Finite element method algorithm for the magma-rock interaction problem

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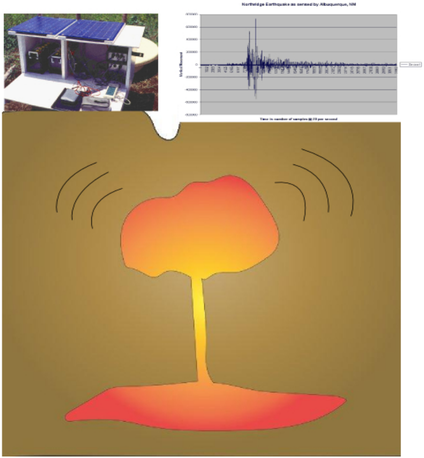
Motivation

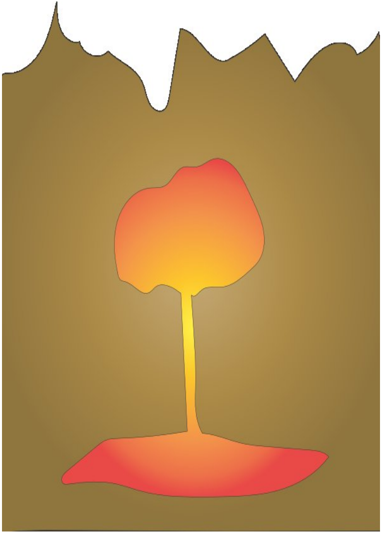


- magma dynamics
- deformation of rocks
- data analysis

Problem Statement

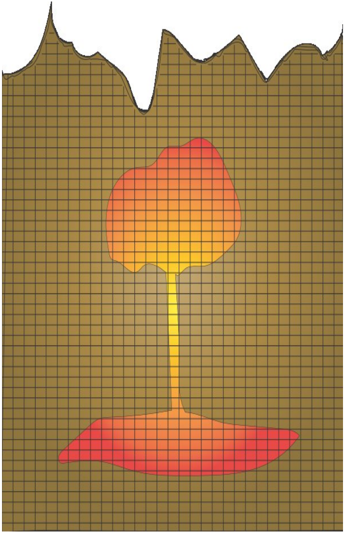
- We are building a two way coupled finite-element model and code for the simulation of dynamics of magma and the surrounding rocks.
- The simulation results will be helpful to understand the link between the deep volcanic processes and the ground geophysical signals registered by the monitoring networks.

Fluid-Structure Interaction



- Fluid (Magma)
- Structure (Rock)
- Fluid-Structure Interface

Eulerian vs Lagrangian



Lagrangian: Each individual node of the computational mesh follows the associated material particle during motion.

Eulerian: The computational mesh is fixed and the continuum moves with respect to the grid.

Eulerian vs Lagrangian

Lagrangian: Easy tracking of free surfaces and interfaces between different materials.

Inability to follow large distortions of the computational domain without remeshing.

Eulerian: No problem of distortion of mesh.

Inability to define precise interface and the resolution of flow domain.

Fluid model

- Magma is a multicomponent fluid mixture described by the weight fraction y_k of its components. $\sum_k y_k = 1$
- Components can be in gaseous and/or liquid state. Phase distribution of each component k is described by weight fraction η_k^π of component k in the phase π.
- Newtonian rheology.
- Space-time discontinuous Galerkin least square approach.

Navier-Stokes Equations

$$(\rho y_k)_{,t} + (\rho u_i y_k)_{,i} = (-J_i^k)_{,i} \qquad \text{for} \quad k = 1, \ldots, n$$

$$(\rho u_j)_{,t} + (\rho u_i u_j + p\delta_{ij})_{,i} = (\tau_{ij})_{,i} + (\rho b_j) \qquad \text{for} \quad j = 1, \ldots, d$$

$$(\rho e_t)_{,t} + (\rho u_i e_t + p u_i)_{,i} = (\tau_{ij} u_j - q_i - \sum_{k=1}^{n} J_i^k h_k)_{,i} + \rho(b_i u_i + r)$$

ρ, y_k, T, u_i, p are variables.

