Study on individual stochastic model of GNSS observations for precise kinematic applications

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1. MOTIVATION

The proper definition of mathematical positioning model, which is defined by **functional** and **stochastic** models, is a prerequisite to obtain the optimal estimation of unknown parameters. Especially important in this definition is realistic modelling of stochastic properties of observations, which are more receiver-dependent and time-varying than deterministic relationships. This is particularly true with respect to precise kinematic applications which are characterized by weakening model strength. In this case, incorrect or simplified definition of stochastic model causes that the performance of ambiguity resolution and accuracy of position estimation can be limited.

In this study we investigate the methods of describing the measurement noise of GNSS observations and its impact to derive precise kinematic positioning model. In particular stochastic modelling of individual components of the variance-covariance matrix of observation noise performed using observations from a zero baseline is analyzed. Experimental test results indicate that the utilizing the individual stochastic model of observations including **elevation dependency** and **cross-correlation** instead of assumption that raw measurements are independent with the same variance improves the performance of **ambiguity resolution** as well as rover **positioning accuracy**. This shows that the proposed stochastic assessment method could be a important part in complex calibration procedure of GNSS equipment.

2. METHODOLOGY

Code (P) and carrier-phase (L) pseudorange model:

$$P=\|\mathbf{R}\|+\delta I+\delta T+\delta m+\delta\rho_{sat}+c(\delta t_{rec}-\delta t_{sat})+\varepsilon_P$$
$$L=\|\mathbf{R}\|-\delta I+\delta T+\delta m+\delta\rho_{sat}+c(\delta t_{rec}-\delta t_{sat})+\lambda N+\varepsilon_L$$

Double-differenced code and carrier-phase observation **residuals** for **a zero baseline**:

$$\boxed{\nabla\Delta P=\nabla\Delta\varepsilon_P} \qquad \boxed{\nabla\Delta \mathrm{L}-\lambda\nabla\Delta N=\nabla\Delta\varepsilon_L}$$

Variance and **covariance** of code and carrier-phase noise have been determined as a average value for **5 degrees** elevation bins, according to formulas:

$$\mathrm{Var}\,(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2 \qquad \mathrm{Cov}\,(X,Y)=\mathrm{E}(X\cdot Y)-\mathrm{E}(X)\mathrm{E}(Y) \qquad \mathrm{Std}\,(P)=(\mathrm{Var}\,(\nabla\Delta P)/4)^{1/2}$$

Observation noise model - **elevation dependency**:

$$\boxed{\mathrm{Var}\,(X)=(a+b/\sin^{n}el)^2}$$

Cross-correlation noise model:

