Source term identification in atmospheric modelling via sparse optimization

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Inverse modelling

- From observations $y \in \mathbb{R}^m$ we try to estimate release $x \in \mathbb{R}^n$
- We have (linear) model $M \in \mathbb{R}^{m \times n}$, which is known as sensitivity matrix
- Applications
  - Nuclear accidents (Fukushima)
  - Volcano eruption
  - Enforcing pollution limits
Solution approaches

- Minimize the distance between $y$ and $Mx$
- Add nonnegativity constraint on $x$
- Problem
  \[
  \begin{aligned}
    & \text{minimize} & \|Mx - y\|_2^2 \\
    & \text{subject to} & x \geq 0.
  \end{aligned}
  \]

- Problem unstable. Some regularization (Tikhonov) is usually added
  \[
  \begin{aligned}
    & \text{minimize} & \|Mx - y\|_2^2 + \alpha \|x\|_2^2 \\
    & \text{subject to} & x \geq 0.
  \end{aligned}
  \]

- Problematic interpretation?
  - Minimizing $\|Mx - y\|$ and $\|x\|$. Why the second part?
  - How to choose $\alpha$?
Denoting $m_i$ columns of $M$, we have $Mx = \sum_{i=1}^{n} x_i m_i$

Small norm of $m_i$ may cause $x_i$ to be huge

Typical column norms of $M$

Idea: omit columns with small norm
Comparison of $\sum_{i=1}^{n} x_i$ for using Tikhonov regularization and removing some columns

![Graph showing comparison of sum(x) vs log10(threshold) for optimal, Tikhonov, and matrix reduction methods.](image)
Sparsity

- Idea: look for a solution with high number of nonzeros
- Good interpretation for both identifying release point and release window
- Define $\|x\|_0$ to be the number of nonzeros and solve

$$\begin{align*}
& \text{minimize} \quad \|Mx - y\|_2^2 \\
& \text{subject to} \quad \|x\|_0 \leq k_{tol}, \\
& \quad x \geq 0.
\end{align*}$$

- Parameter $k_{tol} \in \mathbb{N}$ makes more sense than $\alpha > 0$
(a) Original solution

(b) OLS solution

(c) Sparse solution with $k_{tol} = 5$

(d) Sparse solution with $k_{tol} = 10$

(e) Sparse solution with $k_{tol} = 15$
Conclusion

- We have proposed to use the techniques of sparse optimization to the field of atmospheric modelling.
- We modified the known methods to handle nonnegativity constraints.
- The methods seem to have a good performance.
- Submitted version available at Optimization Online
  www.optimization-online.org/DB_HTML/2015/04/4861.html