

Introduction

Space-geodetic techniques, e.g., Lunar Laser Ranging (LLR), rely on a hierarchy of consistent reference systems. For instance, the Barycentric Celestial Reference System (BCRS) is essential in geodesy through its intimate relation to the Geocentric Celestial Reference System (GCRS). Barycentric ephemerides represent a dynamical realization of the BCRS. In accordance with existing renowned ephemerides (DE, INPOP, EPM), we want to lay the foundation of a new solar-system ephemeris that can beneficially be applied in LLR analysis, for example. Conversely, better knowledge of the Moon enables an improved ephemeris force modeling. So far, our existing LLR analysis software package takes only a limited number of solar system bodies into account. Thus, in a very first step, we refine the dynamical model by the inclusion of many more asteroids. In this respect, we make use of related studies on the significance of certain bodies, carried out by other groups (at JPL, IMCCE, IPA). We present the effect of this refinement on selected derived quantities like helio-/geocentric distances of the major solar-system bodies, and its classical Keplerian elements, in comparison to selected third-party planetary solutions (DE430, INPOP10a).

Dynamical Model

In essence, the dynamical model consists of mutual gravitational interactions between 11 major solar system bodies (Sun, Moon, planets with Pluto) plus a selection of 343 asteroids (same set as used in DE430). The classical Newtonian attraction is supplemented with a considerable number of terms that account for higher order effects, cf. eqs. (1), (2), e.g., relativity via eq. (3). Another example are the figure-figure interactions, especially in the Earth-Moon system, which uses a lunar libration model. The whole system is integrated simultaneously.

$$\ddot{\mathbf{r}}_A = \sum_{B \neq A} \frac{\mu_B}{r_{AB}^3} (\mathbf{r}_B - \mathbf{r}_A) + \sum_{\alpha} \Delta \ddot{\mathbf{r}}_A^{\alpha} \quad (1) \quad \sum_{\alpha} \Delta \ddot{\mathbf{r}}_A^{\alpha} = \Delta \ddot{\mathbf{r}}_A^{\text{rel}} + \Delta \ddot{\mathbf{r}}_A^{\text{fig}} + \dots \quad (2)$$

$$\Delta \ddot{\mathbf{r}}_A^{\text{rel}} = \frac{1}{c^2} \sum_{B \neq A} \frac{\mu_B}{r_{AB}^3} (\mathbf{r}_B - \mathbf{r}_A) \left(-2(\beta + \gamma) \sum_{C \neq A} \frac{\mu_C}{r_{AC}} - (2\beta - 1) \sum_{C \neq B} \frac{\mu_C}{r_{BC}} + \gamma \|\mathbf{v}_A\|^2 \right) + (1 + \gamma) \|\mathbf{v}_B\|^2 - 2(1 + \gamma) \mathbf{v}_A \cdot \mathbf{v}_B - \frac{3}{2} \left(\frac{(\mathbf{r}_A - \mathbf{r}_B) \cdot \mathbf{v}_B}{r_{AB}} \right)^2 + \frac{1}{2} \sum_{C \neq B} \frac{\mu_C}{r_{BC}^3} (\mathbf{r}_C - \mathbf{r}_B) \cdot (\mathbf{r}_B - \mathbf{r}_A) + \frac{1}{c^2} \sum_{B \neq A} \frac{\mu_B}{r_{AB}^3} (\mathbf{v}_A - \mathbf{v}_B) \cdot (\mathbf{r}_A - \mathbf{r}_B) \cdot ((2 + 2\gamma)\mathbf{v}_A - (1 + 2\gamma)\mathbf{v}_B) + \frac{3 + 4\gamma}{2c^2} \sum_{B \neq A} \sum_{C \neq B} \frac{\mu_B \mu_C}{r_{AB} r_{BC}} (\mathbf{r}_C - \mathbf{r}_B) \cdot (\mathbf{r}_A - \mathbf{r}_B) \quad (3)$$

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Numerical Integration of an Initial Value Problem

The integrator itself is an initial value solver of Shampine and Gordon, based on a variable order variable step size Adams-Bashfort-Moulton method. Initial epochs were chosen to be either $t_0 = \text{July 28th 1969}$, or half a year later, a time at which LLR measurement campaigns started. For all test cases, the arclength was set to a few tens of years, between 30 and 50 years. Any computations were performed in quarter precision.