$$\boxed{\mathrm{Cov}\,(XY)=a+b/\sin^{n}el}$$

3. EXPERIMENT DESIGNE

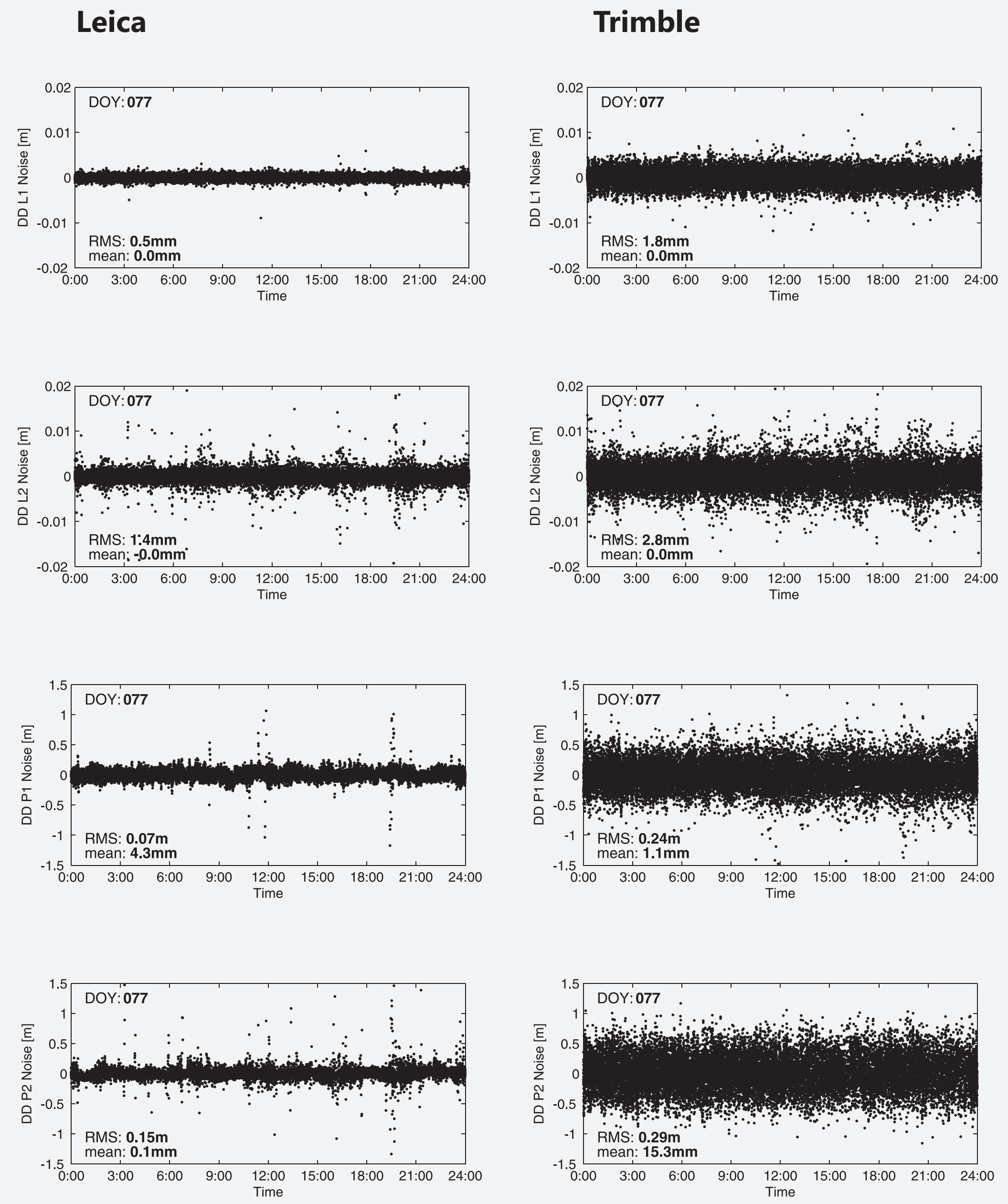
Stochastic properties of observations were determined on **a zero baseline** at WUT1 station located on the roof of Warsaw University of Technology Main Building. Two sets of GPS receivers were tested:
Leica: Leica GX1230GG S. No. 466566, 468950
Trimble: Trimble 4700 S. No. 0220224423, 0220224414

The data used:
Observations: GPS - L1 / L2 / P1 / P2
Time: **5 days** (doy: 073 - 077, 2015) / 24^h / sample rates: 30 sec. (2880 epoch) / cut-off angle: **5 deg**.

Compared models:
#1**Standard**: $\sigma_{L1/2}=0.003$ m; $\sigma_{P1/2}=0.30$ m; $\mathrm{cov}_{P/L}=0$
#2**Elevation**: σ_{Li} / σ_{Pi} → individual determined models; $\mathrm{cov}_{P/L}=0$
#3**Elevation + cross-correlation**: σ_{Li} / σ_{Pi} / $\mathrm{cov}_{L1\,L2}$ / $\mathrm{cov}_{P1\,P2}$ / $\mathrm{cov}_{L1\,P1}$ / $\mathrm{cov}_{L2\,P2}$ → individual determined models

Positioning model:
Functional model: Geometry-Based
Observables: Double-differenced, Reference satellite: max. elevation
Ambiguity resolution: **Instantaneous** (1 epoch)
ILS estimation method: MLAMBDA
ILS validation test: R-ratio (c=2.0)

