Copula-based estimation of large-scale water storage changes: exploiting the dependence structure between hydrological and GRACE data

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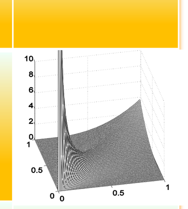
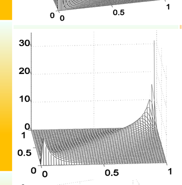
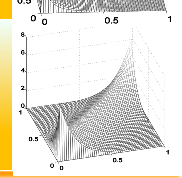
Introduction

Data from the Gravity Recovery and Climate Experiment (GRACE) have significantly improved our knowledge of the terrestrial water cycle. With the availability of GRACE data from 2002, we are now able to even perform climate change studies with respect to water storage variations. However, as GRACE is already after its predicted lifetime by many years, thus increasing the risk of a possible gap in regular observations of the total water storage changes until GRACE Follow-On becomes operational in 2017. We have to find methods for filling the gaps of both past data and also until the launch of GRACE Follow On. One idea could be finding the relation between the water storage anomaly that is derived from GRACE and hydrologic parameters like precipitation to estimate and assimilate the missing data of GRACE using copula-based analysis. In this study, we therefore analyze the potential of copula-based methods for simulating GRACE data from other hydrological data sources.

Methodology

Copula–based approach

Sklar (1959) introduced copula functions as a powerful tool to model the dependency between variables. The copula–based approach exploits linear and non-linear relationships between two or more variables by fitting a theoretical copula function into an empirical bivariate or multivariate distribution function. Finally, new data which is now consistent with the previously derived dependency structure can be simulated by evaluating the conditional distribution function which is given by the theoretical copula. Therefore, it can model complex dependency structures. Nelsen (2010) defined several bivariate Archimedean copulas with parameter θ

	Formula	Parameter	
Clayton	$C_\theta(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$-1 \le \theta$	
Gumbel	$C_\theta(u,v) = e^{-((-\ln u)^\theta + (-\ln v)^\theta)^{1/\theta}}$	$1 \le \theta$	
Frank	$C_\theta(u,v) = -\frac{1}{\theta}\ln(1 + \frac{(e^{-\theta u}-1)(e^{-\theta v})}{e^{-\theta}\ - 1})$	$-\infty < \theta < \infty$	

We assimilate data using copula by the following steps (Vogl et al., 2012).

1. Independent identically distribution (iid)-transformation of input time-series
2. Compute the marginal distribution of the input data
3. Transform data to rank space using the estimated marginal distribution
4. Compute the empirical dependency structure between random variables
5. Fit a theoretical copula to empirical copula
6. Sample random data from the conditional copula CDF
7. Transfer the sample back to the data space using the inverse marginal.

Proof of Concept

The applicability of the copula-based method to spherical harmonic data from GRACE is analyzed. As the approach involves several drawings of random data, we are interested if this random nature has any impact on the results. Based on the derived dependency structure, we generate random data out of unfiltered GRACE coefficients, which is statistically consistent with the filtered .

