Methods for the Analysis of interferometric Time Series Non-linearity

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Interferometric stacking techniques emerged as methods to obtain very precise measurements of small terrain displacements. In particular, the so-called Persistent Scatterers and Small BASeline methods can be considered as the two most representative stacking approaches. In both cases, the exploitation of 20 or more satellite Synthetic Aperture Radar (SAR) acquisitions obtained from the same satellite sensor with similar geometries on the interest area allows to measure average displacement rates with an accuracy in the order of few mm / year, and to derive the full location history of "good" pixels with an accuracy of 1cm or better for every available date.

Although the temporal component of these measurements provides very rich information to investigate the evolution of complex phenomena, this wealth of data can result of difficult interpretation as soon as the area of investigation reaches certain sizes and several millions of valid pixels can be identified. The typical approach is then to focus the analysis on the average displacement rate: one evident advantage is that it can be easily displayed, and regions showing different average behaviours can be easily identified with a simple visual analysis. Limitations of this approach become evident as soon as more complex, non-linear behaviours are to be expected (as natural) in a certain region, and different methods shall be sought to provide a synthetic way to visualise the time series in a synoptic way and to identify areas with similar, non-linear characteristics.

The poster focus on the identification of which could be descriptive parameter(s) that, complementarily to the average displacement rate, could be synthesized from the displacement time series and exploited in this analysis. While asking this it shall be noticed that this approach is of particular applicability to time series obtained with the SBAS method that, due to its algorithm, is less depending on linearity assumptions than the PS method.

A first, simple but very interesting parameter can be identified in the chi-square value, as measure of the goodness of fit of the time series. Let's consider for example a linear fit: the chi-square value will easily distinguish points for which the selected model is appropriate, characterised by a low chi-square value and for which the average displacement rate well represents the temporal behaviour of the time series, from points showing large chi-square values, hence not properly described by a simple linear model. It shall be noticed that a typical standard PS analysis will not identify such points as valid pixels, hence providing no measurement.
The fit can be repeated for pixels showing large chi-square value by, for example, iteratively increasing the order of the fitted polynomial up to a certain reasonable value, identifying if the time series can be well described by a polynomial function, or if other non-linear behaviours (e.g. periodic, seasonal) shall be expected.
In case of good higher-order polynomial fit, parameters like the average acceleration and the curvature sign can be of interest to describe the type of motion, together with the best-fit order (e.g. sign of an acceleration / deceleration for second order, more step-like for third order etc.).
Finally, the chi-square value can be compared with the a-priori displacement standard deviation estimated from the pixel interferometric coherence, to identify if the pixel-specific selected fit model can extract all information intrinsically available in the data.

Classifications schemes can be proposed based on the various parameters to provide a single layer that could identify at a glance areas characterised by different temporal regimes.

This type of analysis can become more and more important with the availability of long and dense time series, as in the forthcoming case of Sentinel-1.

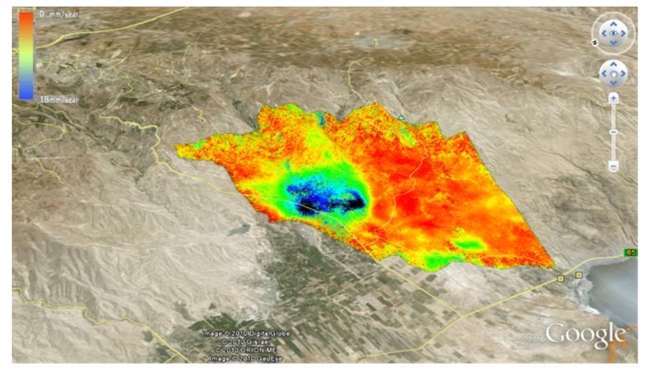
$$\chi^2 = \sum_{\forall i} \left(R_i - \hat{R}_i\right)^2$$

$$\sigma_{\Delta R_i}{}^2 = \left(\frac{\lambda}{4\pi}\right)^2 \frac{1-\gamma^2{}_i}{2\gamma^2{}_i}$$

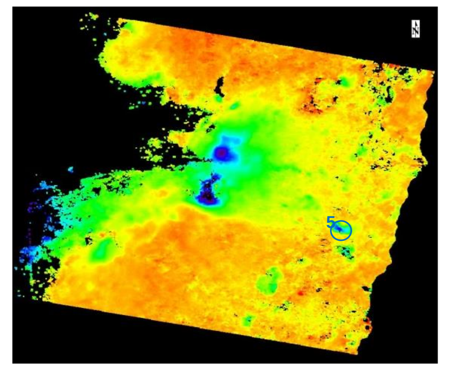
$$\sigma_{\hat{V}}{}^2 = \frac{\sum_{\forall i}\left(\frac{1}{\Delta T_i}\sigma_{\Delta R_i}{}^2\right)}{\sum_{\forall i}\frac{1}{\Delta T_i}}$$

$$\lim_{model \to correct} \frac{\chi^2}{T_{max} - T_{min}} = \sigma_{\hat{V}}{}^2$$