Comparison with other Ephemerides

The effect of any changes in our own dynamical model was studied via comparisons to the planetary solutions DE430, INPOP10a, and our own solution versions, respectively. All checks were performed either directly w.r.t. the major bodies' resulting state vectors and corresponding Keplerian elements, or via derived quantities like geocentric planetary distances. The latter are of greater interest, especially in view of subsequent adjustments to Earth-based planetary observations. For the DE430 comparisons, we applied identical initial conditions to our own numerical integrations, and model parameters or assumptions as close as possible to their documentation. Regarding a LLR application of ephemerides, the Earth-Moon distance r and its change $\delta r = r_{i+1} - r_i$ are major quality measures. The "old" IFE solution is already in good agreement with other ephemerides, cf. Fig. 2. Differences Δ in δr do not exceed the cm-level over a few decades.

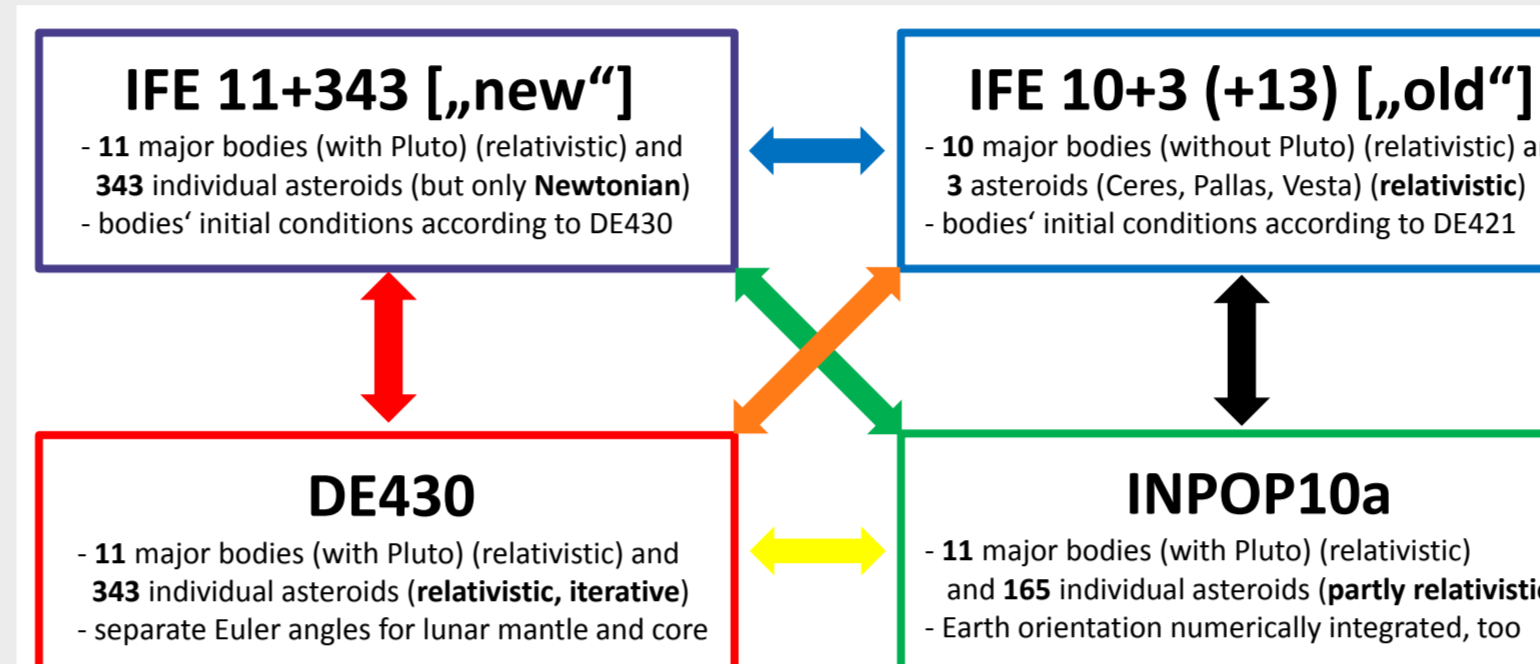


Fig. 1: Color-coding for the cross-comparison of planetary solutions (see Fig. 2).

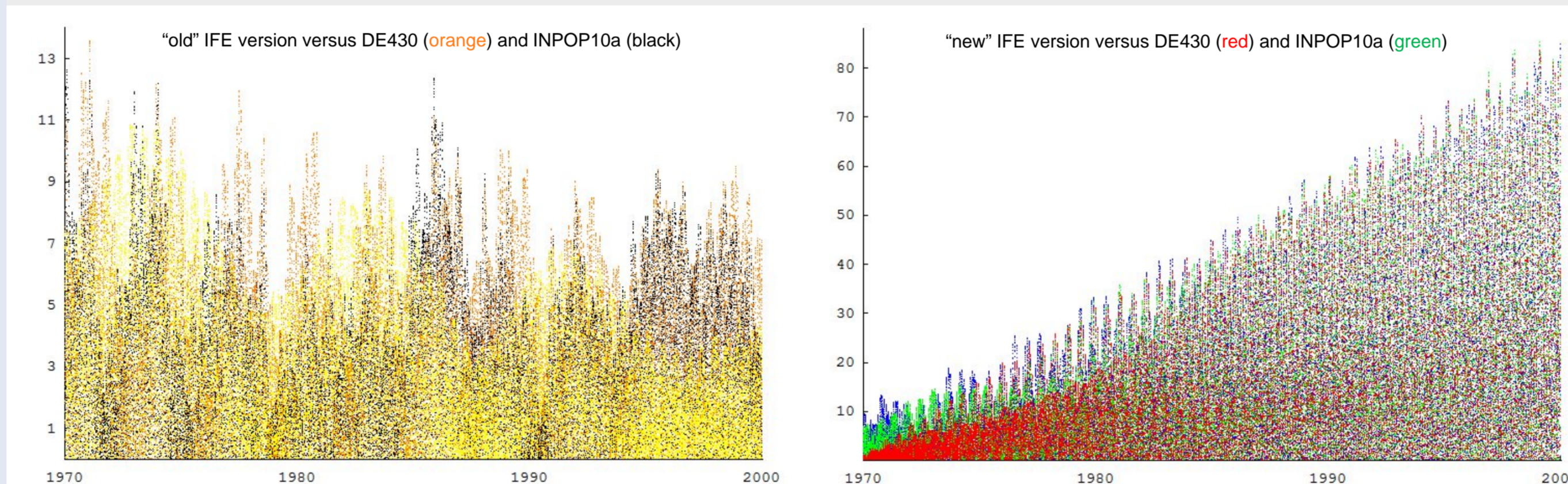


Fig. 2: Differences in the change of the Earth-Moon distance $[\Delta(\delta r)]$ in mm (dynamical model of various IFE versions versus final planetary solutions DE430 and INPOP10a, color-coding see Fig. 1). For LLR, a careful adjustment of the initial conditions and model parameters to the observations, as well as retaining more relativistic interactions are seemingly more important than adding a larger number of minor asteroids.

The significance of a careful asteroid modeling for the orbits of selected planets is depicted in figures 3 and 4. We obtained improvements by orders of magnitude. IFE11+16 implies 16 asteroids (but also relativistic).

