Sensitivity of fine sediment source apportionment to mixing model assumptions

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Outline

Sensitivity to error assumptions (“2nd order uncertainty”)

- Prior parameter distributions in a Bayesian framework
- Fixing source & error means & covariances at ML estimates
- Covariance of source fingerprints & instrument errors
- Time-variance of sources
- Common implicit error assumptions

Cooper et al. 2015. High-temporal resolution fluvial sediment source fingerprinting with uncertainty: a Bayesian approach. Earth Surf Process Landforms 40

Cooper et al. 2014. Sensitivity of Fluvial Sediment Source Apportionment to Mixing Model Assumptions: A Bayesian Model Comparison. Water Resour Res 50

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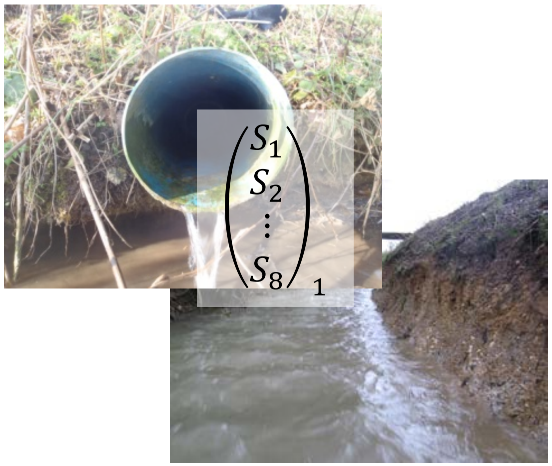
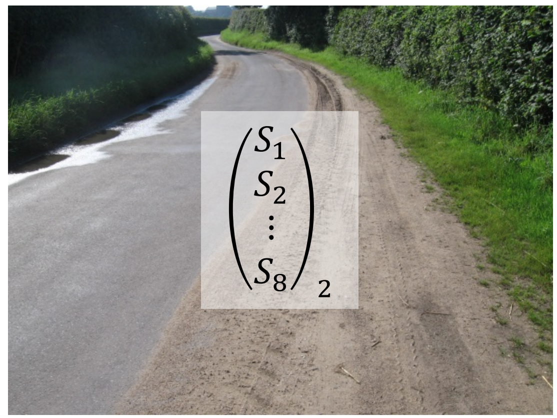
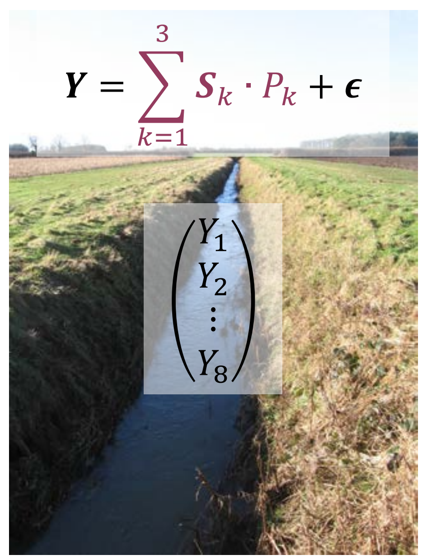
The mixing model







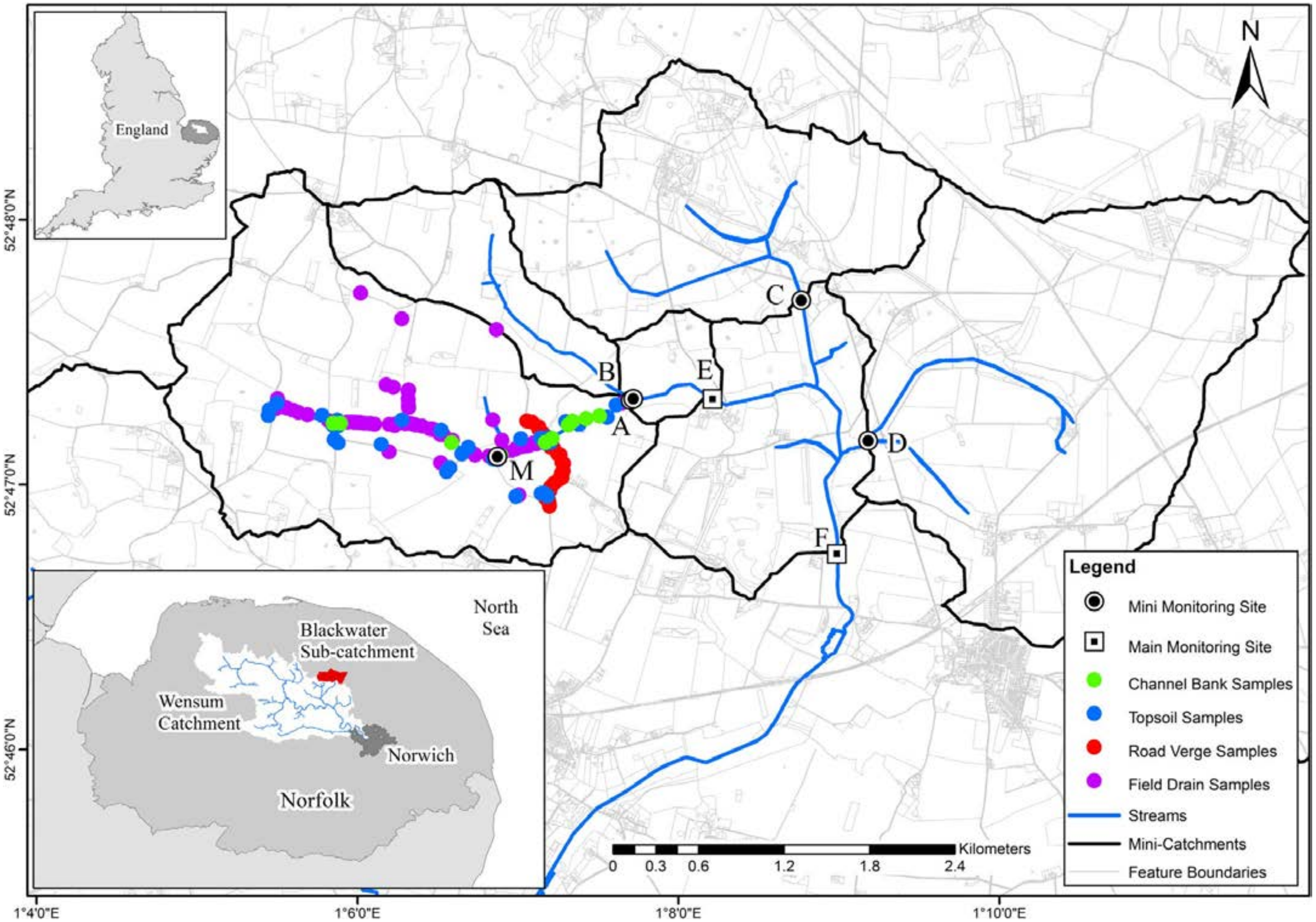
Ordinary Least Squares:

$$p(\boldsymbol{\epsilon}) = N(\boldsymbol{\mu} = 0, \sigma^2)$$

$$p(\boldsymbol{Y}) = N\left(\boldsymbol{\mu} = \sum_{k=1}^{3} \boldsymbol{S}_k \cdot P_k , \sigma^2\right)$$

