Mathematics of tsunamis: modelling and identification

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1. Inverse problem statement

The operationally refining the tsunami source parameters is the most important for providing real information about expected tsunami waves on shore. **The combined inverse tsunami problem** consists in determining an initial tsunami perturbation $q(x,y)$ in $\Omega = (0, L_x) \times (0, L_y)$ described by linear shallow water equations

$$\begin{cases} \mathcal{L}\eta \equiv \eta_{tt} - \operatorname{div}(gH(x,y)\operatorname{grad}\eta) = 0; \\ \eta|_{t=0} = q(x,y), \quad \eta_t|_{t=0} = 0; \\ \eta|_{\partial\Omega} = 0, \quad t \in (0,T) \end{cases} \tag{1}$$

using combined underwater systems [1]

$$\eta(x,y,t) = f_m^\varepsilon(x,y,t), \; t \in (T_{m_1}, T_{m_2}), \\ (x,y) \in \omega_m^\varepsilon, m = \overline{1,M}, \varepsilon > 0, \tag{2}$$

and satellite altimeters data

$$\eta(x,y,T) = f(x,y), \; (x,y) \in \omega \subset \Omega. \tag{3}$$

Here $\eta(x,y,t)$ is the free surface, $H(x,y) > 0$ is a known function describing the bottom relief (bathymetry), $g = 9.8\ [m/s^2]$ is the acceleration of gravity, $\omega_m^\varepsilon := (x_m - \varepsilon, x_m + \varepsilon) \times (y_m - \varepsilon, y_m + \varepsilon)$, $\omega := (l_x^{(1)}, l_x^{(2)}) \times (l_y^{(1)}, l_y^{(2)})$.

We show that using of combined data (2) and (3) allows one to increase the stability and efficiency of tsunami source reconstruction.

2. Variational approach and gradient method

We **regularize** inverse problem (1)-(3) using cut Fourier series $q(x,y) = \sum_{n=1}^{N} q_n(x) \sin\frac{2\pi n}{L_y} y$ [2]. The combined inverse tsunami problem $Aq = (f_1^\varepsilon, \ldots, f_M^\varepsilon, f)$ reduces to minimization problem of a cost function $\mathbf{J(q)} = \beta J_1(q) + (1-\beta) J_2(q)$, $\beta \in [0,1]$. Here

$$J_1(q) = \sum_{m=1}^{M} \int_{T_{m_1}}^{T_{m_2}} \iint_{\omega_m^\varepsilon} [\eta(x,y,t) - f_m^\varepsilon(x,y,t)]^2 \, dx dy dt$$

and $J_2(q) = \iint_\omega (\eta(x,y,T) - f(x,y))^2 \, dx dy$.

The gradient of a cost function is $\mathbf{J'q} = \beta J_1' q + (1-\beta) J_2' q$ [3]. Here $J_1' q = \psi_{1_t}(x,y,0)$ [4] and $J_2' q = \psi_{2_t}(x,y,0)$, where ψ_i are solutions of corresponding conjugate problems, $i = 1,2$.

For solving inverse problem (1)-(3) numerically we apply **conjugate gradient method** that consists in finding an approximate solution $q_{n+1} = q_n - \alpha_n p_n$. Here $\alpha_n = 0.5\,(J'q_n, p_n)/\|Ap_n\|^2$, $p_n = J'q_n + p_{n-1}\|J'q_n\|^2/\|J'q_{n-1}\|^2$.

3. Numerical experiment

We put $L_x = 50$ km, $L_y = 100$ km, $T = 60$ min, $\varepsilon = 125$ m, $N_x = 750$, $N_y = 500$, $N_t = 600$. The bottom is assumed to be one-dimensional with the highest $H_{\max} = 6$ km average depth of the ocean. Noise data (2) and (3) are generated from the discrete numerical solution of the problem (1) in six points (x_m, y_m) equally-spaced on the interval $((40,15);(47,89))$ and for $\omega = 25 \times 50$ kms. We choose $q_0 = H_{\max}$ which corresponds to an unperturbed sea surface. The reconstructed solution q_4 of inverse problem (1)-(3) from the random noisy output data with $\gamma = 3\%$, $\beta = 0.3$, is demonstrated on fig. 2 for $n = 4$ iterations.