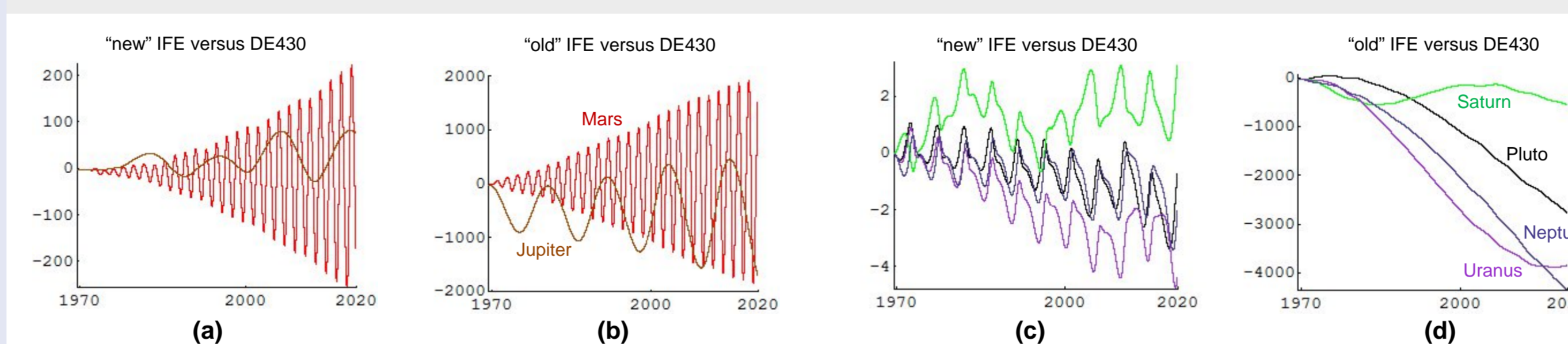


Fig. 3: $\Delta r_{\text{Sun Planet}}$ in m for IFE11+343 (subfigures (a), (c)) or IFE11+16 (subfigures (b), (d)) versus DE430, for Mars, Jupiter, Saturn, Uranus, Neptune, Pluto.

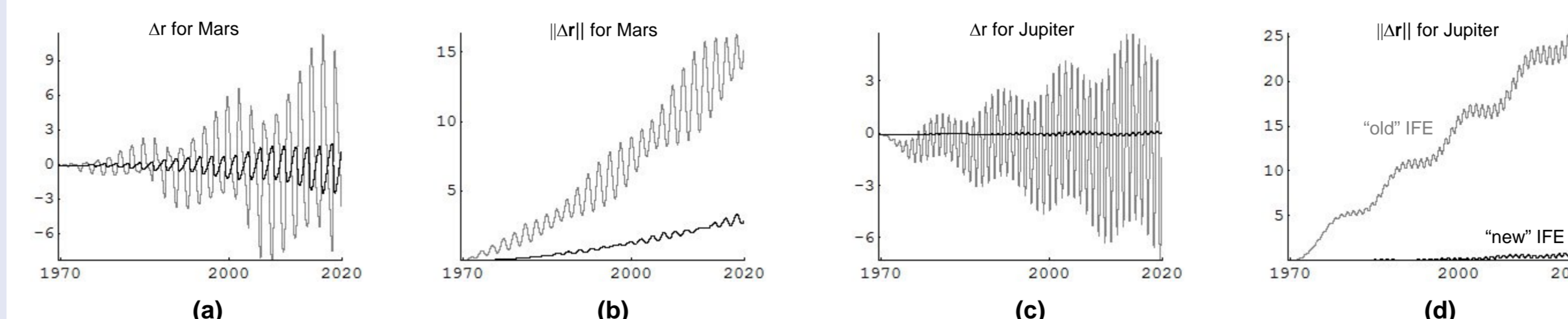


Fig. 4: Δr_{Planet} ((a), (c)) and $\|\Delta r_{\text{Planet}}\|$ ((b), (d)) in km for IFE11+343 or IFE11+16 (gray) versus DE430, for Mars ((a), (b)) and Jupiter ((c), (d)).