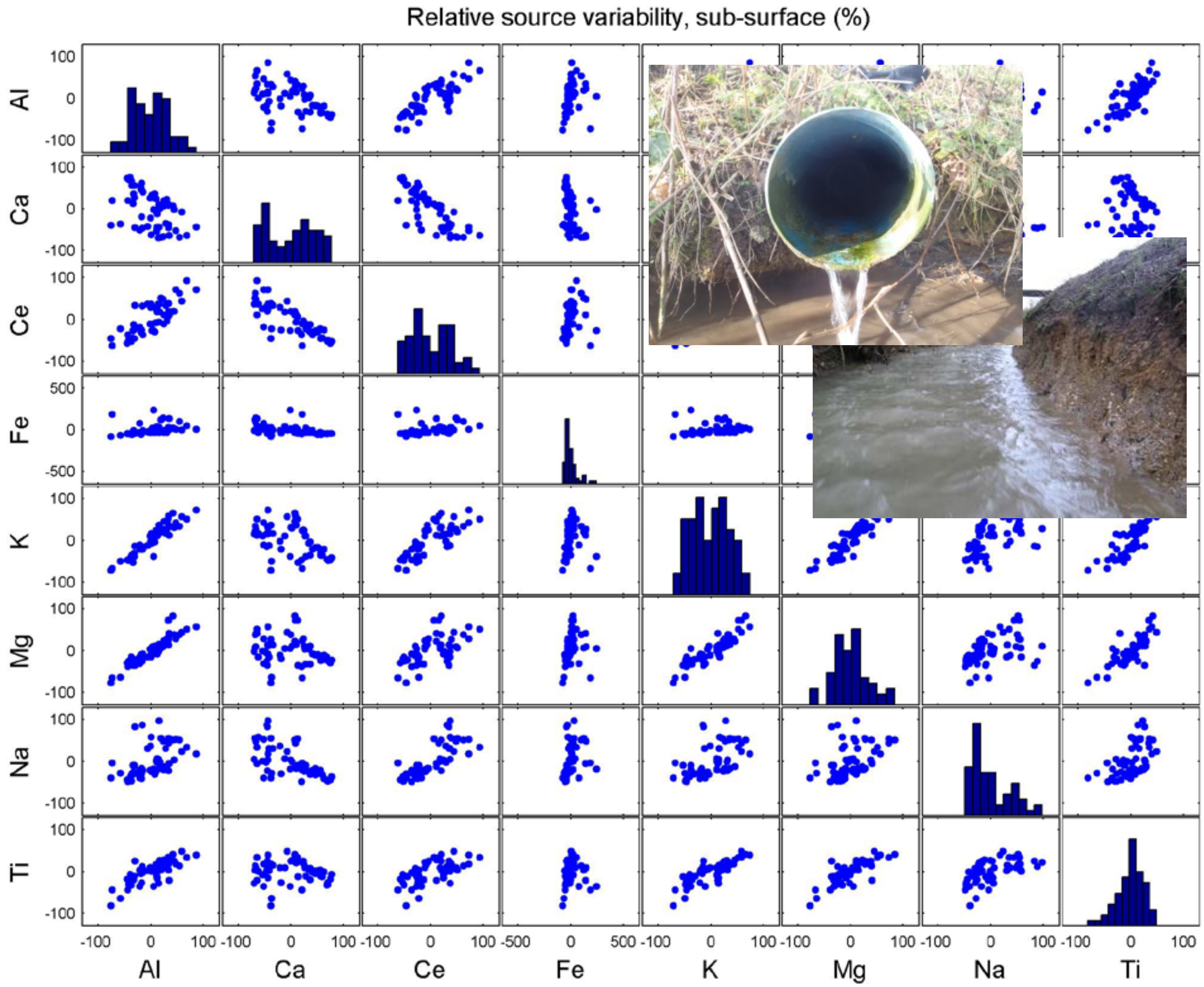
$$\begin{pmatrix} Al \\ Ca \\ Ce \\ Fe \\ K \\ Mg \\ Na \\ Ti \end{pmatrix} \text{ / \% by mass}$$

Study site



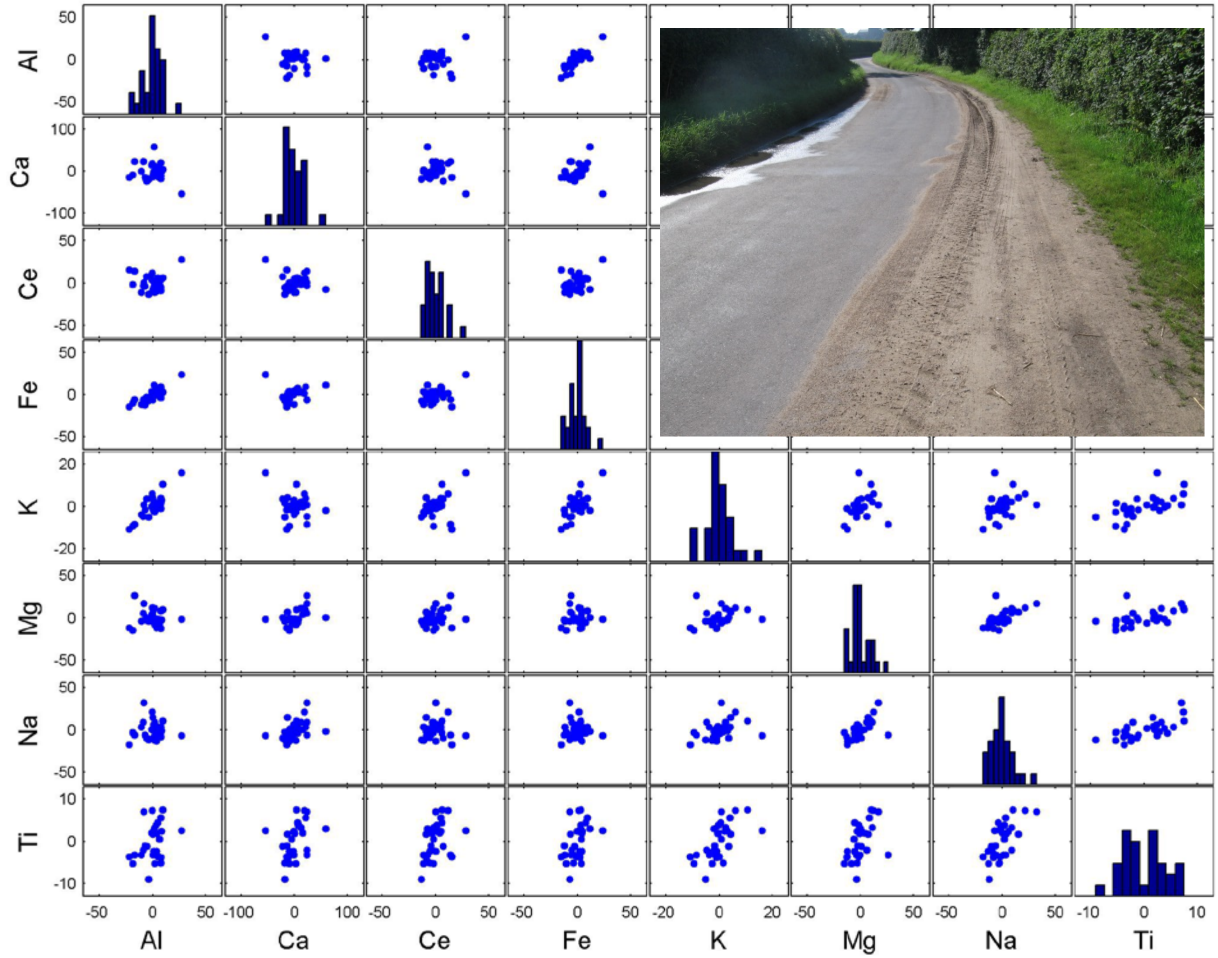
England
North Sea
Blackwater Sub-catchment
Wensum Catchment
Norwich
Norfolk
N
A
B
C
D
E
F
M
52°48'0"N
52°47'0"N
52°46'0"N
1°4'0"E
1°6'0"E
1°8'0"E
1°10'0"E
0 0.3 0.6 1.2 1.8 2.4 Kilometers
Legend
Mini Monitoring Site
Main Monitoring Site
Channel Bank Samples
Topsoil Samples
Road Verge Samples
Field Drain Samples
Streams
Mini-Catchments
Feature Boundaries

