterns?

What are the physical mechanisms controlling predictability?

How to evaluate the quality of the hydrological models by dividing inherent and model-related predictability limits?

Representation of a hydrological system as dynamical systems subjected to the effect of noise (stochastic-dynamical system) provides possible way to such conceptualization

Objective

To propose a method for assessing potential predictability through a procedure of convergence of the system state probabilistic measure to its stable value (if the latter exists)

Theoretical basis (Zeeman, 1988; Dymnikov, 2007; Klyatskin, 2005)

$$\frac{dW}{dt} = L(W) + \varepsilon(t)$$

$$W\big|_{t=0} = W_0$$

$$\left\langle \varepsilon_i(t) \times \varepsilon_j(t') \right\rangle = 2 d_{ij} \delta(t - t')$$

$$d_{ij} \equiv d$$

Fokker-Planck equation for p.d.f.

$$\frac{d\rho}{dt} + div\big(L(W)\rho\big) = d\Delta\rho$$

$$\rho \geq 0, \int \rho dW = 1$$

$$\rho\big|_{t=0} = \delta\big(W - W_0\big)$$

In process of time, probability function will be converge to an invariant measure $\bar{\rho}$ and information on the initial state will be lost

<u>The time of the convergence, i.e. the time interval during which the system losses memory about its initial state, defines limit of the potential predictability of the system.</u>

Dymnikov, V. P. (2007) Stability and predictability of macro-scale atmospheric processes (in Russian)

Klyatskin, V.I. (2005) Stochastic equations through the eye of the physicist

Zeeman E.S. (1988) Stability of dynamical systems

Scalar stochastic-dynamical equation as a very simple parameterization of soil moisture dynamics

$$\frac{dW}{dt} + \lambda W = \varepsilon$$

$$\langle \varepsilon(t) \times \varepsilon(t') \rangle = d \exp\left(-\left|t - t'\right| / \tau\right),\ \tau << \lambda^{-1}$$

$$W\big|_{t=0} = W_0$$

Measure of convergence is the variance of the process

$$\boxed{W\big|_{t=0} = W_0 = const}$$

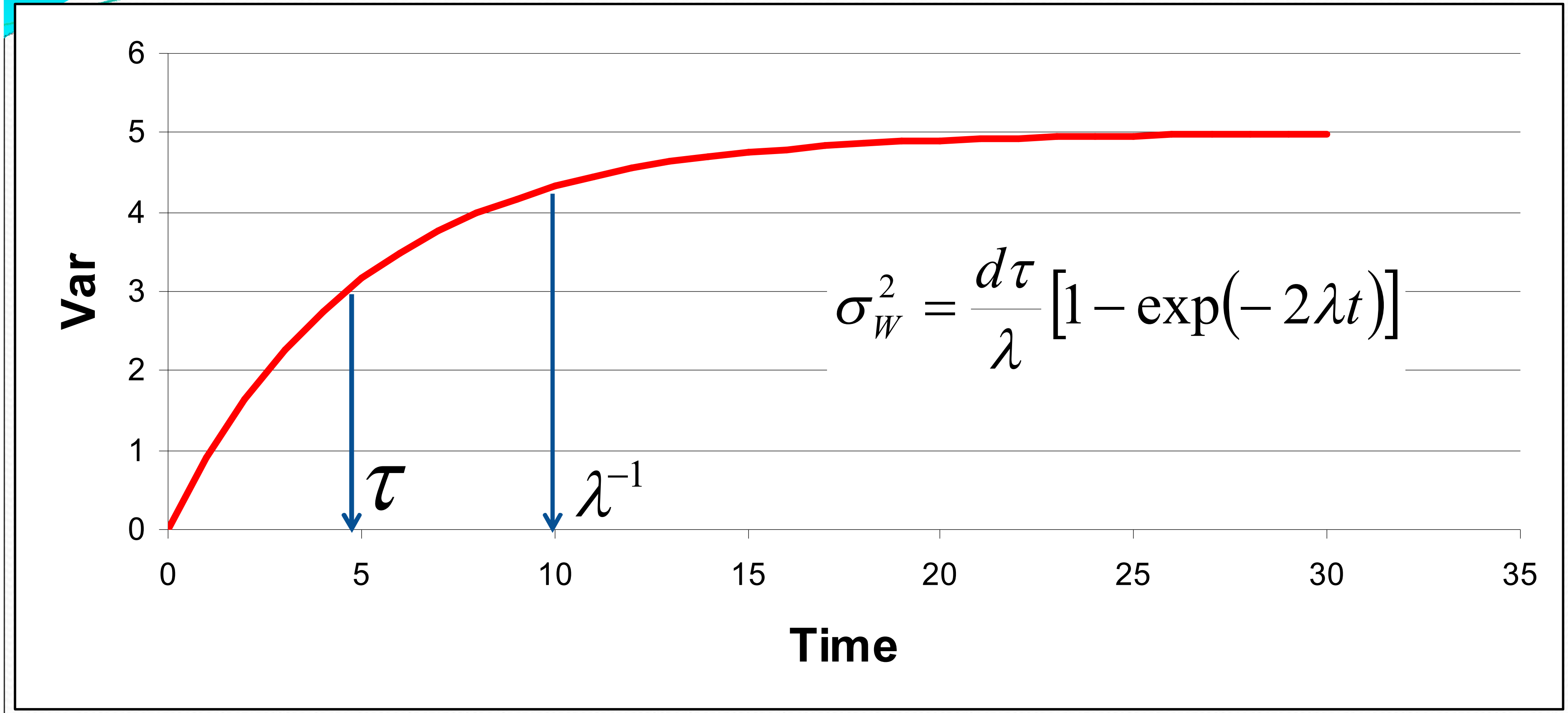
$$\boxed{\sigma_W^2 = \frac{d\tau}{\lambda}\left[1 - \exp\left(-2\lambda t\right)\right]}$$

$$\boxed{W\big|_{t=0} = W_0 \in N\left(0; \sigma_{W_0}\right)}$$

$$\boxed{\sigma_W^2 = \frac{d\tau}{\lambda}\left[1 - \exp\left(-2\lambda t\right)\right] + \sigma_{W_0}^2 \exp\left(-2\lambda t\right)}$$