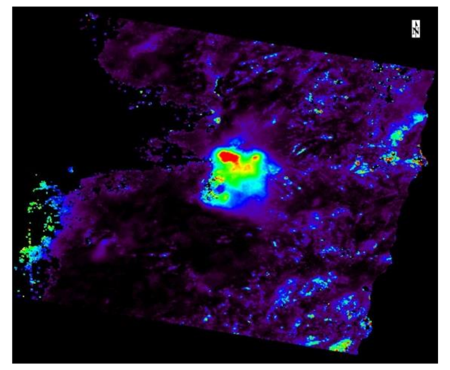
The Madaba (Jordan) case – water extraction



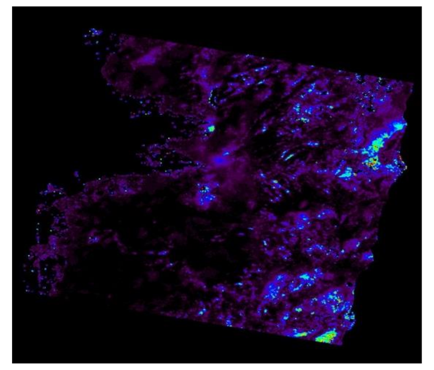
Average displacement rate [mm/year]
between 2003 and 2010 with SBAS
exploiting 31 Envisat ASAR acquisitions



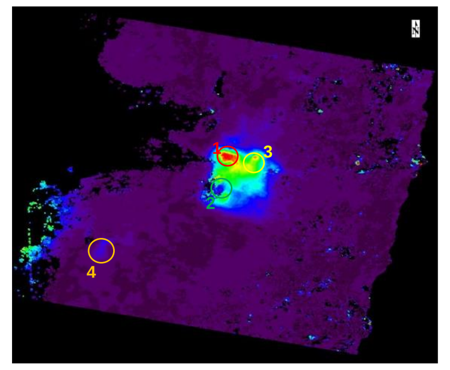
Average displacement rate from linear model



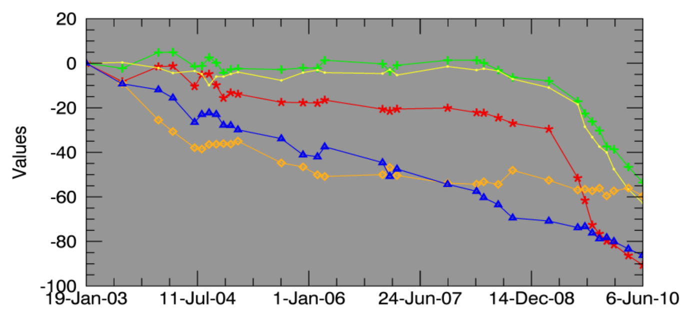
χ2 of linear model



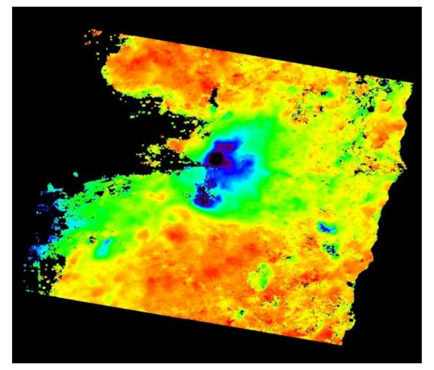
χ2 of cubic model



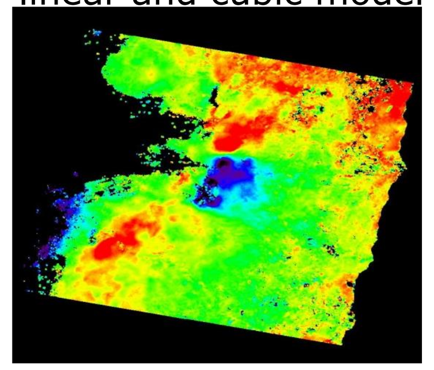
Difference of χ2 between linear and cubic model