Source variability

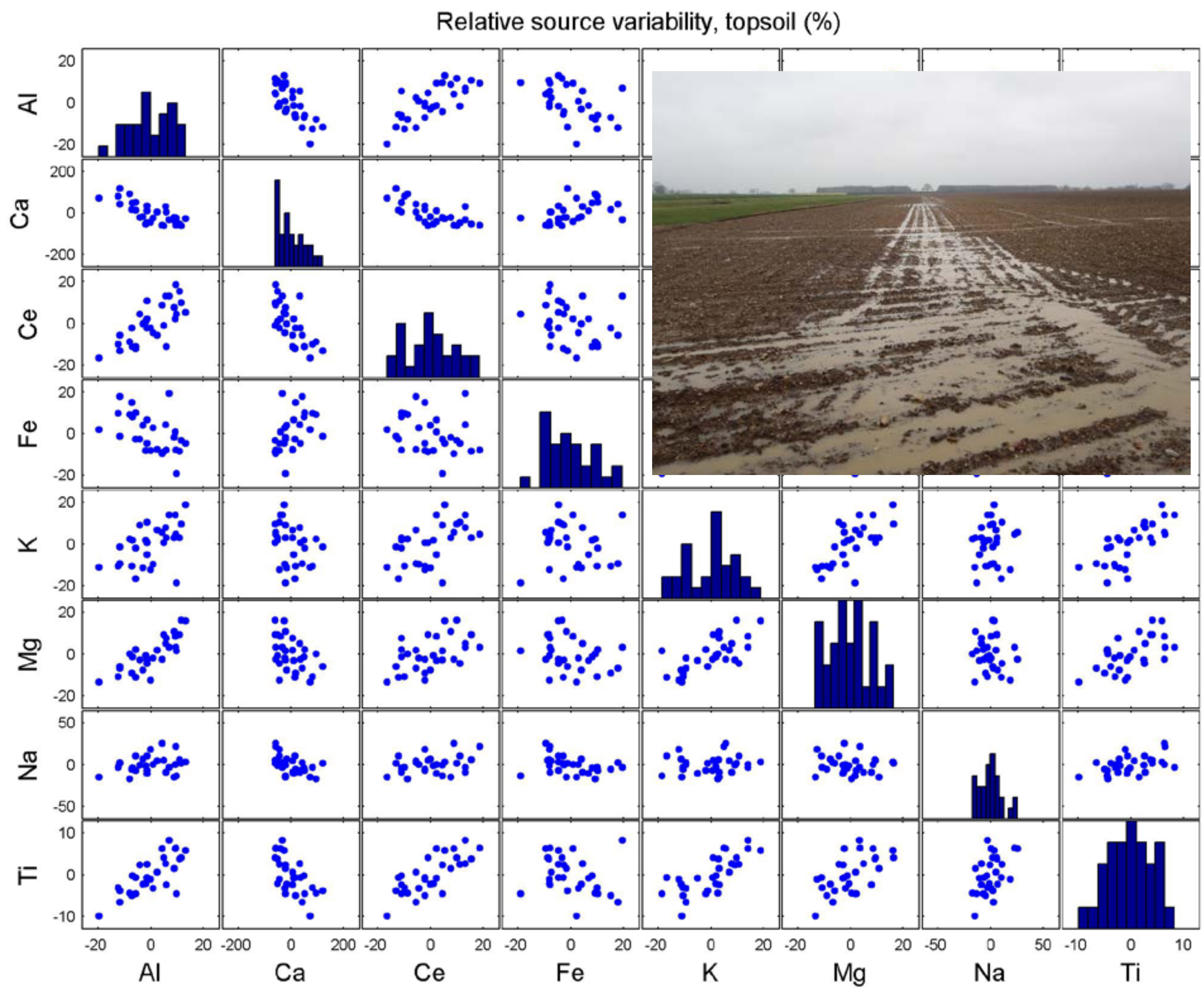


Source variability

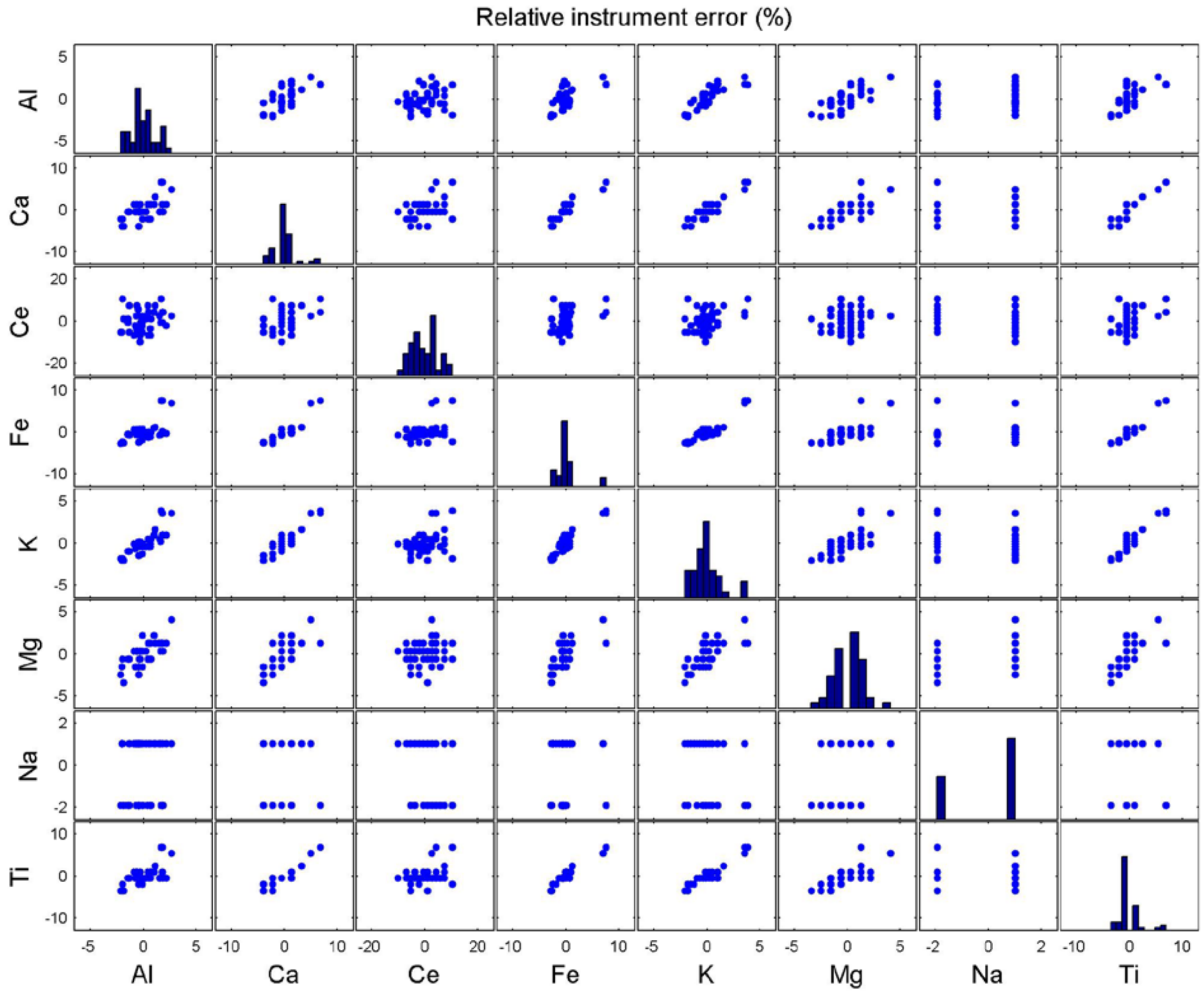
Relative source variability, road verges (%)