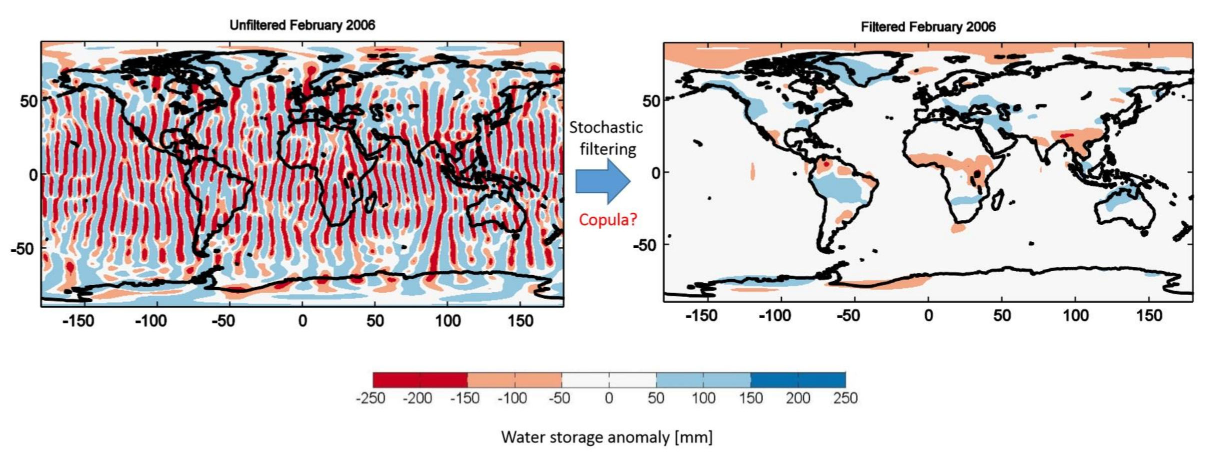


Figure 1: Unfiltered and filtered GRACE data in February 2006. The GRACE data are filtered by a regularization-type filter.

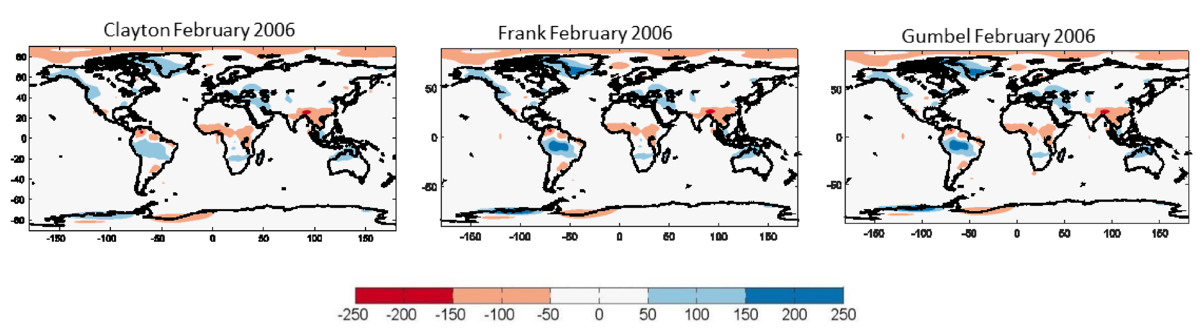


Figure 2: The filtered data generated out of unfiltered GRACE coefficients using Archimedean copulas in February 2006.

The results with Clayton (left), Frank, (center), and Gumbel (right) copulas are shown in Figure 2. Compared to filtered GRACE data, the copula filtered data show similar patterns.

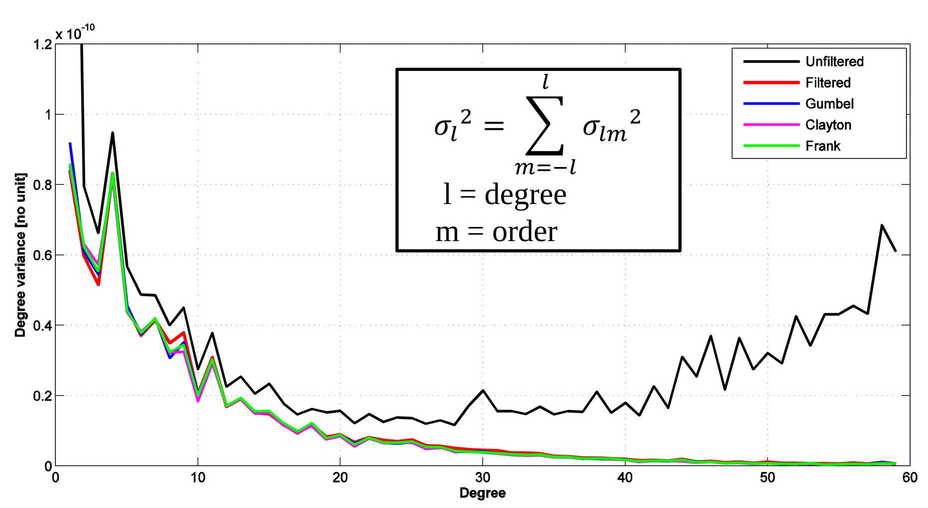


Figure 3: Degree variance of unfiltered and filtered GRACE data and the filtered data using Archimedean copulas in February 2006.

The comparison between degree variance of copula filtered and the filtered data shows very good agreement.

