



Correlation analysis for long time series by robustly estimated autoregressive stochastic porcesses

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Magic Square -Spectral Estimation

Data Model -Robust Estimation

GOCE application

Resume

Motivation

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GOCE-TIM05 results & products





consistent modeling of the error budget variance/covariance information available

Brockmann et.al. (2014): EGMTIMRL05: An Independent Geoid with Centimeter Accuracy Purely Based on the GOCE Mission. *Geophysical Research Letters* 41(22), 8089-8099







- huge number of measurements
- time variable behavior
- strongly correlated measurement noise
- conspicuous data especially in the low orbit operation campaign

GOCE-TIM data segments

440 mio observations, eff. 1273 days, 88 segments











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- time variable behavior
- strongly correlated measurement noise
- conspicuous data especially in the low orbit operation campaign

GOCE-TIM data segments

440 mio observations, eff. 1273 days, 88 segments



01-Jan-2010 01-Jul-2010 01-Jan-2011 01-Jul-2011 01-Jan-2012 01-Jul-2012 01-Jan-2013 01-Jul-2013 01-Jan-2014



signal vs. noise

-1000 01-Nov-2009 23:55:54

1500

02-Nov-2009 04:25:05









02-Nov-2009 04:25:05

huge number of measurements

- time variable behavior
- strongly correlated measurement noise
- conspicuous data especially in the low orbit operation campaign

GOCE-TIM data segments





conspicuous data



signal vs. noise

residuals

.V_{zz} model – GRS80 – mean V_{zz} measurements – GRS80 – mear

1500

1000

500

-50

-1000 01-Nov-2009 23:55:54

Vzz [mE]

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Goals



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Goals of this presentation:

- Approach for an optimal description of the stochastic behavior of GOCE data
- Identification of conspicuous data
- Demonstration of first results with real GOCE data







Magic Square -Spectral Estimation

Magic Square

GOCE noise

stochastic signal

discrete signal

AR process

Data Model -Robust Estimation

GOCE application

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Magic Square - Spectral Estimation





Magic Square





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Representation of GOCE noise in different states

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Magic Square -Spectral Estimation

Magic Square

GOCE noise

stochastic signal discrete signal AR process

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Magic Square - stationary stochastic signal

???

frequency domain











Magic Square - stationary stochastic signal









Magic Square - stationary stochastic signal





Krasbutter et.al (2015): Magic Square of Real Spectral and Time Series Analysis with an Application to Moving Average Processes. *IAG Symposia* 140, 9-14, Springer





Magic Square - estimation process





Magic Square -Spectral Estimation Magic Square

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GOCE application

Resume



covariance stationary discrete equispaced stochastic signal







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Data Model -Robust Estimation

GOCE application

Resume



covariance stationary discrete equispaced stochastic signal





















Magic square - AR(p) process



parametric spectral estimation

Motivation

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Magic Square

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stochastic signal

discrete signal

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Data Model -Robust Estimation

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AR(p)-auto-regressive process (equivalent to MA or ARMA representation — Wold (1938) Wold (1938): A study in the analysis of stationary time series. *Almqvist & Wiksell*









AR(p)-auto-regressive process — Magic Square Schuh et.al. (2014): Korrelierte Messung - was nun? In: Neuner (Ed): Zeitabhängige Messgrößen - Ihre Daten haben (Mehr-)Wert, Wißner, Augsburg, 74, 85 - 101









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Magic Square -Spectral Estimation

Data Model -Robust Estimation

Data Model

discrete signal

robust estimation

GOCE application

Resume

Data Model - Robust Estimation









Data model



Observed Quantities:

Motivation

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Data Model -**Robust Estimation**

Data Model

discrete signal

robust estimation

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 $\mathcal{L}_k = \mathcal{S}_k + \mathcal{N}_k$

 S_k ... stochastic process in AR representation p

$$\mathcal{S}_k = \sum_{j=1} \alpha_j \mathcal{S}_{k-j} + \mathcal{E}_k$$

 \mathcal{E}_k ... innovation process

 \mathcal{N}_k ... additive process (noise)





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Data Model -Robust Estimation

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GOCE application

Resume

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Observed Quantities:

 $\mathcal{L}_k = \mathcal{S}_k + \mathcal{N}_k$ \mathcal{S}_k ... stochastic process in AR representation

$$\mathcal{S}_k = \sum_{j=1}^r \alpha_j \mathcal{S}_{k-j} + \mathcal{E}_k$$

 \mathcal{E}_k ... innovation process \mathcal{N}_k ... additive process (noise)

Contamined real observed time series (e.g. GOCE)

 $\mathcal{E}_k \sim F_{\mathcal{E}_k}(x)$ heavy-tailed distribution (e.g. Student distribution $F_{\mathcal{E}_{k}}^{t_{r}}(x)$)

 $\mathcal{N}_k \sim F_{\mathcal{N}_k}(x)$ patchy outlier distribution $F_{\mathcal{N}_k}(x) = \alpha \delta_0 + \beta F_{pattern}(x) + (1 - \alpha - \beta) F_{\mathcal{O}}^{\mathcal{N}(0,\sigma_{\mathcal{O}}^2)}(x)$... degenerated Dirac distribution δΩ $F_{pattern}(x)$... degenerated fix pattern distribution





Magic Square -Spectral Estimation

Data Model -**Robust Estimation**

Data Model

discrete signal robust estimation

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Resume

Data model



Observed Quantities:

 $\mathcal{L}_k = \mathcal{S}_k + \mathcal{N}_k$ \mathcal{S}_k ... stochastic process in AR representation

$$S_k = \sum_{j=1}^{P} \alpha_j S_{k-j} + \mathcal{E}_k$$

$$\mathcal{E}_k \quad \dots \quad \text{innovation process}$$

 \mathcal{N}_k ... additive process (noise)

Contamined real observed time series (e.g. GOCE)

 $\mathcal{E}_k \sim F_{\mathcal{E}_k}(x)$ heavy-tailed distribution (e.g. Student distribution $F_{\mathcal{E}_{L}}^{t_{r}}(x)$)

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\Rightarrow robust estimation of the stochastic process is essential

Kleiner et.al. (1979): Robust Estimation of Power Spectra. Journal of the Royal Statistical Society B-41, 313-351























non-parametric spectral estimation parametric spectral estimation Motivation Magic Square -Spectral Estimation $\ell_k e^{-i2\pi\nu k\Delta t}$ Data Model data fitting **Robust Estimation** Data Model approach $\{\ell_k\}_{\Delta k}$ discrete signal averaging periodogram robust estimation Welch) approach **GOCE** application Burg Resume approach Yule-Walker smoothed periodogram approach approach $\cos(2\pi\nu)$ Blackman-Tukey approach $\{g_k\}$ psd(u $g_k = \int psd(\nu)\cos(2\pi\nu k\Delta t)d\nu$ time domain frequency domain















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discrete signal

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measurement l_k is a single realization of the random variable \mathcal{L}_k Observation equations:

$$_{k} + v_{k} = \sum_{j=1}^{p} \alpha_{j} l_{k-j} = \begin{bmatrix} l_{k-1} \dots l_{k-p} \end{bmatrix} \begin{vmatrix} \alpha_{1} \\ \vdots \\ \alpha_{p} \end{vmatrix} := \boldsymbol{a}_{k}^{T} \boldsymbol{\alpha}$$

Optimization principle:

$$\inf_{\widetilde{\alpha}} \sum_{i=p+1}^{\#obs} \rho\left(\frac{v_k}{s_{\mathcal{E}}}\right)$$

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Normal equations:

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$$\sum_{i=p+1}^{\#obs} a_k \psi\left(\frac{v_k}{s_{\mathcal{E}}}\right)$$

 $ho\left(ar{v}_k
ight)$. . . loss function

$$\psi(\bar{v}_k) = \frac{\partial \rho(\bar{v}_k)}{\partial \boldsymbol{\alpha}} \dots$$
 influence function







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Influence function:

/____

$$\begin{split} \psi_{L2}(v_k) &= v_k & \text{L2-estimator} \\ \psi_{L1}(\overline{v_k}) &= \operatorname{sign}(\overline{v_k}) & \text{L1-estimator} \\ \psi_H(\overline{v_k}) &= \begin{cases} \overline{v_k} & |\overline{v_k}| < k \\ \operatorname{sign}(\overline{v_k}) & |\overline{v_k}| \ge k \end{cases} & \text{Huber-estimator} \\ \psi_R(\overline{v_k}) &= \begin{cases} \overline{v_k} & |\overline{v_k}| < k \\ 0 & |\overline{v_k}| \ge k \end{cases} & k\sigma\text{-rejection-estimator} \end{split}$$







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Data Model

discrete signal

robust estimation

GOCE application

Resume

Influence function:

$$\begin{split} \psi_{L2}\left(\overline{v_{k}}\right) &= \overline{v_{k}} & \text{L2-estimator} \\ \psi_{L1}\left(\overline{v_{k}}\right) &= \operatorname{sign}(\overline{v_{k}}) & \text{L1-estimator} \\ \psi_{H}\left(\overline{v_{k}}\right) &= \begin{cases} \overline{v_{k}} & |\overline{v_{k}}| < k \\ \operatorname{sign}(\overline{v_{k}}) & |\overline{v_{k}}| \geq k \end{cases} & \text{Huber-estimator} \\ \psi_{R}\left(\overline{v_{k}}\right) &= \begin{cases} \overline{v_{k}} & |\overline{v_{k}}| < k \\ 0 & |\overline{v_{k}}| > k \end{cases} & k\sigma\text{-rejection-estimator} \end{split}$$

GOCE-LOOC: $k\sigma$ -rejection-estimator (single and area-mean) AR(p) processes up to degree 2000 ARI(i,p) integrated AR-processes - filter cascades







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GOCE application

filter psd conspicuous data first results

Resume

Application to GOCE residual time series

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residuals AR (outlier:-5,1,0,0)



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residuals AR (outlier:-5,1,0,0)



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Filter estimation: Segment 68-yy

residuals AR (outlier:-5,1,0,0)



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Spectral Estimation

Data Model -Robust Estimation

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Motivation

Spectral Estimation

Data Model -Robust Estimation

GOCE application filter psd conspicuous data

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Resume

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Motivation

Magic Square -

Data Model -

filter psd

first results

Resume

Spectral Estimation

Robust Estimation

GOCE application

conspicuous data

Filter estimation: Segment 68-yy

residuals AR (outlier:-5,1,0,0)



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first results

Resume

















GOCE AR-filter psd





power spectral density

Welch σ = 28.8

FT(emp. covariances) σ = 28.5

10⁻¹

- filter(5168) $\sigma_{t=16rev} = 131.3$

-filter(5868) $\sigma_{t=16rev}^{t=16rev} = 33.4$

10⁻³ 10⁻² frequency (Hz)

segment:

10

10¹¹

10⁸

10⁶

10

10[°]

er/Hz





















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Segment 68-yy: AR filter psd







outlier 680

mean rev/50 2028

mean rev/40 348

mean rev/30 345

3.5

x 10⁵

3

outlier 2413

mean rev/50 2291

mean rev/40 874

mean rev/30 991

7

outlier 26783

mean rev/50 52465

mean rev/40 8337

mean rev/30 8901

8

x 10⁵

4037

cluster 2

daps

x 10⁵

GOCE conspicuous data identification



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time

6

8

x 10⁵

4

0

2

o

2

4

6

time

8

x 10⁵



2

4

time

6

8

x 10⁵

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6

2

4

time













4.1

6.1

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5.8



First Reslust: rms 59xx-EGM08 51xx-EGM08 universitätbonn

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conspicuous data

first results

Resume



expected gain of accuracy for the reprocessed data (5-15%) Brockmann et.al. (EGU2015-12307): A case study on the potential of robust decorrelation filter design for a reprocessing of a gravity field model from GOCE data - presentation on Tuesday







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Resume

advanced outlier detection:

- much more conspicuous data (6% \Rightarrow 18%)
- ⊢ separation of effects (outlier, gap, disruption, ...)

optimized AR filter:

- + optimal relative weighting between the components
- + gain of accuracy for the GOCE-TIM models ($\sim 10\%$)

quality vs. quantity







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Resume

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quality vs. quantity Thank you for your attention