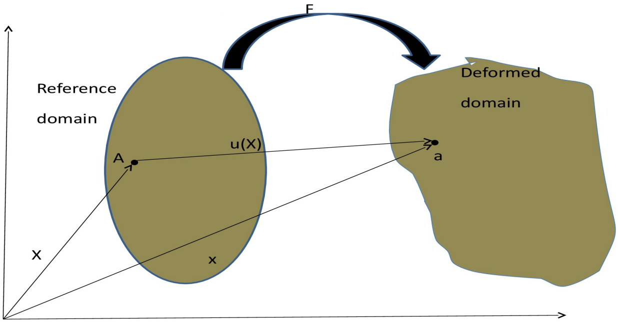
Structural model

- Rock is treated as an elastic isotropic material.
- Lagrangian frame of work in reference domain.
- Space-time discontinuous Galerkin approach.

Elastodynamics equation

The Elastodynamics equation for motion of rock is given by

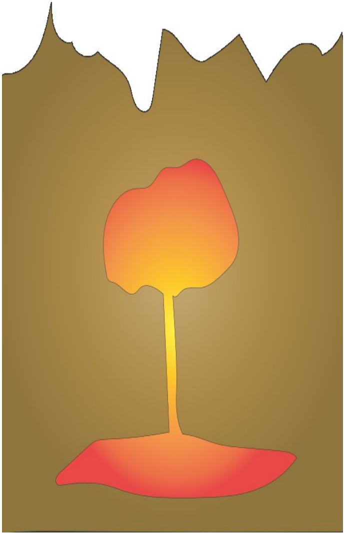
$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + \rho g$$



$$\rho_0 \frac{\partial^2 u}{\partial t^2} = \nabla \cdot P + \rho_0 g$$

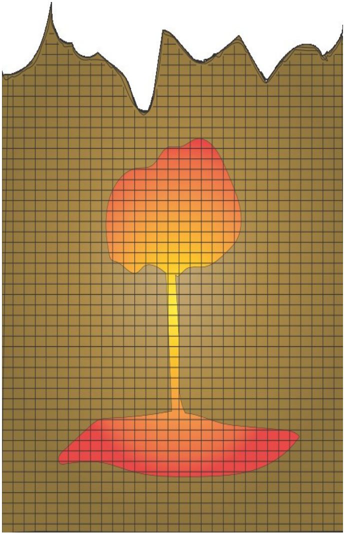
- $F = I + \nabla u$, is the deformation gradient.
- $E = \frac{1}{2}(F^T F - I) = \frac{1}{2}[(\nabla u)^T + \nabla u + (\nabla u)^T \nabla u]$
- $S = 2\mu E + \lambda tr(E) I$ is second Piola-Kirchhoff stress tensor.
- μ and λ are Lamè coefficients
- $P = FS - (\rho g y) I$

Algorithm



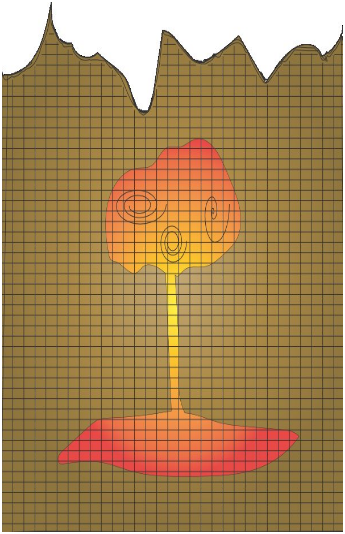
- Define domains for Magma chamber and rock.