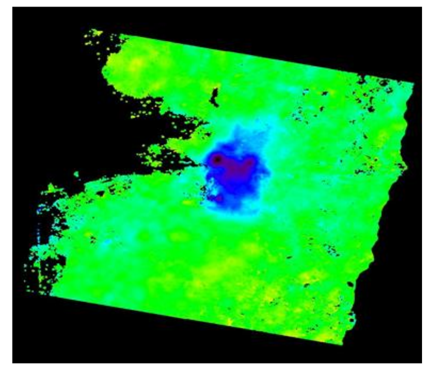
Displacement time series in [mm] from areas 1 to 5



Average displacement rate from cubic model

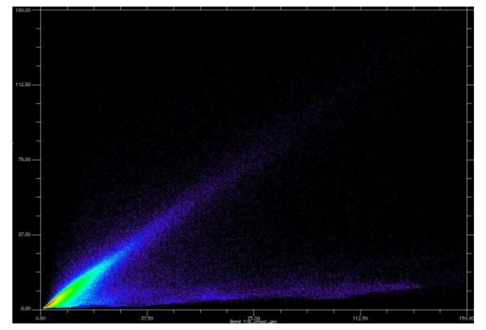


Average acceleration rate from cubic model

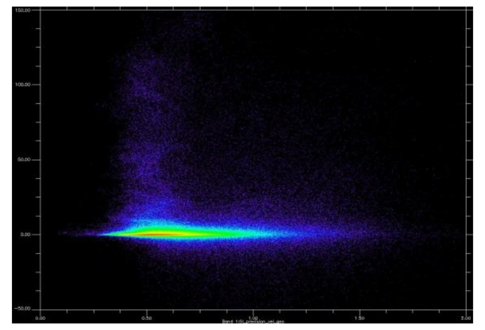


Acceleration variation from cubic model

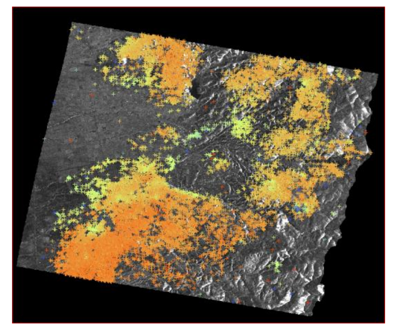
Different from the coefficients of th 3rd order polynomial!



χ2 from linear model against χ2 from cubic model

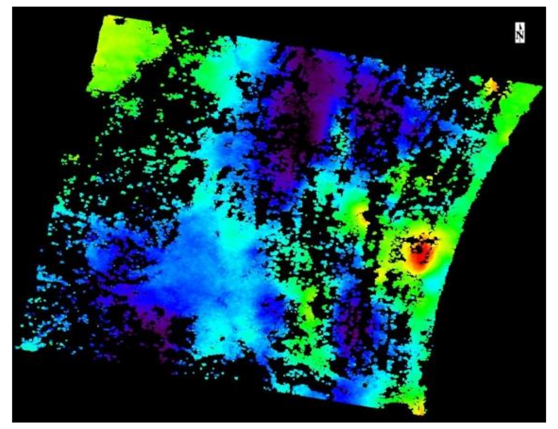


Expected displacement rate precision against χ2 decrease from linear to cubic model

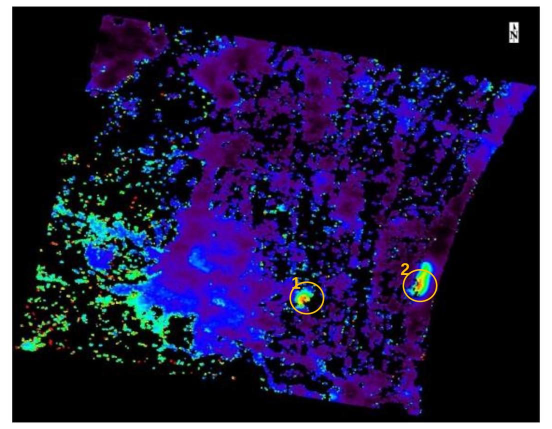


PS average displacement rate

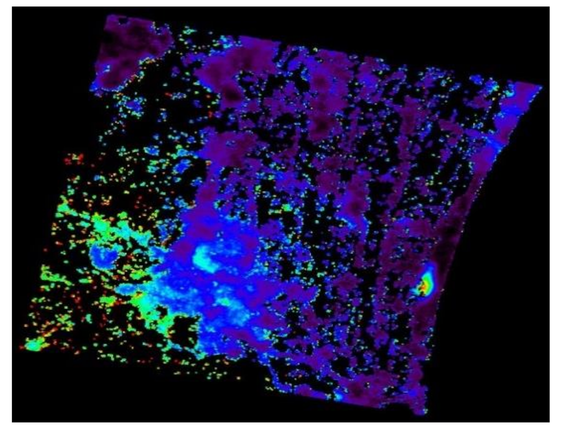
The Togane (Japan) case – gas reservoir exploitation