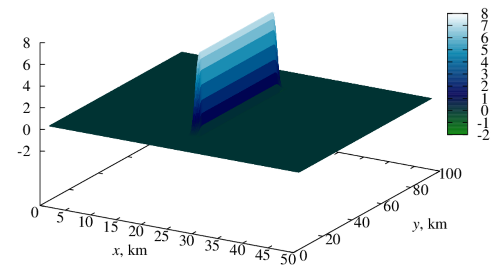


Figure 1: The exact solution $q_e(x,y)$.

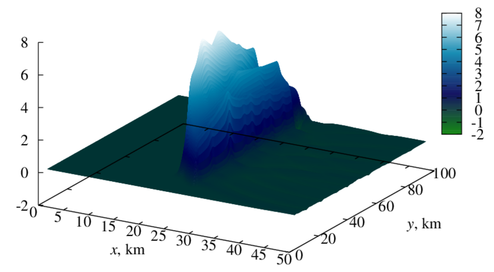
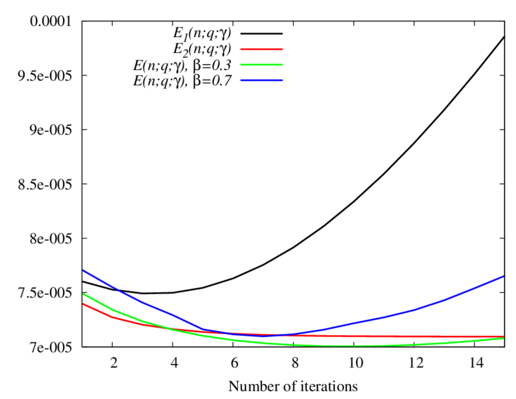


Figure 2: The reconstructed solution q_4.



The relative accuracy errors $E_i(n;q;\gamma)$ for inverse problems: **1** (1),(2), **2** (1),(3) and **3** (1)-(3). Note, that curves E_3 are located below curves E_i.

4. ITRIS: tsunami database

The Integrated Tsunami Research and Information System (ITRIS) was developed for reducing risk due to natural and man-maid hazards and for rescue planning after disasters [5]. There are built-in catalogues with set of interfaces for data managing. Fig. 3 presents visualization of earthquake epicenters around Japan and fig. 4 shows historical tsunami source locations.

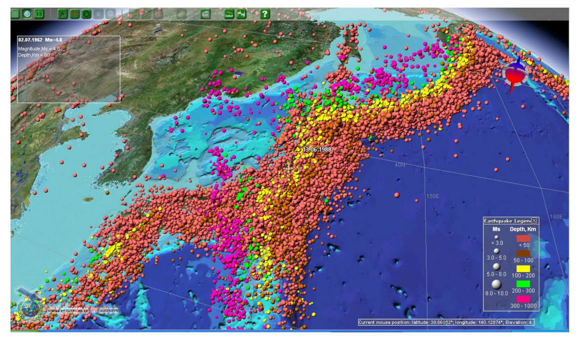


Figure 3: Visualization of the available seismic data around Japan.

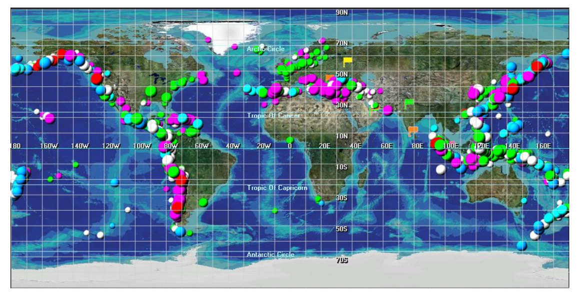


Figure 4: Projection map of historical tsunami source locations from 1628 B.C. to the present.

5. ITRIS: tsunami simulation

We simulate the Simushir tsunami using ITRIS (fig. 5). GIS methods will be used to create and combine the different inundation and flooding map components, which will be practically GIS-layers containing bathymetric, topographic, land use and inundation projections (fig. 6).

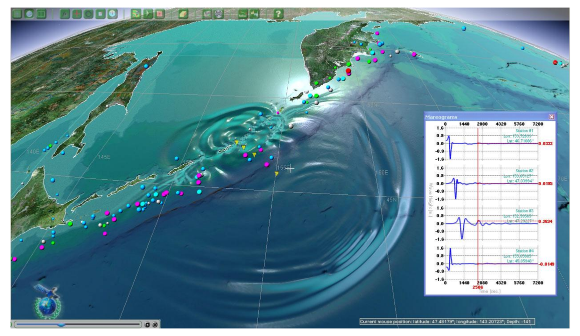


Figure 5: Numerical modeling of the Simushir tsunami 15.11.2006.