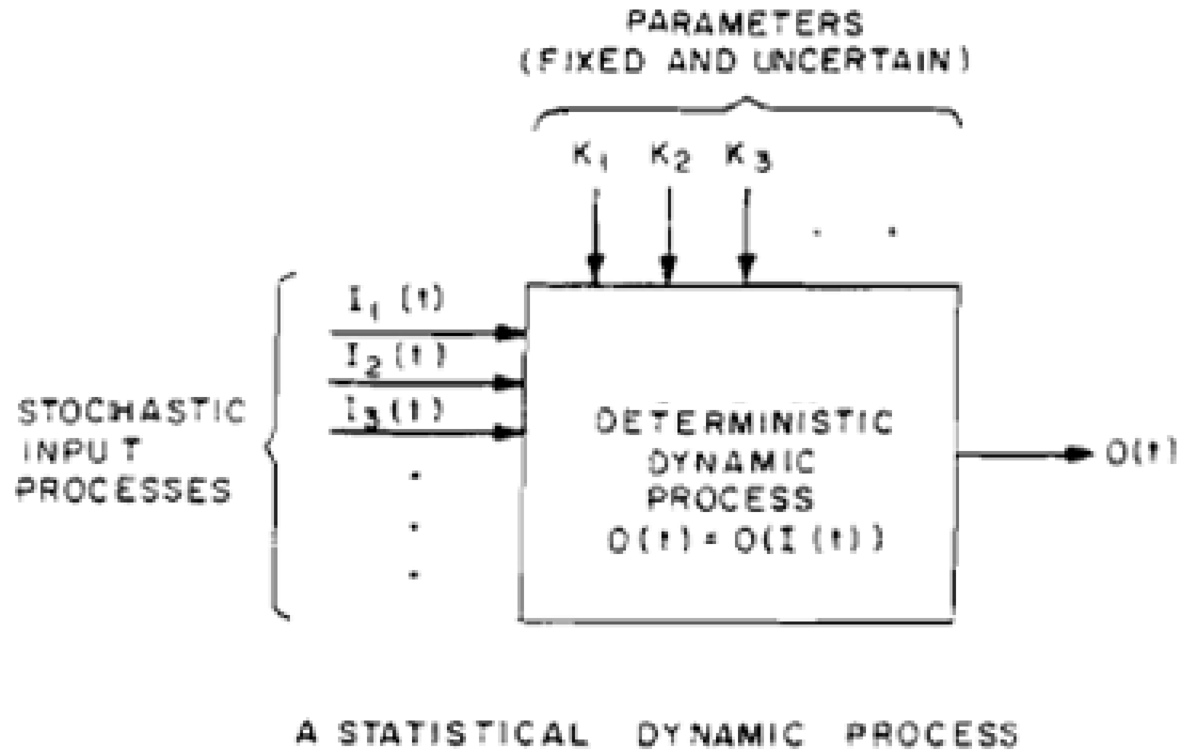
$$t >> \lambda^{-1}$$

$$\sigma_W^2 = \frac{d\tau}{\lambda}$$



Rate of convergence depends on the dissipation parameter λ^{-1} and <u>does not depend on the variance of forcing process</u>

Potential predictability of hydrological processes and physical mechanisms controlling the predictability can be studied by numerical dynamic-stochastic modeling of the processes



Schematic of a dynamic-stochastic model

(from P.S. Eagleson "Climate, Soil and Vegetation: Introduction to Water Balance Dynamics")

Water and heat transfer in a frozen soil (Gelfan, 2006)

$$\frac{\partial \theta}{\partial t} = -\frac{\rho_i}{\rho_w}\frac{\partial I}{\partial t} + \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z} + D_I \frac{\partial I}{\partial z} - K \right)$$

$$c_T \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}(\lambda \frac{\partial T}{\partial z}) + \rho_w c_w (D\frac{\partial \theta}{\partial z} + D_I \frac{\partial I}{\partial z} - K)\frac{\partial T}{\partial z} + \rho_w \chi \frac{\partial W}{\partial t}$$

$$-\left(D\frac{\partial \theta}{\partial z} + D_I \frac{\partial I}{\partial z} - K \right)_{z=0} = \begin{cases} R - E, snow = 0 \\ R_{melt}, otherwise \end{cases}$$

$$-\lambda \frac{\partial T}{\partial z}\bigg|_{z=0} = Q_g$$

or

$$T(0,t) = T_a(t)$$

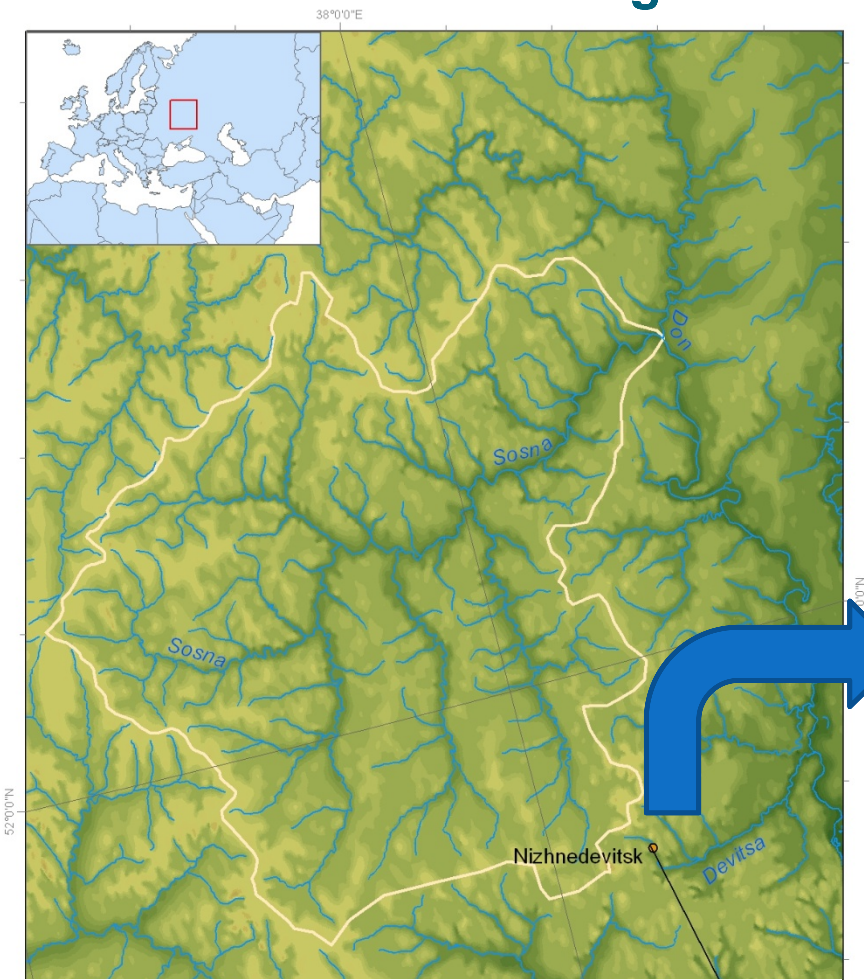
Stochastic processes simulated by a weather generator (Gelfan, 2010)

