Pattern formation throught spatial interactions in a modified Daisyworld model

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1. Introduction

According to the Daisyworld model, the interactions between an Earth-like planet, its biosphere and the solar incoming radiation are enough to self-regulate the global climate on the planet, thus maintaining the conditions for life, even for relatively small variations of the incoming radiation. Here we investigate a modified version of the Daisyworld model where a spatial dependency on latitude is introduced, and both a variable heat diffusivity along latitudes and a simple greenhouse model are included. We show that the spatial interactions between the variables of the system can generate some equilibrium dynamics which can locally stabilize the coexistence of the two vegetation types. The feedback on albedo is able to generate new equilibrium solutions which can efficiently self-regulate the planet climate, even for values of the solar luminosity relatively far from the current Earth conditions.

2. The spatial dependence in Daisyworld model

The model, in its essence [1,2], is based on a hypothetical planet, like the Earth, which receives the radiant energy coming from a Sun-like star, and populated by two kinds of identical plants differing by their colour: white daisies reflecting light and black daisies absorbing light.
[3] The extension to spatial Daisyworld gives room to the possibility of inhomogeneous solar forcing in a curved planet, with explicit differences between poles and equator and the direct use of the heat diffusion equation.

1. To describe a spherical planet, we consider the temperature $T(\theta, t)$ and the surface coverage as depending only on time and on latitude θ $(-90^\circ \leq \theta \leq 90^\circ)$
2. We also consider the greenhouse effect in the model, the process by which outgoing infrared radiation is partly screened by greenhouse gases, which can be described by relaxing the black-body radiation hypothesis and by considering a grayness function $g(T)$ [4]

$$g(T) = 1 - \frac{1}{2}\tanh\left(\frac{T}{T_0}\right)^6$$

3. Finally, we consider a latitude dependence of the Earth's conductivity, $\chi = \chi(\theta)$

Considering these terms, using spherical coordinates and symmetry with respect to θ, the modified Daisyworld equations reduce to the following set of equations

$$\frac{\partial \alpha_w}{\partial t} = \alpha_w\left[(1-\alpha_w-\alpha_b)\beta(T)-\gamma\right] \quad (1)$$

$$\frac{\partial \alpha_b}{\partial t} = \alpha_b\left[(1-\alpha_w-\alpha_b)\beta(T)-\gamma\right] \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[1-A(\theta,t)\right]R(\theta) - \frac{\sigma}{\rho c_p} g(T)T^4 + \frac{1}{r_E^2\cos\theta}\frac{\partial}{\partial\theta}\left[\kappa(\theta)\cos\theta\frac{\partial T}{\partial\theta}\right] \quad (3)$$

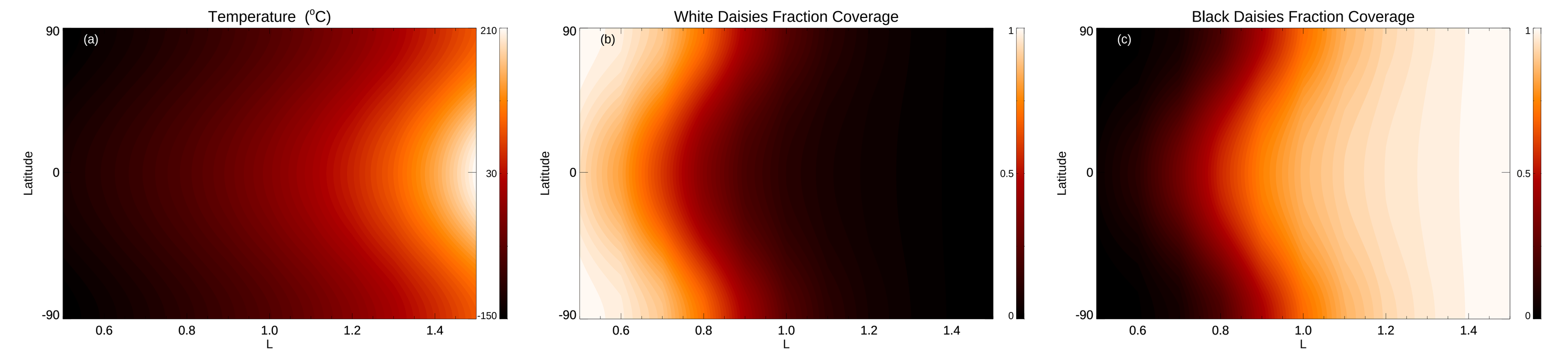
where $\alpha_{w,b}$ are the daisy coverages of each species, $\beta(T)$ and γ are the growth rate and the death rate per unit of time of daisies respectively, $A(\theta, t)$ is the albedo of the Earth, $R(\theta)$ describes the incident radiation, ρ is mass density of the atmosphere, c_P is the heat capacity, $\kappa = \chi/\rho c_p$, r_E is the Earth's radius and in which we use the expression of the Laplace operator in spherical coordinates.
Eq.s (1-3) have been solved numerically by using a second order Runge-Kutta scheme for time integration and spectral methods for integration on latitude θ. The poles ($\theta = \pm 90^\circ$) are singular points for the Laplace operator in spherical coordinates. There we assume a free-flux boundary condition, corresponding to zero derivatives for all variables. In this way the continuity of the laplacian in such points is ensured, providing that suitable parity boundary conditions are imposed.