Source variability



Instrument error

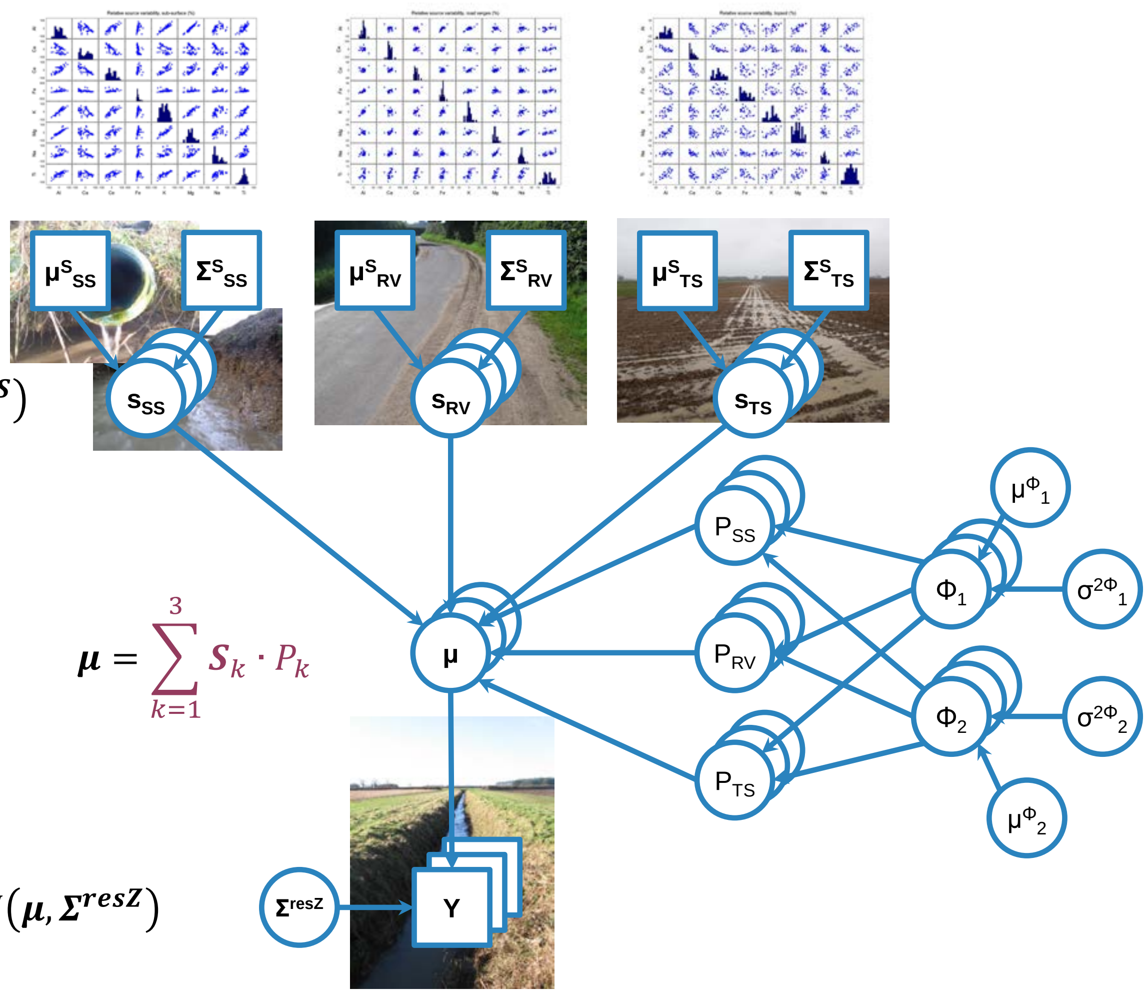


The Bayesian statistical approach

$$p(\boldsymbol{S}) = MVN(\boldsymbol{\mu}^{S}, \boldsymbol{\Sigma}^{S})$$

$$\boldsymbol{\mu} = \sum_{k=1}^{3} \boldsymbol{S}_k \cdot P_k$$