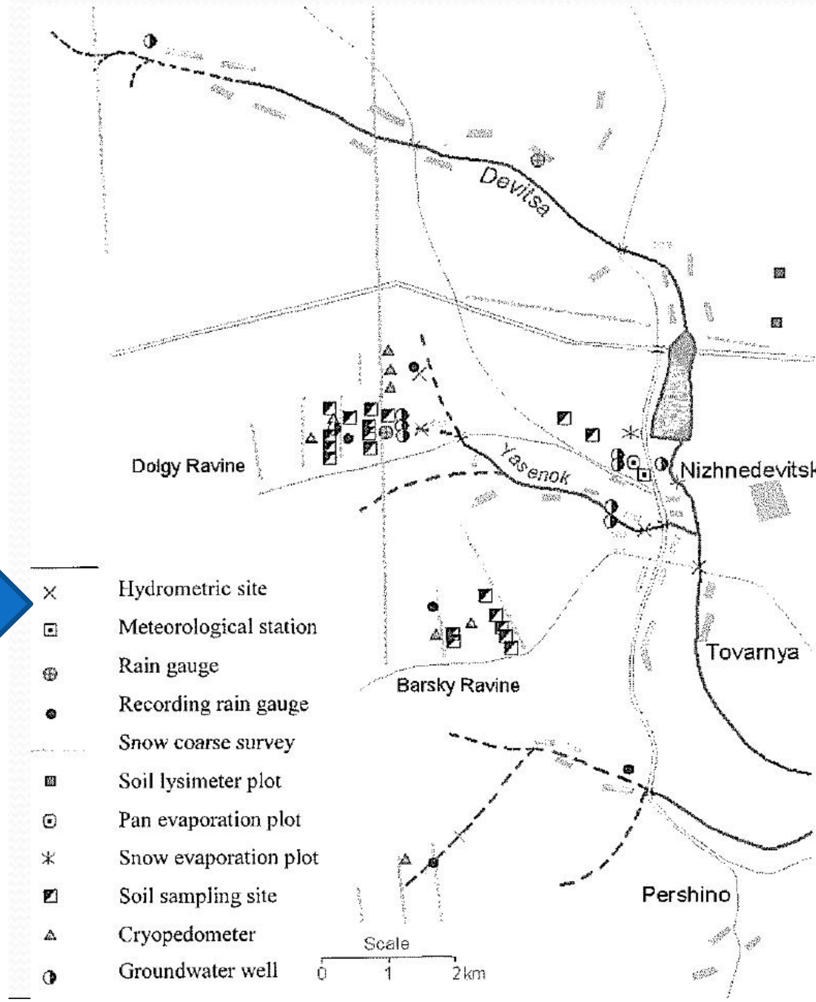
Gelfan A.N. (2010) Extreme snowmelt floods: frequency assessment and analysis of genesis on the basis of the dynamic-stochastic approach. J. Hydrology, 388, 85-99

Gelfan A. N. (2006) Physically based model of heat and water transfer in frozen soil and its parametrization by basic soil data. IAHS Publ., 303, pp. 293-304

Case Study

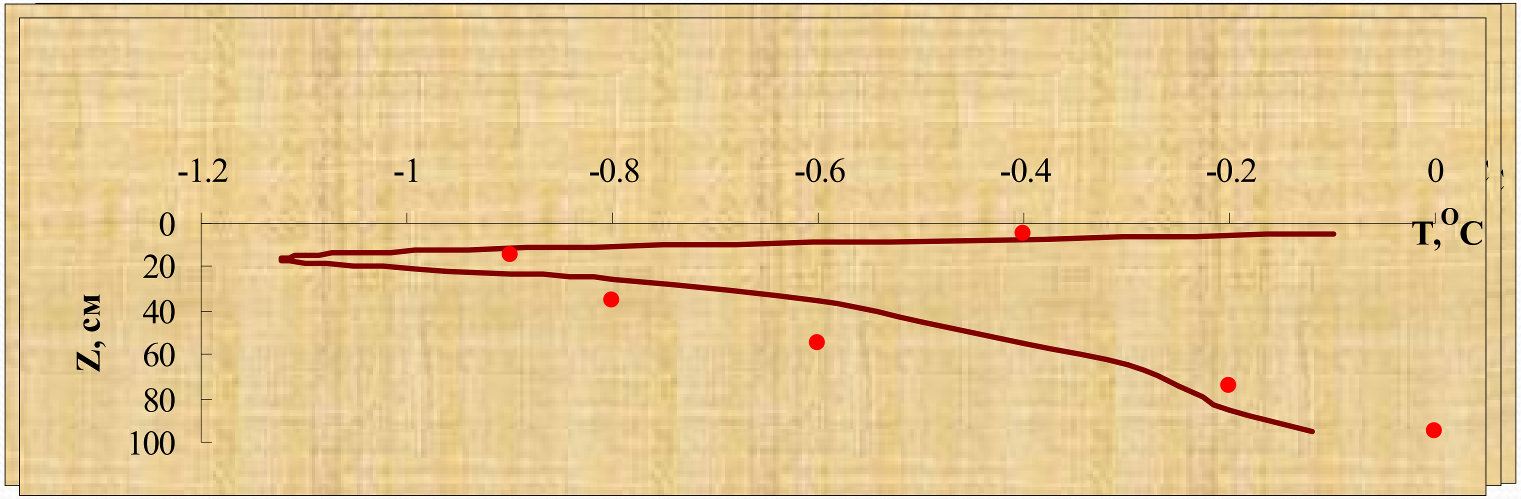
Nizhnedevitskaya water balance station (51°31′N; 38°23′E) is located in the upper part of the Devitsa River basin draining east into the Don River. Relief is flat and the dominant soils are chernozems with some podzol. The bottom water-bearing horizon of 25-30 m depth is the main aquifer