Algorithm



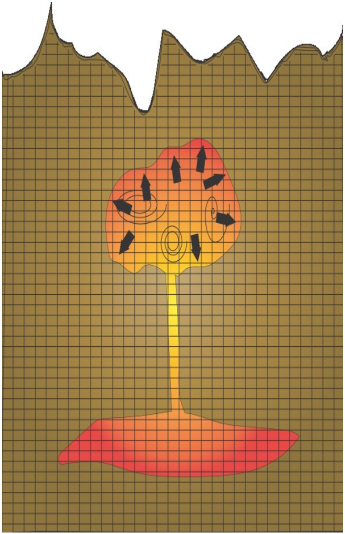
- Discretize the domains by making meshes.

Algorithm



- Set initial conditions.
- Solve Navier stokes Equations for magma.
- Compute ρ, y_k, T, u_i, p.

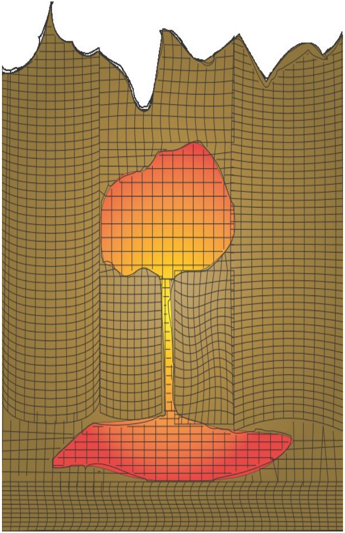
Algorithm



- Compute stress tensor of fluid
  $\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i})$

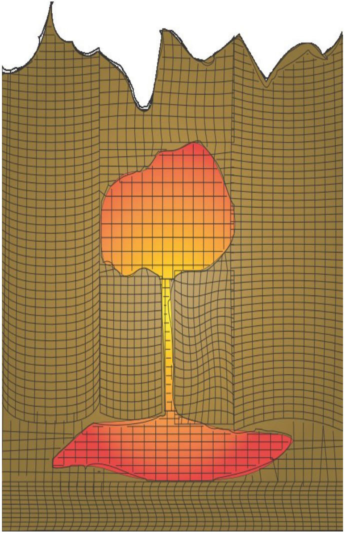
- Compute traction
  $t = (\sigma - pI) \cdot \hat{n}$

Algorithm



- Solve Elastodynamics equation, passing fluid traction as the neumann boundary conditions on interface.
- Compute u and v.

Algorithm

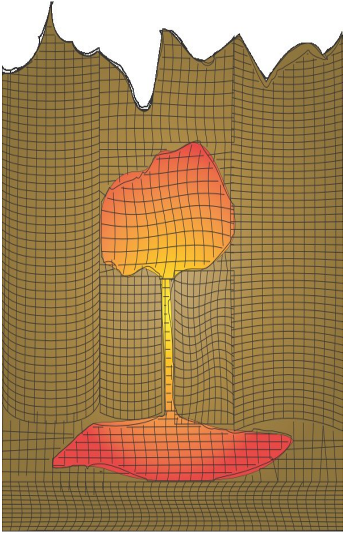


- Solve Elastostatic equation to compute movement of fluid mesh, passing structural deformation as dirichlet boundary conditions on interface.

$$\nabla \cdot \sigma(\epsilon(u)) = 0$$

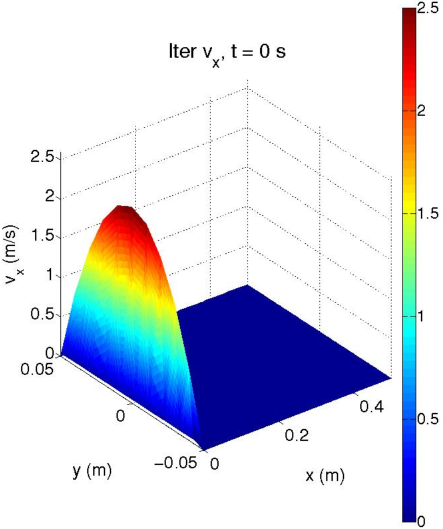
- compute u.
- update mesh

Algorithm



- Solve fluid equations on updated mesh, passing structural velocities as dirichlet boundary conditions on interface.
- Continue until convergence is reached.

Channel flow



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Thank You !!!