Catchment-based analysis

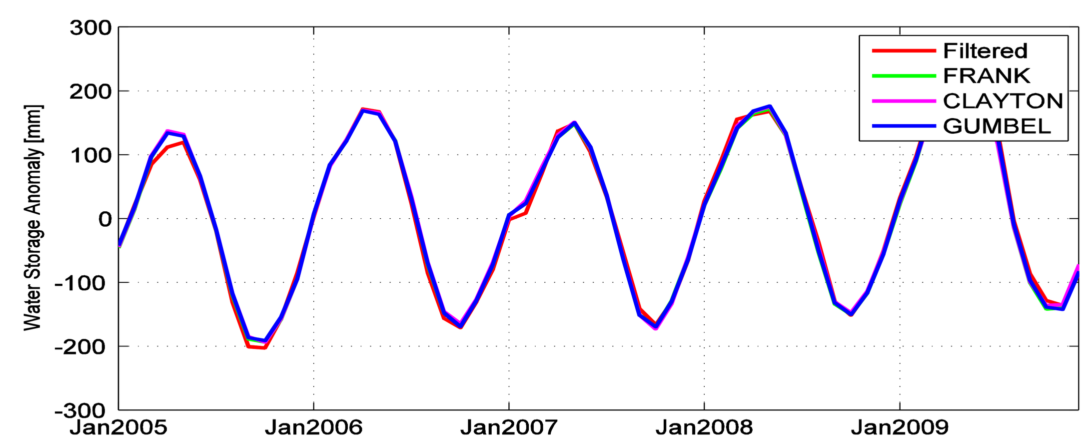


Figure 4: Time series of copula filtered and filtered data in Amazon from January 2005 until December 2009.

	RMS [mm]	Correlation with filtered
Filtered	123.9	-
Clayton	121.1	0.99
Gumbel	122.4	0.99
Frank	121.4	0.99

The result of Amazon basin is shown as an example in this poster. We got the same result for other catchments e.g. Mississippi.

Water storage anomaly from precipitation

We also want to evaluate if copula-based methods are able to estimate reliable water storage changes from independent hydrological data. Therefore, we derive the dependency structure between filtered water storage changes from GRACE and global gridded precipitation data from the Global Precipitation Climatology Centre GPCC.

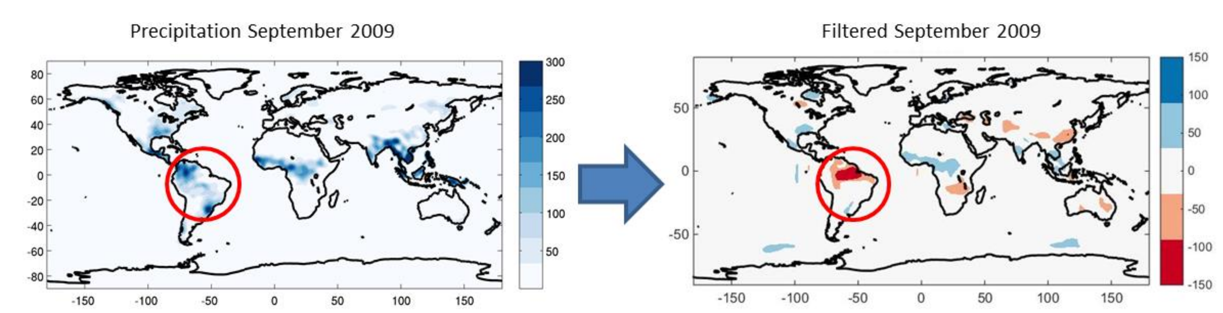


Figure 5: Precipitation and filtered GRACE data in September 2009. They show a different pattern in South America.

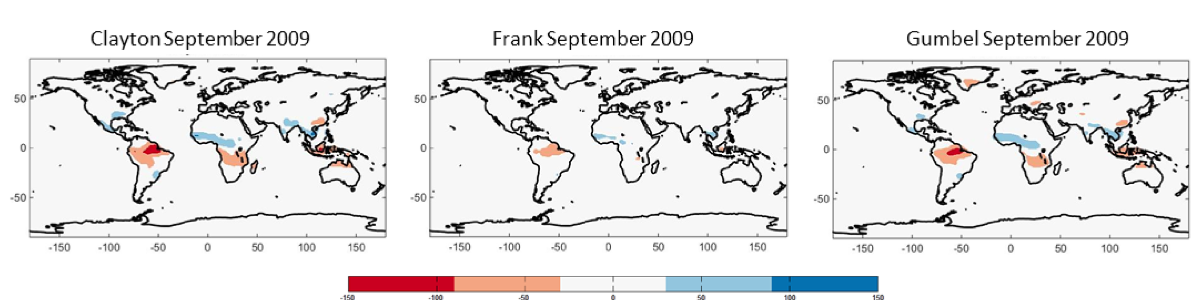


Figure 6: The copula filtered data generated out of precipitation using Archimedean copulas in September 2009.