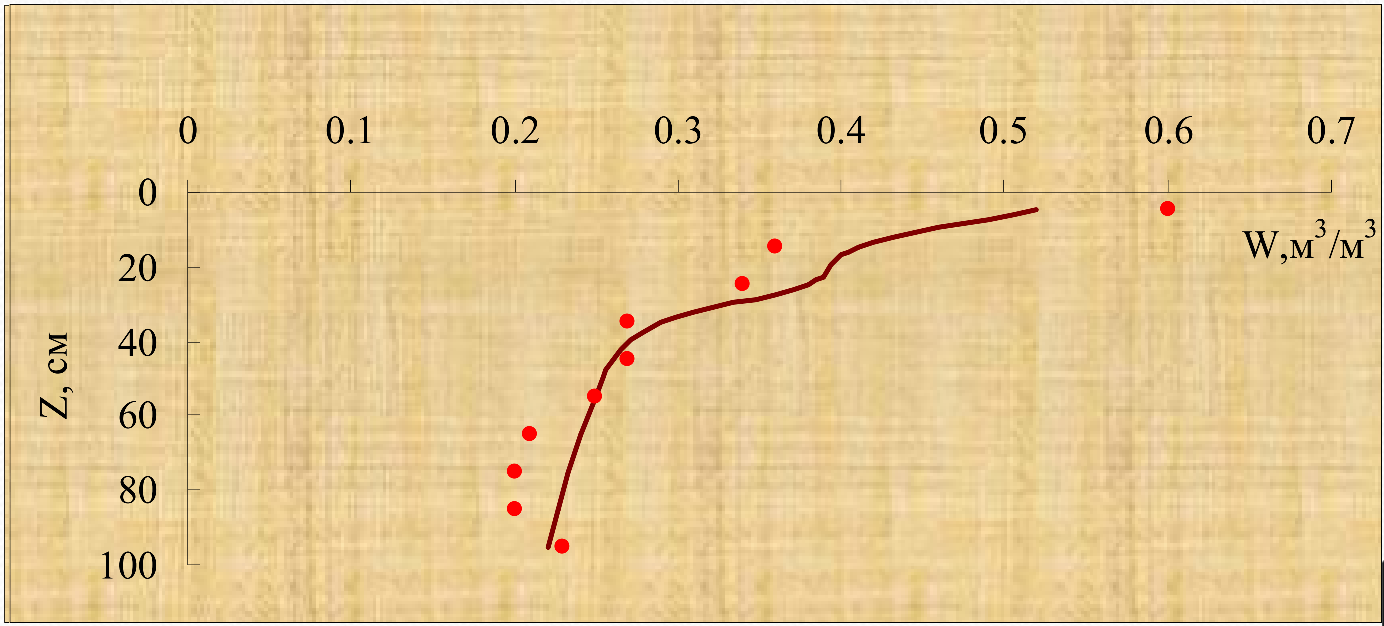


Examples of the model application

Calculated and measured profiles of soil temperature (snowmelt period; spring of 1981 г.)

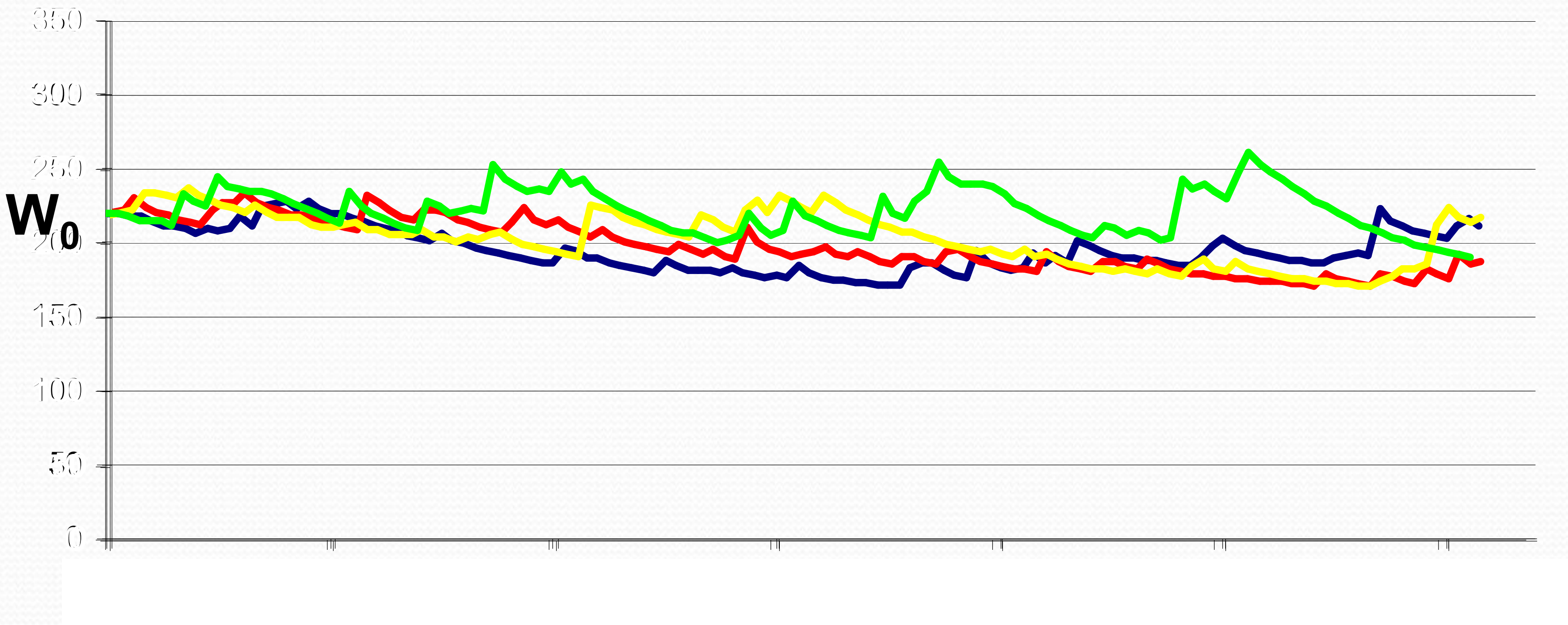


Calculated and measured profiles of soil moisture (snowmelt period; spring of 1981 г.)