$$p(\boldsymbol{Y}) = MVN(\boldsymbol{\mu}, \boldsymbol{\Sigma}^{resZ})$$



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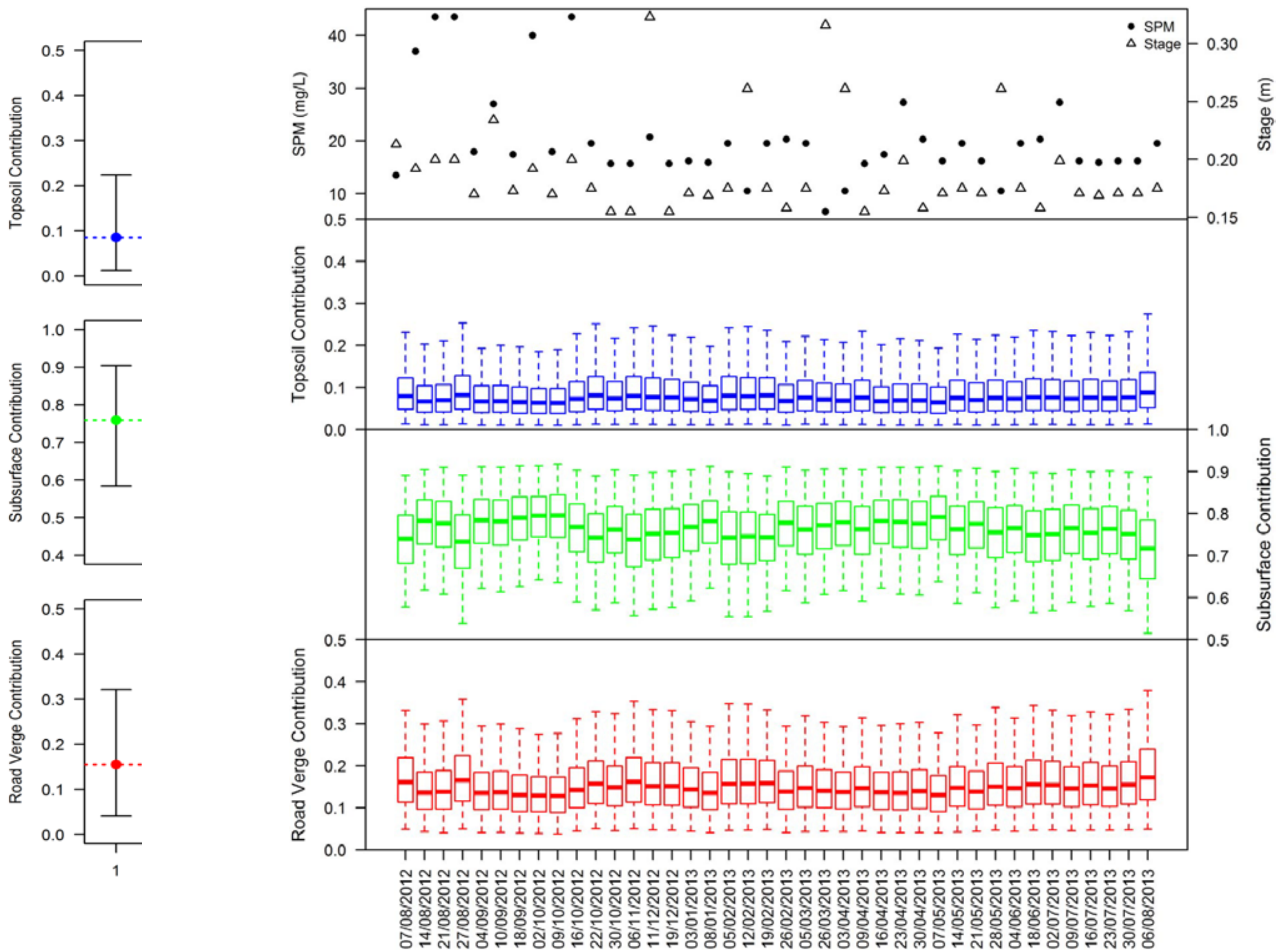
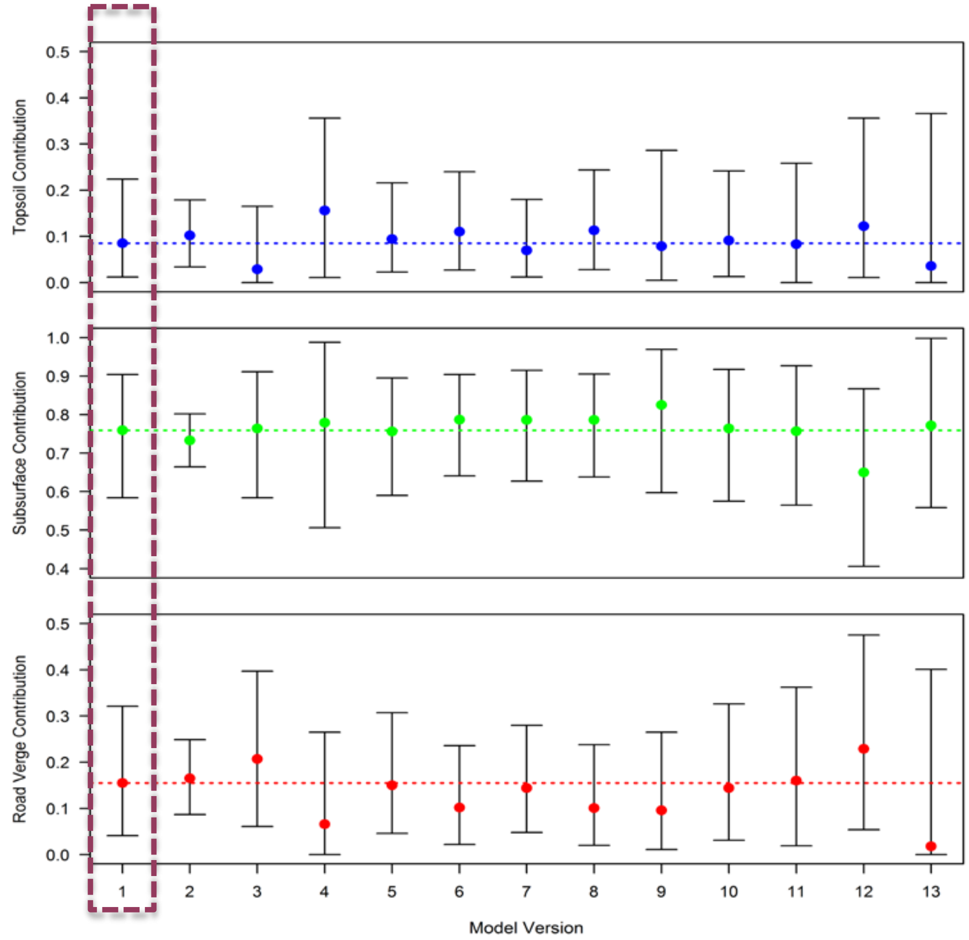
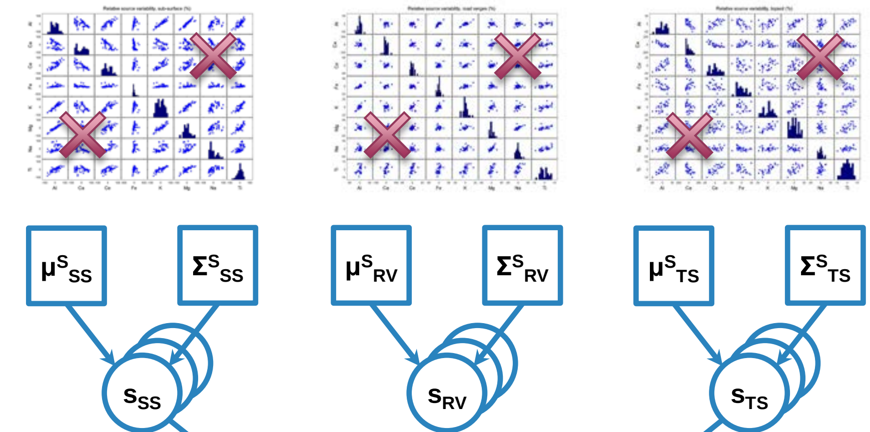
Baseflow results