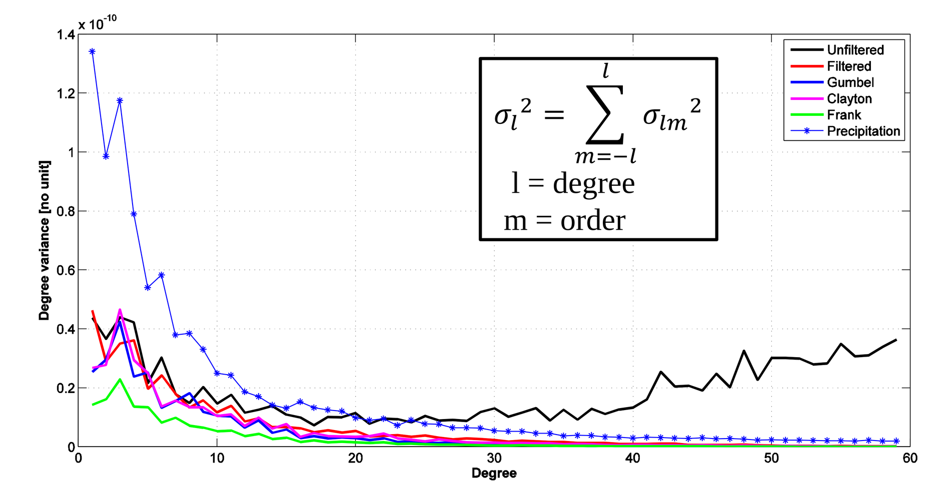


Figure 7: Degree variance of Precipitation, unfiltered and filtered GRACE data and the random data simulated by copula in September 2009.

Catchment-based analysis

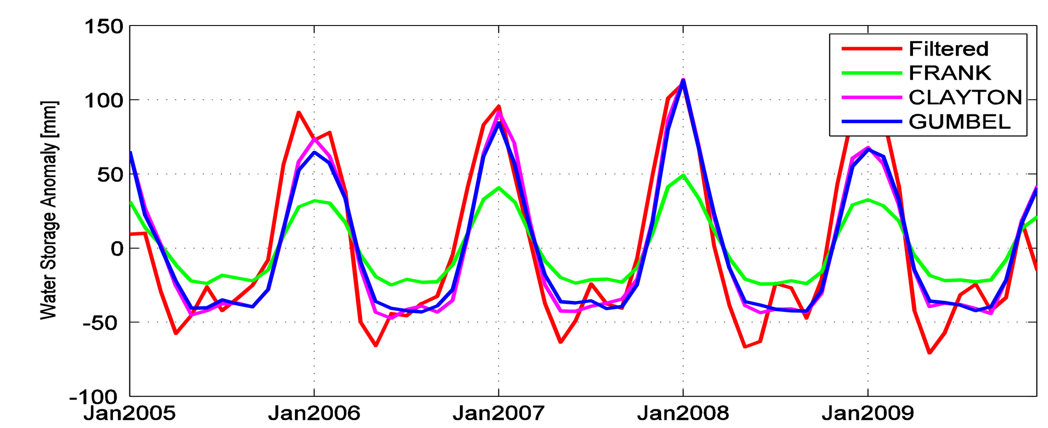


Figure 8: Time series of water storage change which is derived from precipitation data and filtered data in Zambezi from January 2005 until December 2009.

	RMS [mm]	Correlation with filtered
Filtered	52.5	-
Clayton	45.6	0.91
Gumbel	43.3	0.90
Frank	22.5	0.90

Conclusion

First, we analysed the applicability of the proposed method to spherical harmonic coefficients from GRACE. Second, the performance of water storage change prediction from precipitation is evaluated. The copula-based estimates are compared with filtered GRACE data in the spectral and spatial and time domain. The analysis shows that our estimates and the filtered GRACE data are in very good agreement. Thus, we conclude that the copula-based method is indeed able to fill the missing months in the GRACE-dataset and to even extend the time-series until the launch of GRACE Follow On.

Reference

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