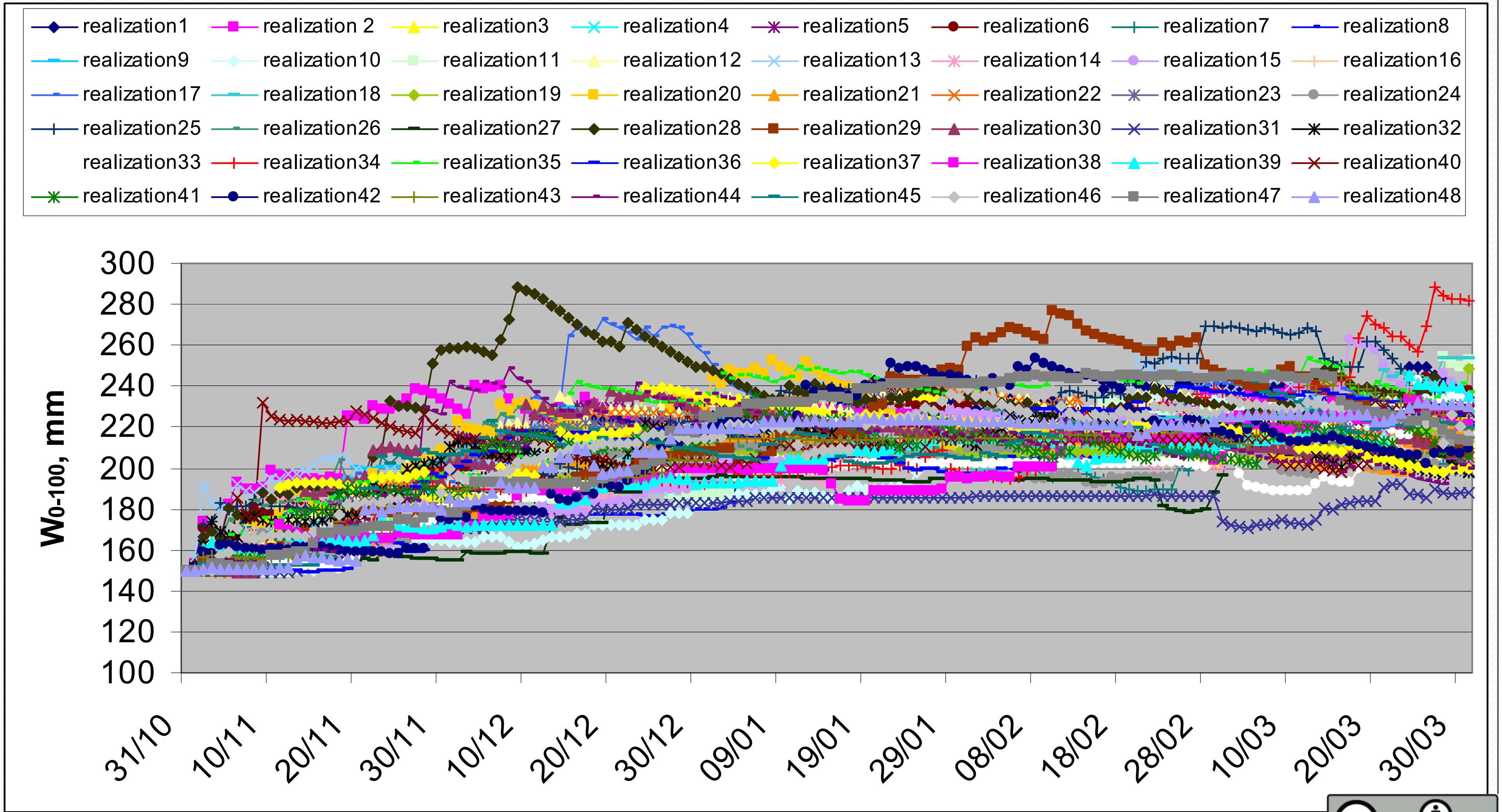


Numerical experiments: design and results

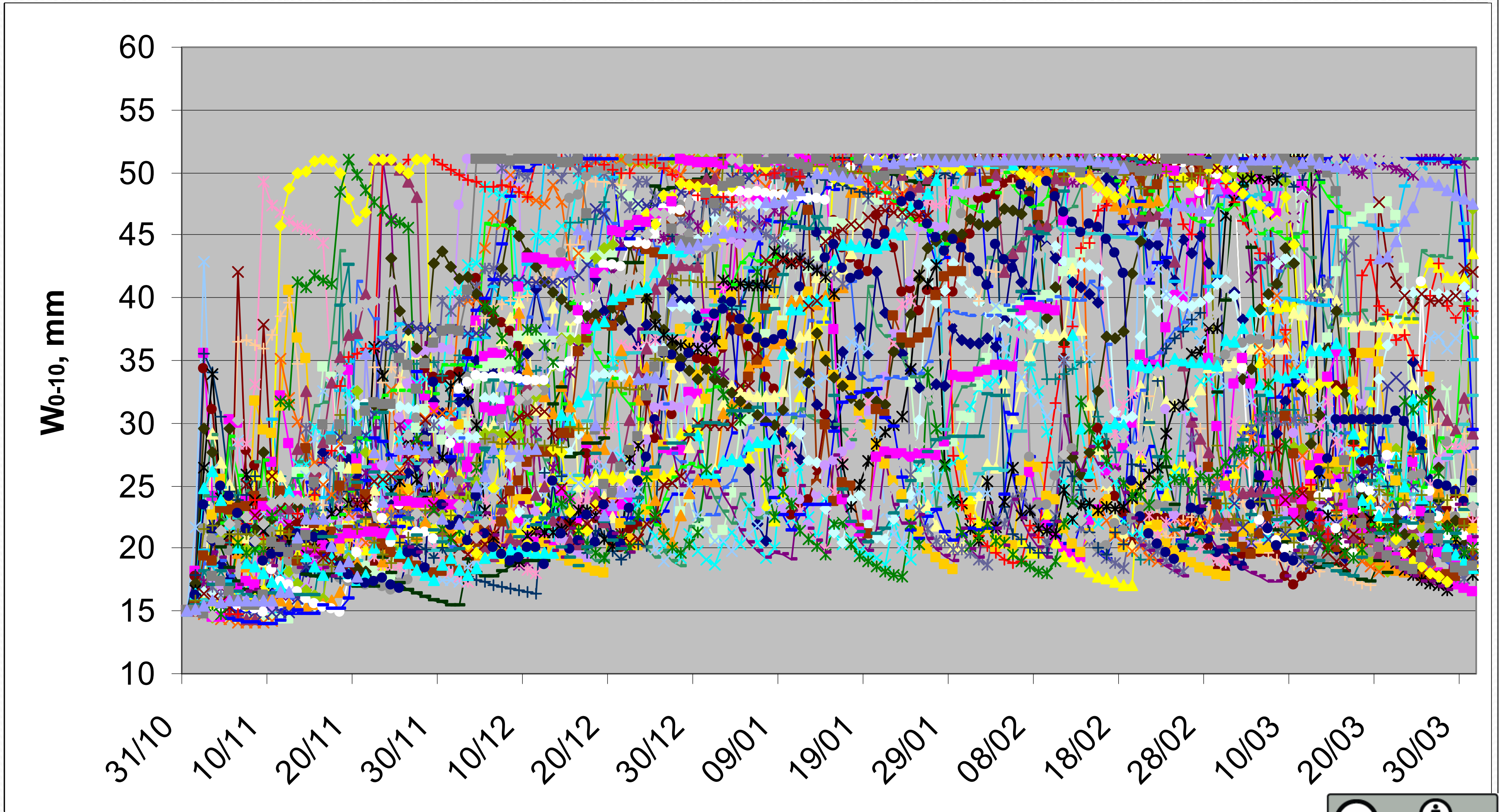
1. Ensemble of thousand 4-month meteorological scenarios is Monte-Carlo generated and used as input into the deterministic model. Output is the ensemble of 1000 trajectories of soil moisture characteristics (water content of soil column, moisture of different soil layers)