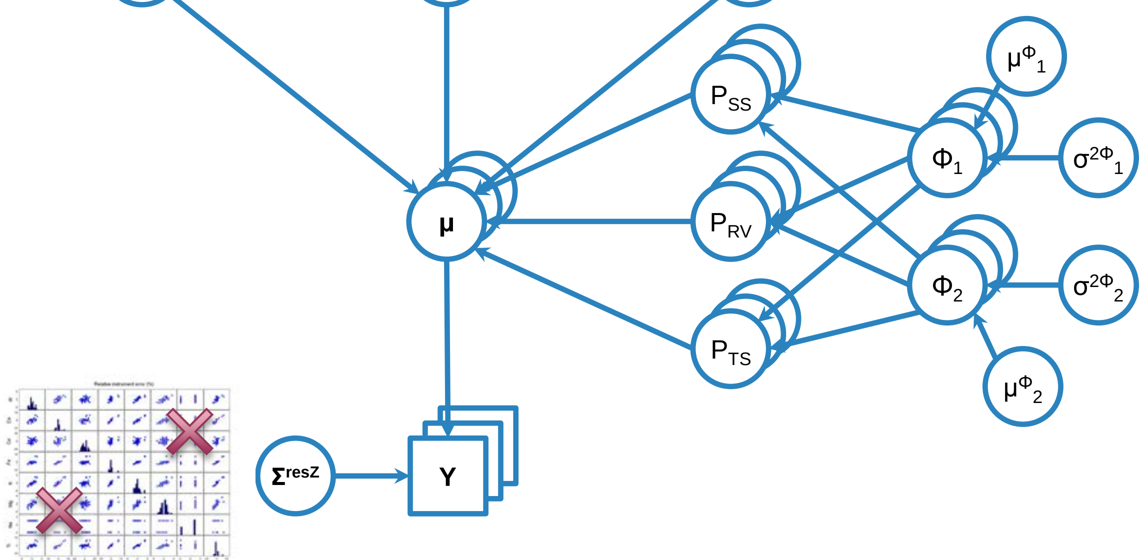


Baseflow results



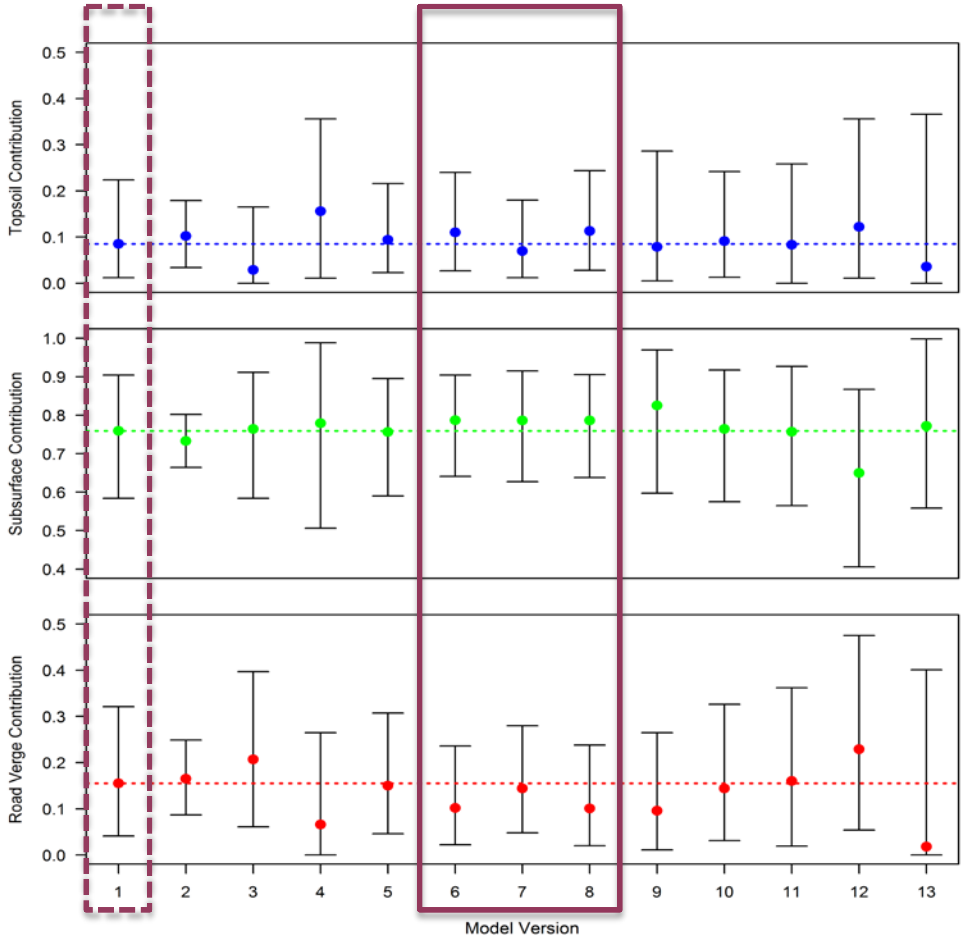
Sensitivity to covariance terms (S & Y)