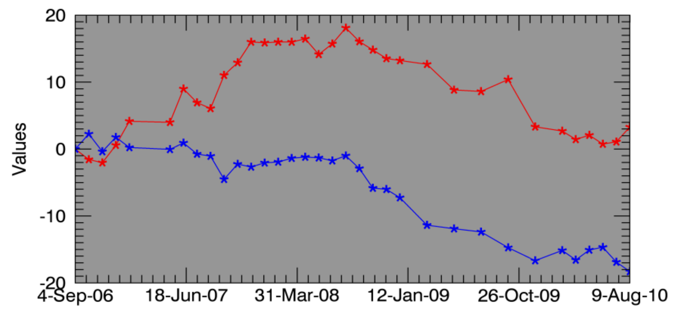
Average displacement rate [mm/year]
between 2006 and 2010 with SBAS exploiting 34 Envisat ASAR acquisitions



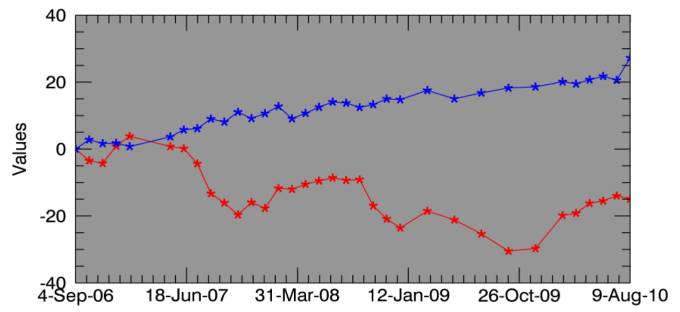
χ2 of linear model



χ2 of cubic model

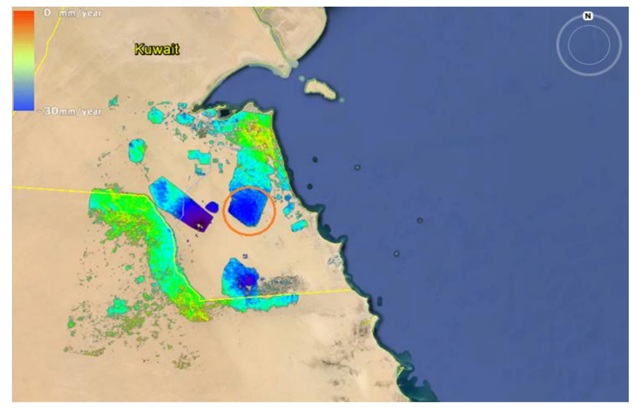


Displacement time series in [mm] from area 1

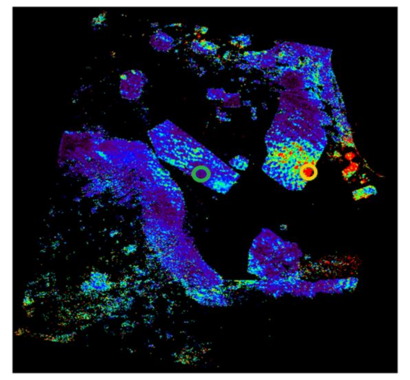


Displacement time series in [mm] from area 2

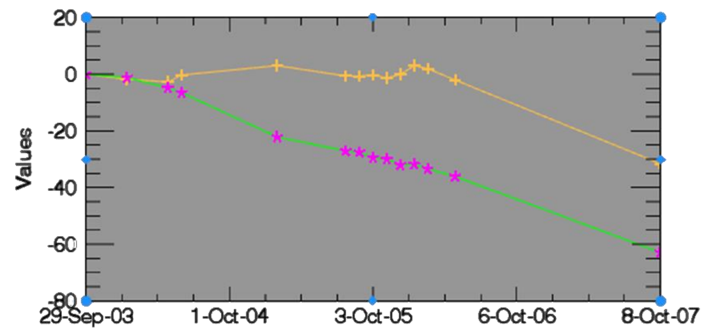
The Al-Ahmadi (Kuwait) case – oil reservoir exploitation



Average displacement rate [mm/year]
between 2003 and 2007 with SBAS
exploiting 14 Envisat ASAR acquisitions.



χ2 of linear model



Displacement time series in [mm] from areas 1 to 5