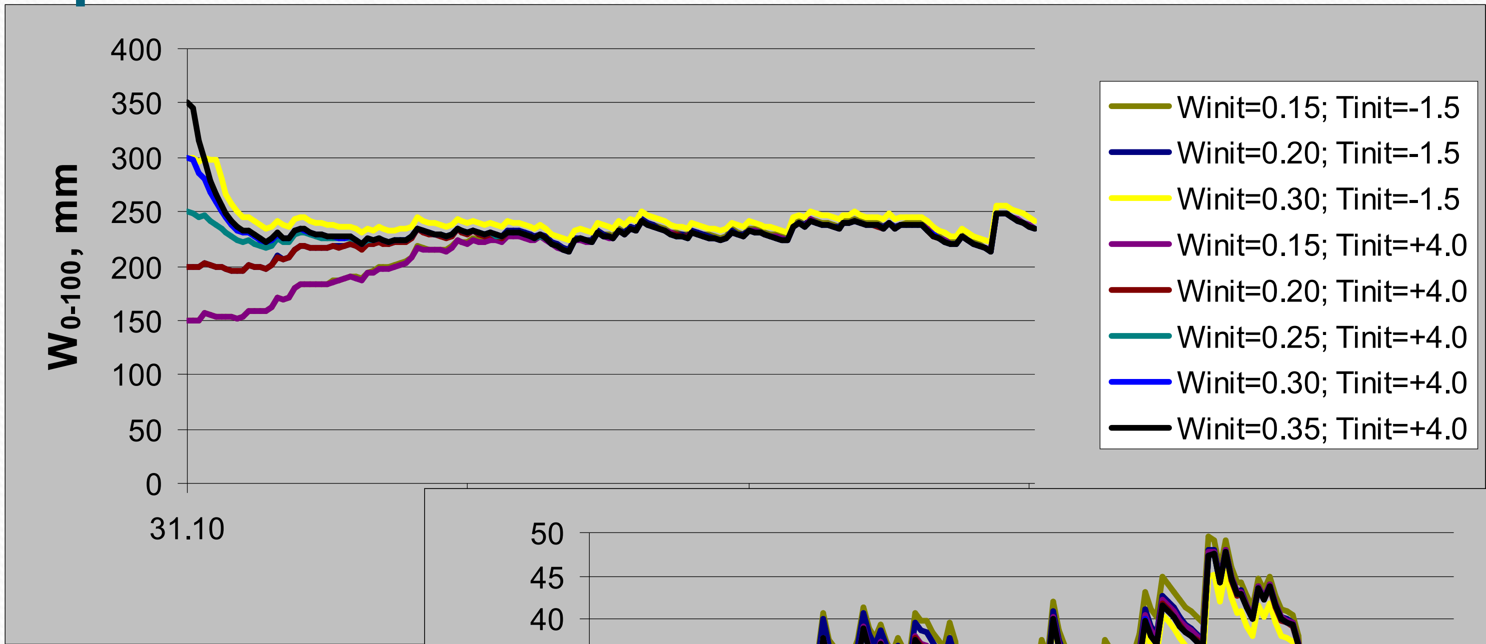
Example: Changes of soil water content (0-100 cm) simulated under 50 Monte-Carlo generated meteorological scenarios (W_0=W(z,0)=0.15)

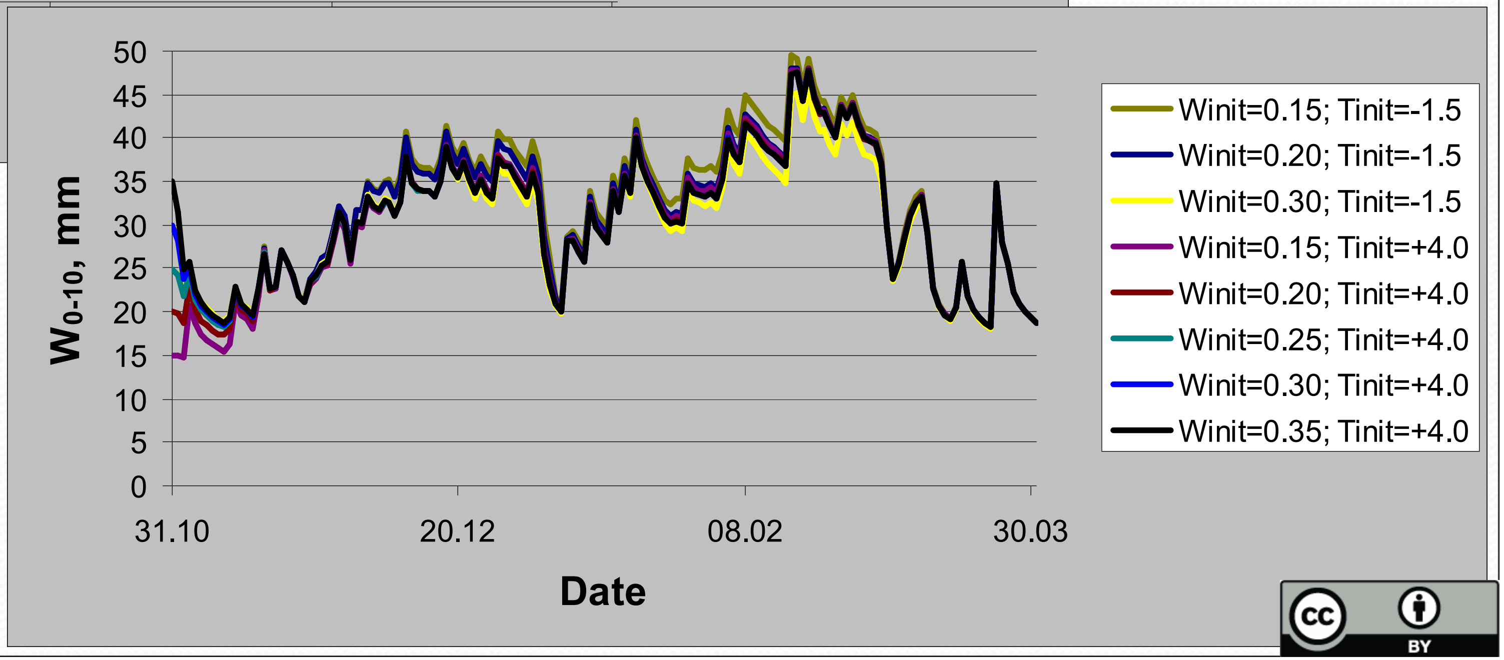


Example: Changes of soil water content (0-10 cm) simulated under 50 Monte-Carlo generated meteorological scenarios ($W_0=W(z,0)=0.15$)