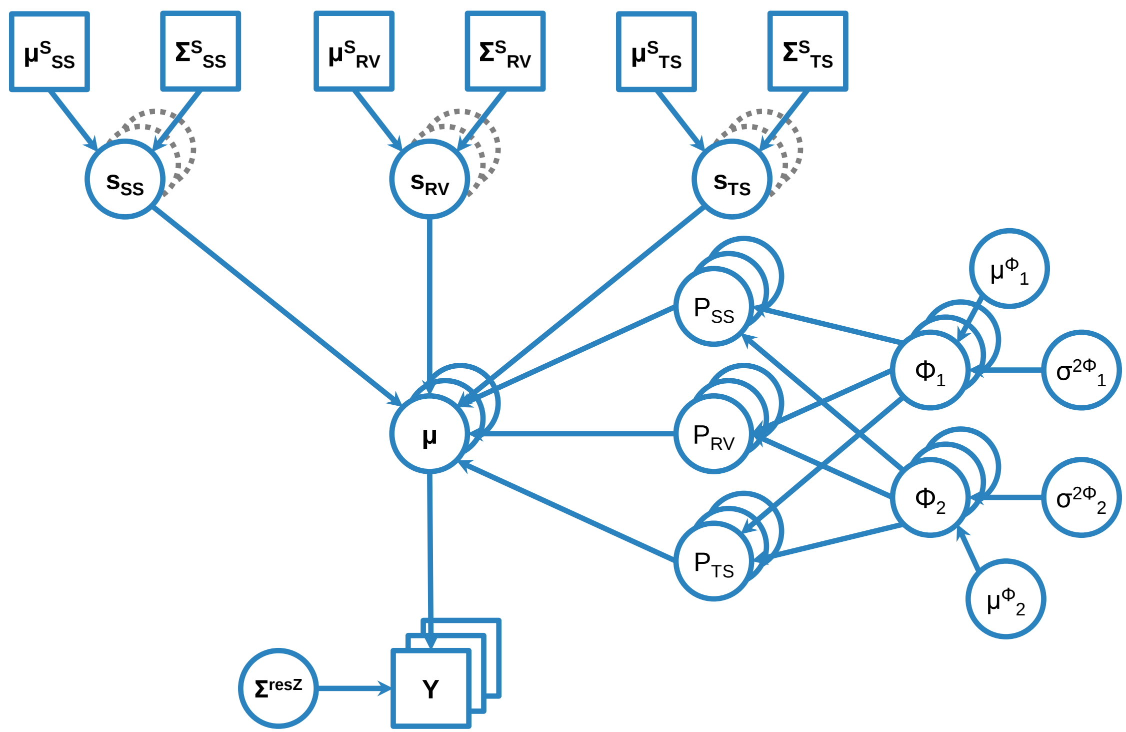


	Σ^S	Σ^{resZ}
M6	n	y
M7	y	n
M8	n	n

Sensitivity to covariance terms (S & Y)



Time-variant vs. time-invariant sources (S)



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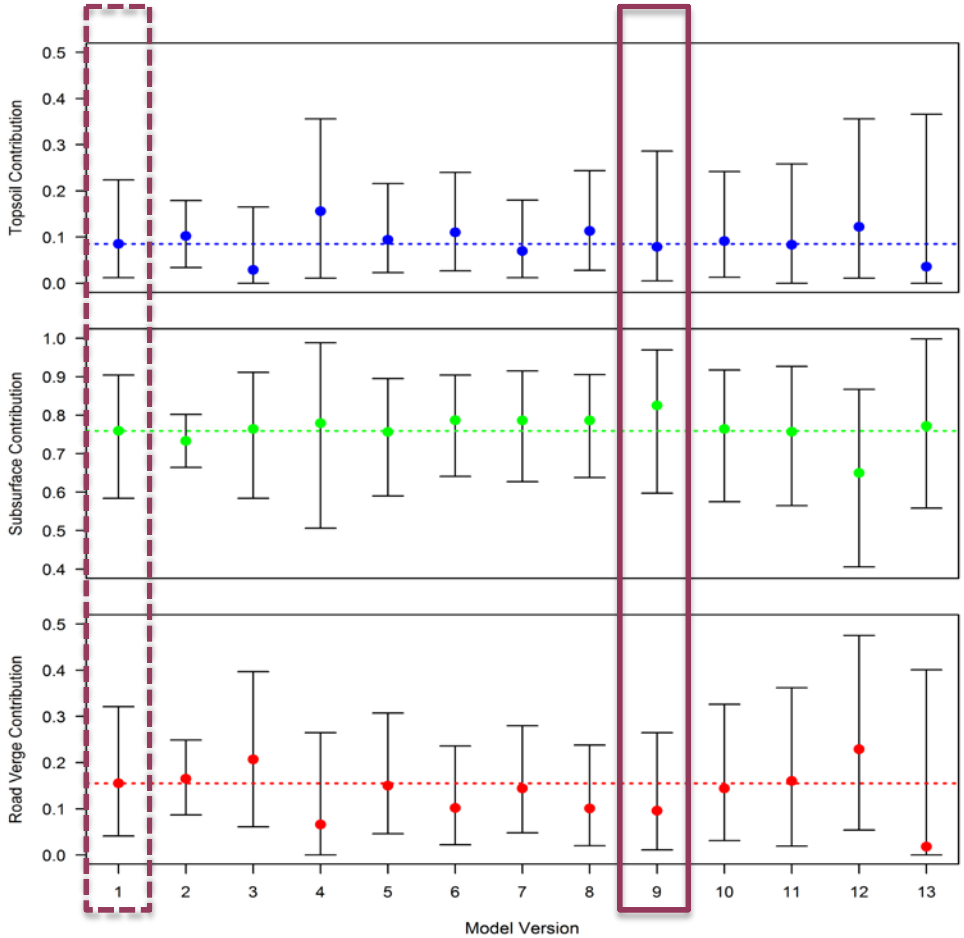
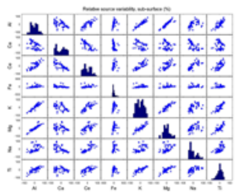
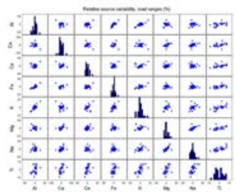
Time-variant vs. time-invariant sources (S)

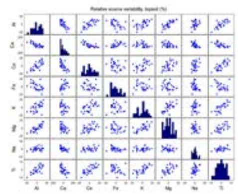


Effect of common error assumptions

Sample randomly from sources (S)







Optimise proportions (P) by Least Squares

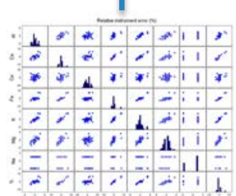
$$min\left(\sum_{j=1}^{8}\left(Y_j-\sum_{k=1}^{3}\boldsymbol{S}_{jk}\cdot P_k\right)^2\right)$$

$$\sum_{k=1}^{3}P_k=1;\ P_k\geq 0$$

Apply goodness-of-fit criterion

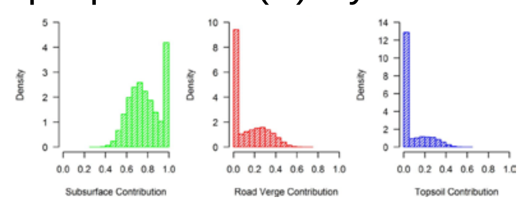
$$\frac{1}{8}\cdot\sum_{j=1}^{8}\frac{Y_j-\sum_{k=1}^{3}\boldsymbol{S}_{jk}\cdot P_k}{Y_j}<0.05$$

$$p(\boldsymbol{Y})=N\left(\boldsymbol{\mu}=\sum_{k=1}^{3}\boldsymbol{S}_k\cdot P_k\,,\sigma^2\right)$$