3. Results

Case 1. $\kappa(\theta) = 1$, $g(T) = 1$

Initial conditions:

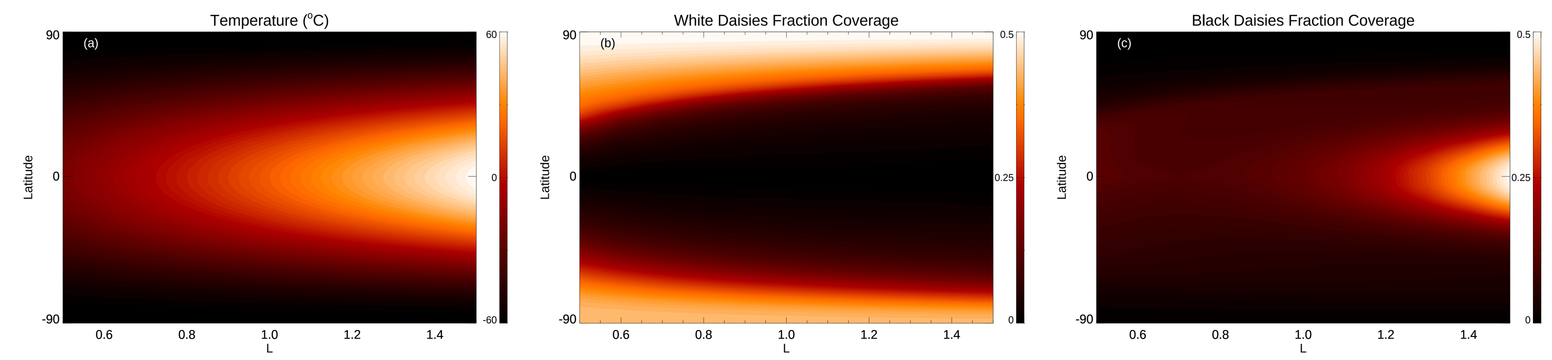
- $T(\theta, 0) = -20 + 40L\cos^2(\theta)$
- $\alpha_w(\theta, 0) = \alpha_b(\theta, 0) = 0.5$



Case 2. $\kappa(\theta) = \frac{1+\cos(2\theta)}{2}$, $g(T) = 1$

Initial conditions:

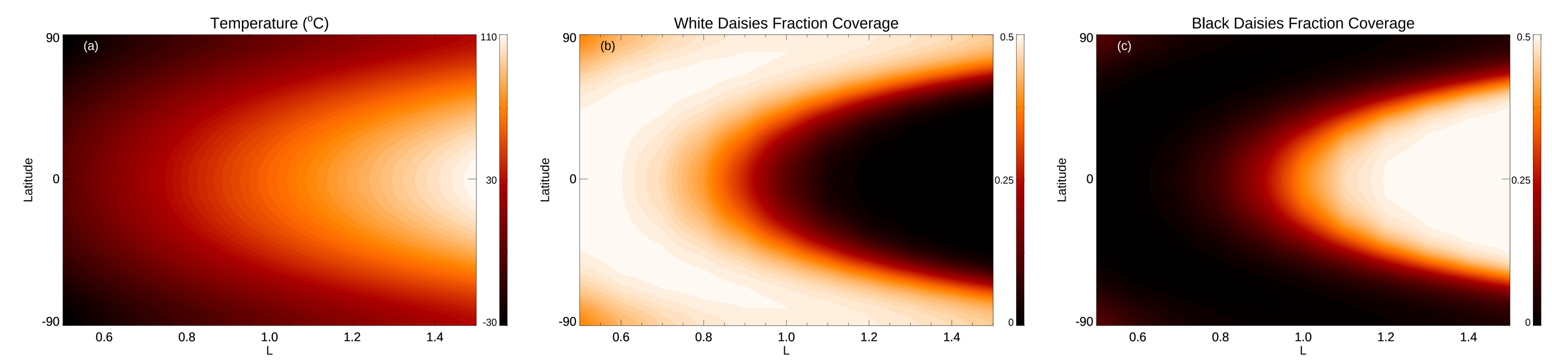
- $T(\theta, 0) = -20 + 40L\cos^2(\theta)$
- $\alpha_w(\theta, 0) = \sin^2(\theta)$
- $\alpha_b(\theta, 0) = \cos^2(\theta)$



Case 3. $\kappa(\theta) = \frac{1+\cos(2\theta)}{2}$, $g(T) = 1 - \frac{1}{2}\tanh\left(\frac{T}{T_0}\right)^6$

Initial conditions:

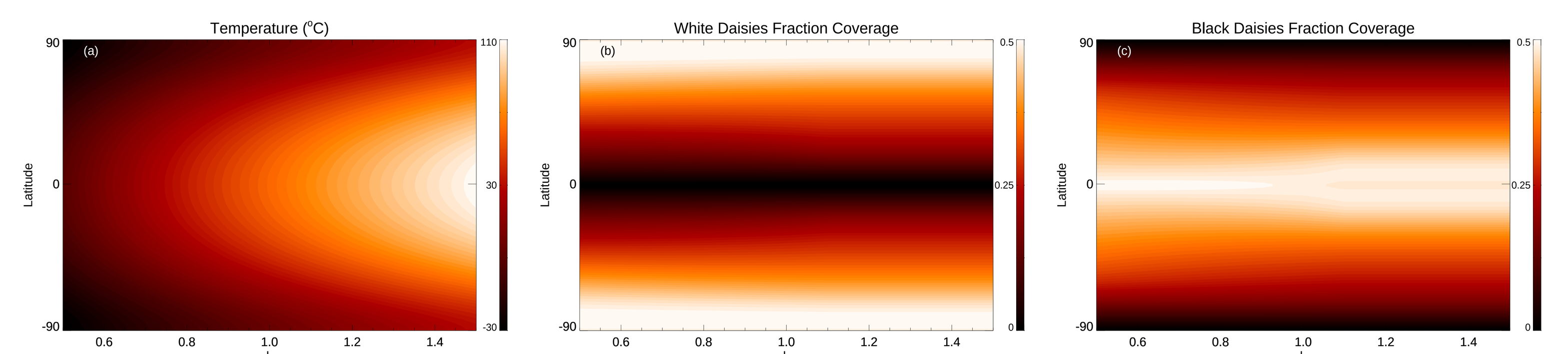
- $T(\theta, 0) = -20 + 40L\cos^2(\theta)$
- $\alpha_w(\theta, 0) = \sin^2(\theta)$
- $\alpha_b(\theta, 0) = \cos^2(\theta)$



Case 4. $\kappa(\theta) = \frac{1+\cos(2\theta)}{2}$, $g(T) = 1 - \frac{1}{2}\tanh\left(\frac{T}{T_0}\right)^6$

Initial conditions:

- $T(\theta, 0) = -20 + 40L\cos^2(\theta)$
- $\alpha_w(\theta, 0) = \begin{cases} \left|\frac{\theta}{\pi}\right| & \text{if } |\theta| < 85^\circ \\ 0.5 & \text{otherwise} \end{cases}$
- $\alpha_b(\theta, 0) = 0.5 - \alpha_w(\theta, 0)$



4. Discussion

- Case 1.

1. The increase/decrease of L corresponds to a global enhancement/decrease of temperature (for $L = 0.5$, $T_{max}(\theta = 0^\circ) \simeq -120^\circ$C and for $L = 1.5$, $T_{min}(\theta = \pm 90^\circ) \simeq 100^\circ$C)
2. An increase of luminosity L produces an increase of the planetary surface covered by black daisies and a decrease of the area covered by white daisies
3. Black daisies withstand high temperatures while the growth of white flowers is favoured by low temperatures

- Case 2.

1. The temperature variations with L are smaller with respect to the previous case in which the thermal diffusivity is not included
2. Larger differences in daisy fraction between polar and equatorial regions
3. For low L values the black daisy coverage increases in the region $0 < \theta \lesssim 40^\circ$ and for all L values the polar areas ($|\theta| \gtrsim 60^\circ$) are mainly populated by white daisies
4. We note an asymmetry in daisy coverage profiles not observed in the previous case which is due to the Laplace operator consisting in two different terms: the first one is symmetric while the second is not symmetric since proportional to the derivative of $\kappa(\theta)$ with respect to θ.

- Case 3.

1. The grayness function is able to control the temperature variations and, in particular, when L decreases the equatorial temperature is well above 10°C and when L increases polar temperatures are about 30°C
2. There is an increase of black daisy coverage in the region $|\theta| \lesssim 40^\circ$
3. The symmetry is recovered because when the grayness function is considered the diffusion and the Stefan-Boltzmann terms are of the same order of magnitude and opposite in sign

- Case 4.

1. The temperature profile remains unchanged
2. For each value of L black daisies are completely absent at the poles while white daisies do not bloom at the equator for high L values
3. The evolution of the fraction coverage for both species is symmetric but show different shapes and values with respect to the previous case

5. Conclusions

Summarizing, we found that:

1. the system is able to self-regulate even in presence of values of the incident luminosity which are far from the current Sun-Earth conditions
2. the mutual exclusion of the two vegetation type, observed in previous models, is never observed in our case
3. the grayness function affects temperature evolution in space and time and it contributes to self-regulate the Earth system
4. the diffusion process is able to destabilize the system and to inhibit the pattern formation process. Moreover, it plays an important role to set the simmetry with respect to the equator

References

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