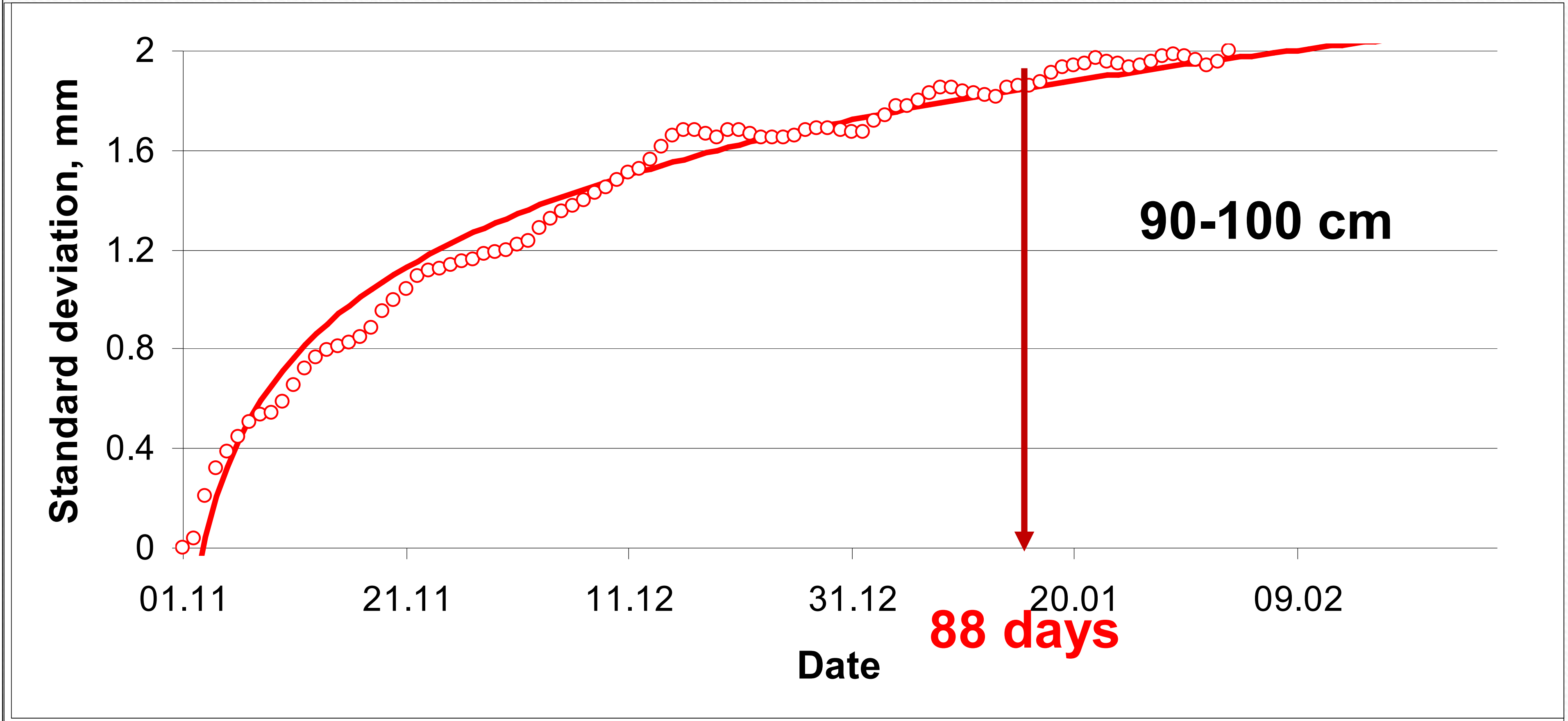


2. Sensitivity to perturbation in the initial condition
Soil moisture trajectories are simulated beginning from the different initial conditions and under the same meteorological input scenario