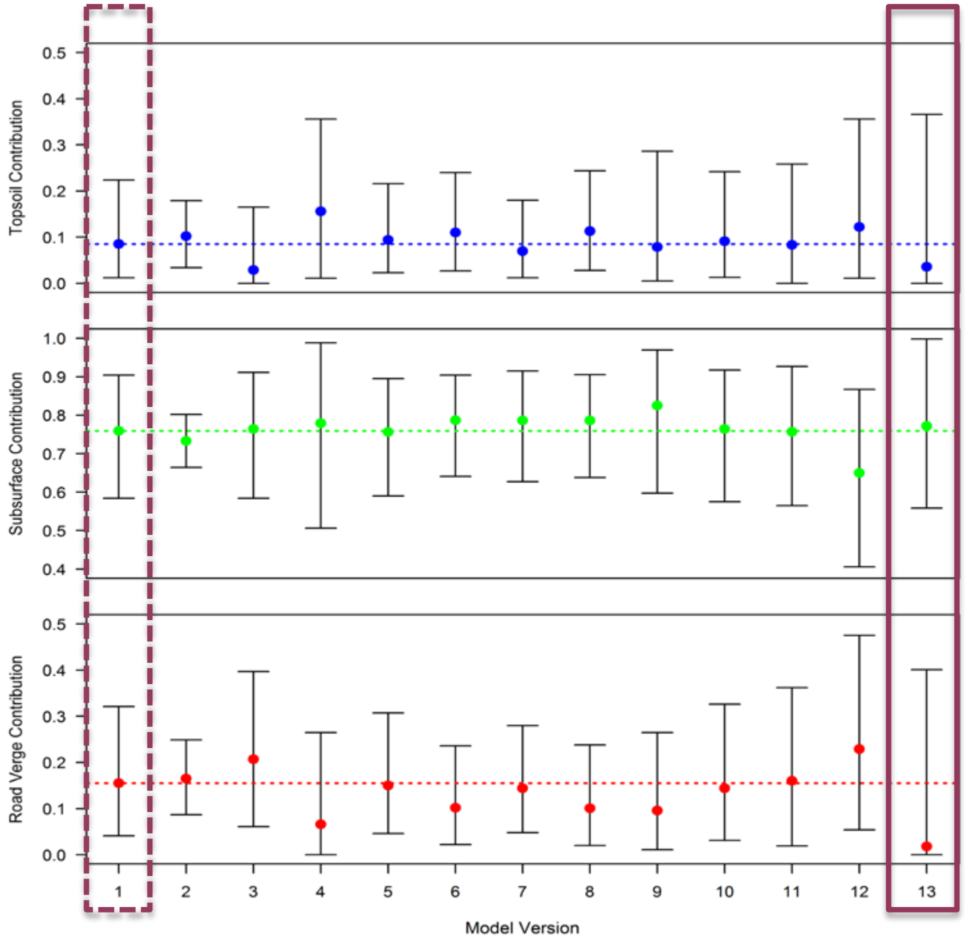


Sample randomly from target (Y)

Summarise proportions (P) by relative frequency



Effect of common error assumptions



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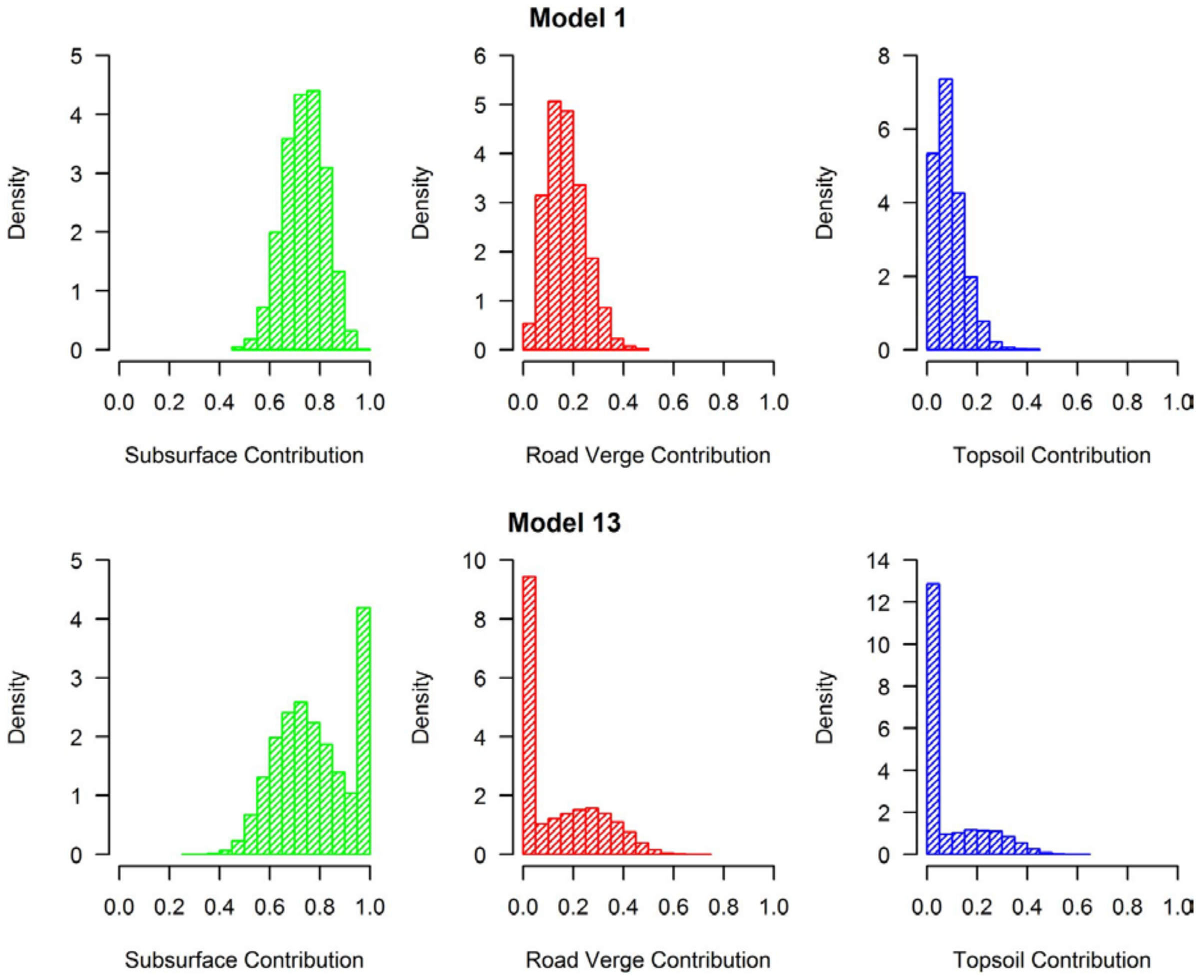
Edge artefacts of the common approach



Conclusions

- Mixing model error assumptions impacted significantly on source apportionment estimates
- Omitting fingerprint covariance resulted in biased source apportionment estimates
- Time-variant sources may account for erodability & connectivity differences in space & time within each source
- If these played a role then omitting them biased the results
- The common source apportionment approach was prone to edge artefacts resulting in overly skewed distributions
- Source apportionment studies should carefully consider & justify their (implicit) mixing model error assumptions

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