Data Collection

The construction of an ephemeris comprises regular checks on the truth via planetary observations. We collected any kind of such data (e.g., historical optical transit observations, cf. Fig. 5) that are public available. Today, a huge amount of very precise tracking data of planetary orbiters or landers enables a reliable verification of the planetary positions.

In future, with the successful launch of the GAIA spacecraft, dedicated to astrometry, an even more precise direct localization of solar-system objects, will become available, e.g., for hundred thousands of asteroids.

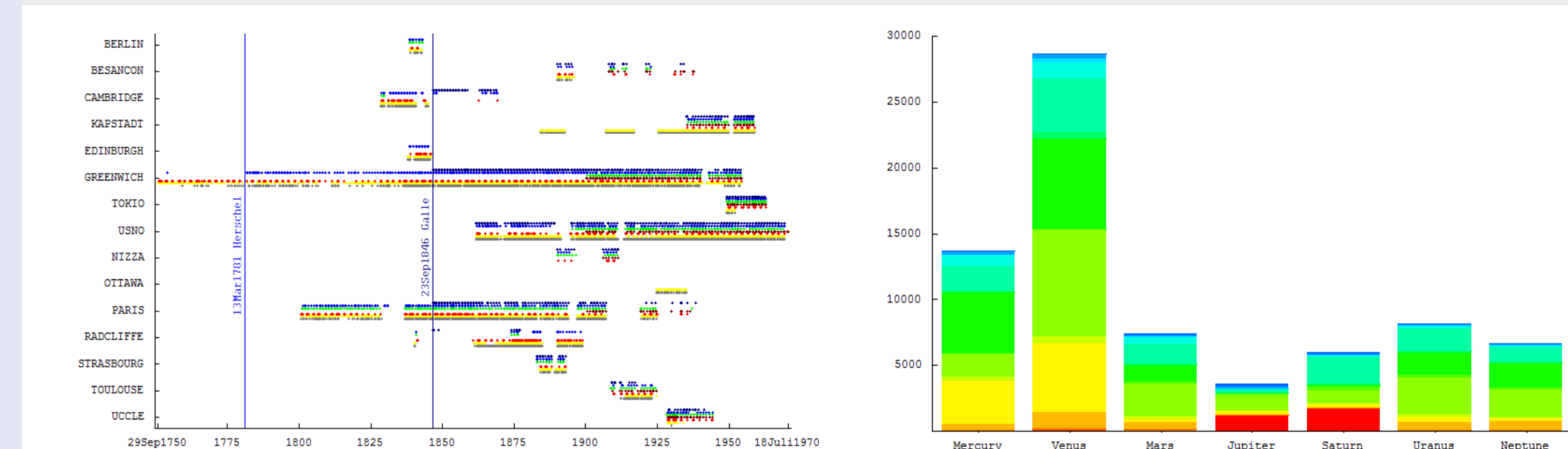


Fig. 5: Example for available records on optical planetary transit observations. Temporal distribution (left) with color-coded planets (from Mercury in gray to Neptune in dark blue), and absolute frequency (right) with color-coded astronomical observatories (e.g., USNO in red).

Outlook

The realization of a truly ephemeris requires many additional steps, e.g.,

- refinement of the dynamical model: simultaneous integration of the ODE for TT-TDB, taking into account additional minor bodies in the solar system (vast number of smaller asteroids via ring model(s), and possibly a few trans-Neptunian objects), employment of the latest lunar libration model, etc.,
- introduction of all kinds of available planetary observations in combination with consistent data reduction models,
- parameter estimation with appropriate data weighting schemes,
- enhancement of the efficiency of the numerical computation.

The last issue is important as we intend to apply new strategies for parameter estimation apart from classical least-squares adjustments.

References

Müller et al. (2014) Lunar Laser Ranging and Relativity. In: Frontiers in Relativistic Celestial Mechanics – Applications and Experiments. Sergei M. Kopeikin (Ed.), De Gruyter, pp.103-156, in press.

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