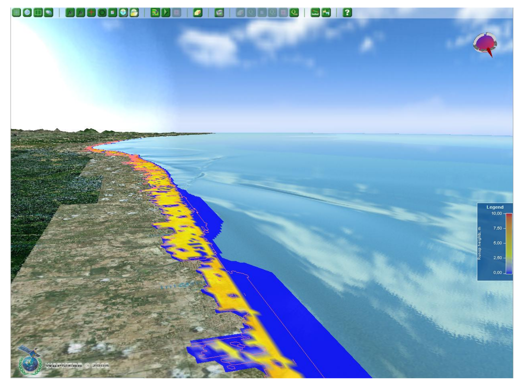
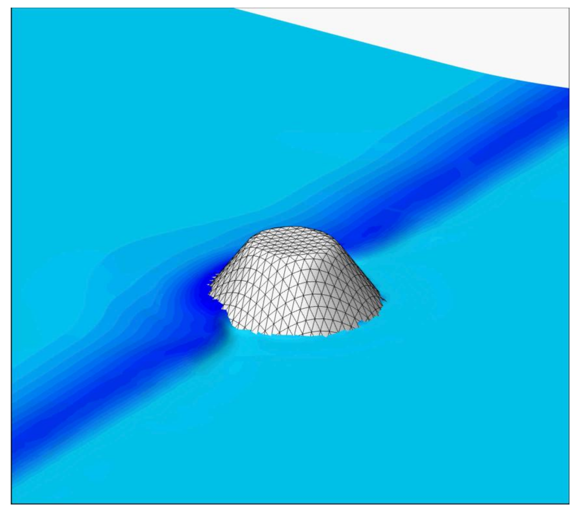


Figure 6: Flooding map of Nagapattinam, India.

6. Evaluation of results

We use the model of the nonlinear shallow water equations for modeling tsunami run-ups. We apply **finite volume method** (FVM) to calculate run-up accurately. Advantages of FVM over finite difference method in application to tsunami modeling are stability, usage nested triangular meshes and obvious parallelization.

We compare the numerical modeling of tsunami run-up with the laboratory experiments of NOAA (National Oceanic and Atmospheric Administration). In the physical model, a 62.5 cm-high, 7.2 m toe-diameter, and 2.2 m crest-diameter circular island was located in a 30 m-wide, 25 m-long, and 60 cm-deep wave basin (fig. 7). The solitary wave height equal to 0.045 at 32 cm water depths. The deviation of the fluid from rest is fixed at three points. The first point is located in front of the incident wave to the side of the island of 2.6 meters in front of the center, the second – 2.6 meters from the side, the third – a 2.6 meters behind the center.



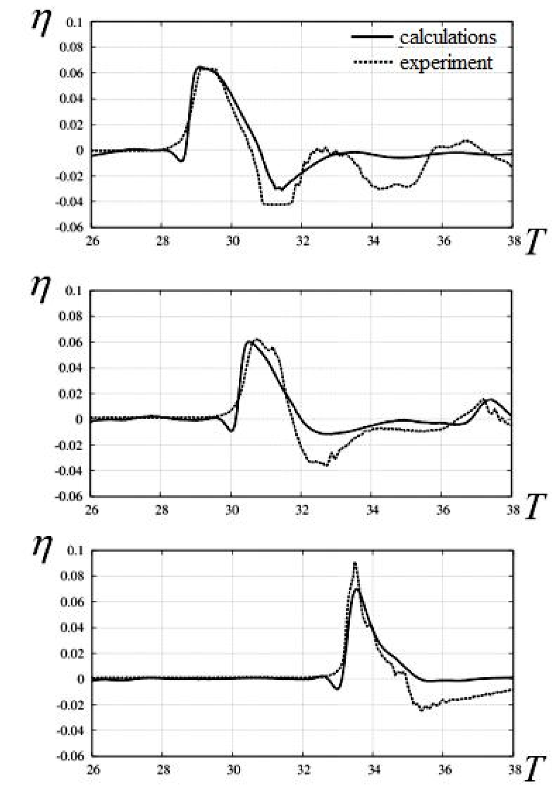


Figure 7: Mathematical modeling of run-up on the island. Comparison of the calculated and experiment marigrams in three points (right).

References

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