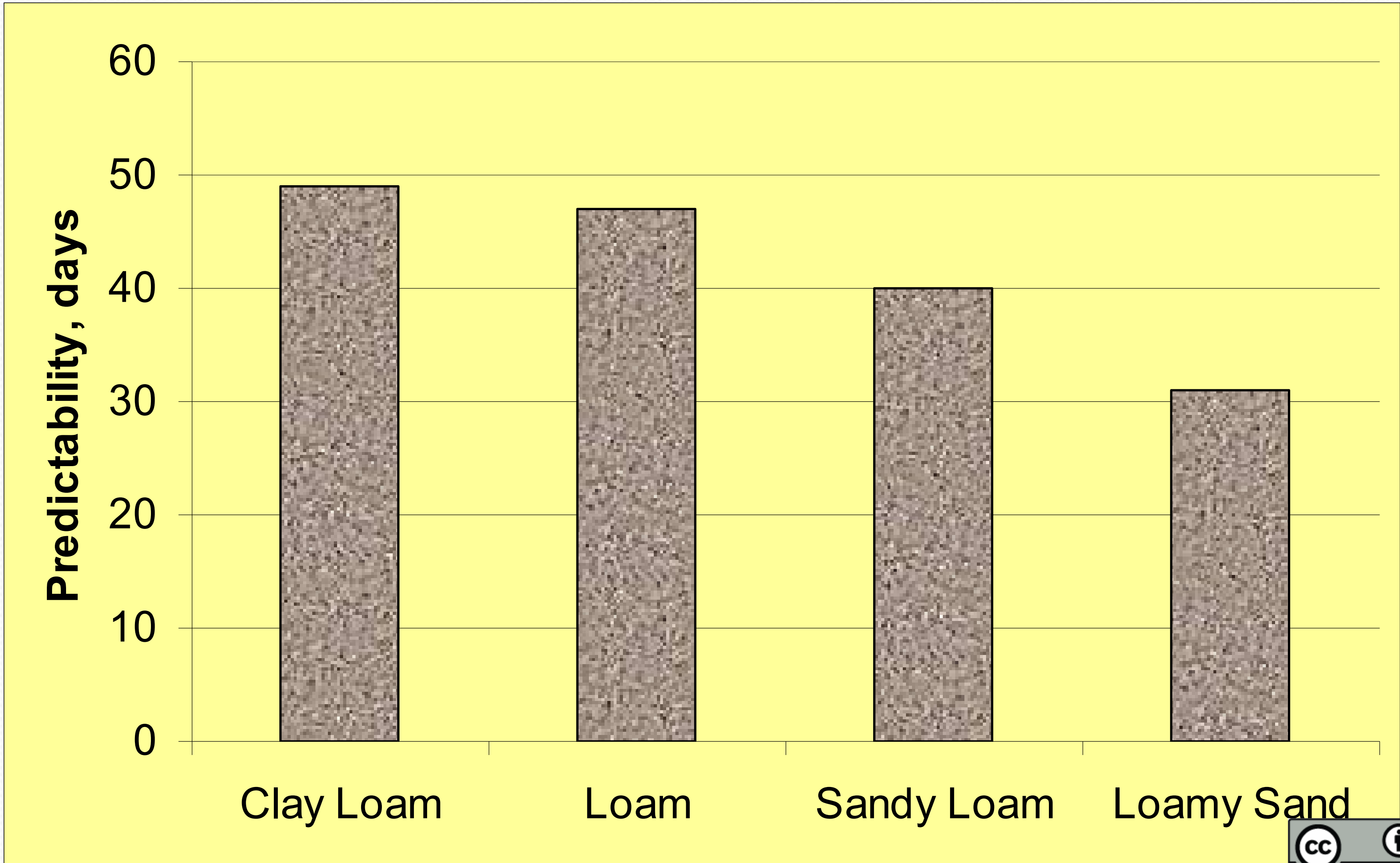




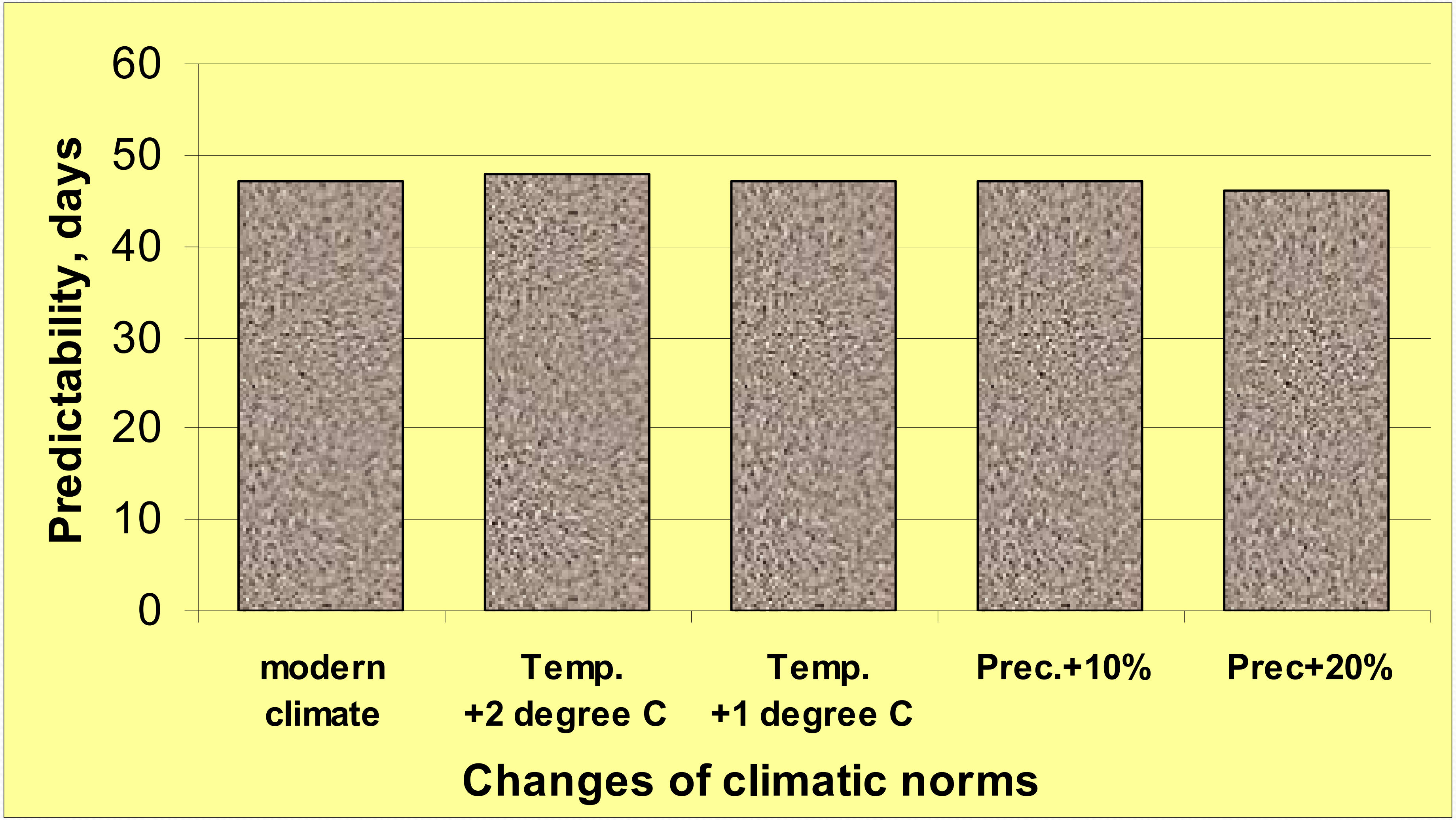
3. Assessing predictability of moisture content in the different soil layers (Sandy Loam)



Predictability of soil moisture content in dependence on the soil texture (layer of 0-100 cm)



Predictability of soil moisture content under the different climatic norms of air temperature and precipitation



Conclusions

An approach to assessing potential predictability through a procedure of convergence of the system state probabilistic measure (variance) to its stable value has been proposed and applied to moisture content of frozen soil

Numerical experiments with the dynamic-stochastic model of hydrothermal regime of frozen soil has been shown that in the steppe physiographic conditions:

- ***soil moisture dynamics are slightly sensitive to perturbation of the initial soil water content and temperature***
- ***predictability of soil moisture increases with increasing thickness of soil layer and depth of the layer***
- ***predictability of soil moisture decreases when soil texture is becoming more coarser***
- ***predictabaility is much more sensitive to changes of soil properties than to climatic changes of mean seasonal air temperature and precipitation***

The presented study is contribution of the Working Group “Physics of Hydrological Predictability” to the Panta Rhei Research Initiative of IAHS.

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Title of the Working Group

Physics of Hydrological Predictability

Abstract of the proposed research activity

The main objective of the Working Group (WG) is to advance our understanding interconnection of predictability aspects of hydrological, weather and climate components of the Earth System.

WG science questions include:

1. What are the predictability limits of different water cycle processes and what metrics can be

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