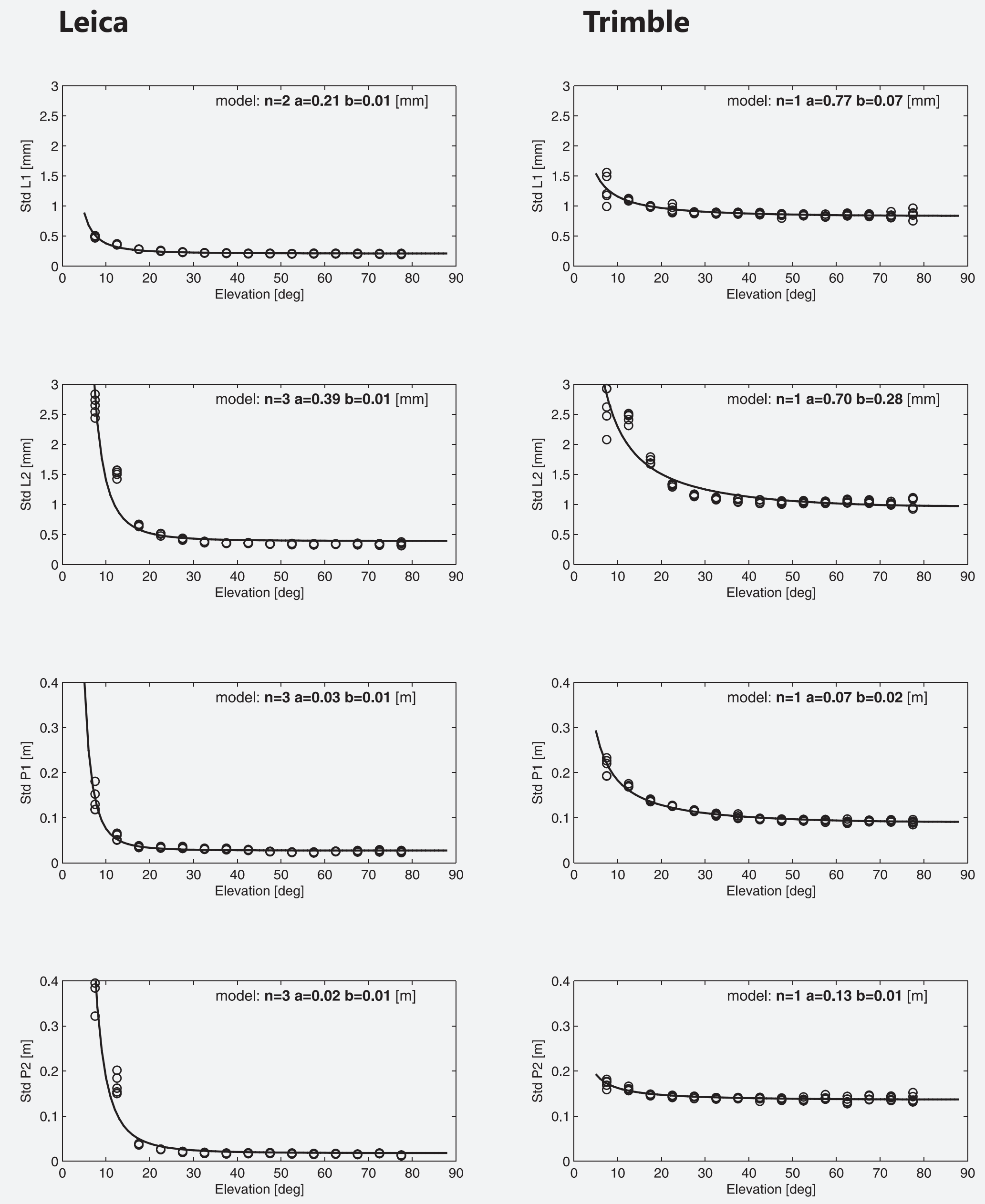
4. TEST RESULTS



RMS of DD observation **noise** (cut-off angle: 10 deg., 5 days)

	RMS [mm]	
DD Noise	**Leica**	**Trimble**
L1	0.5	1.8
L2	1.4	2.8
P1	71.6	237.3
P2	135.2	288.0

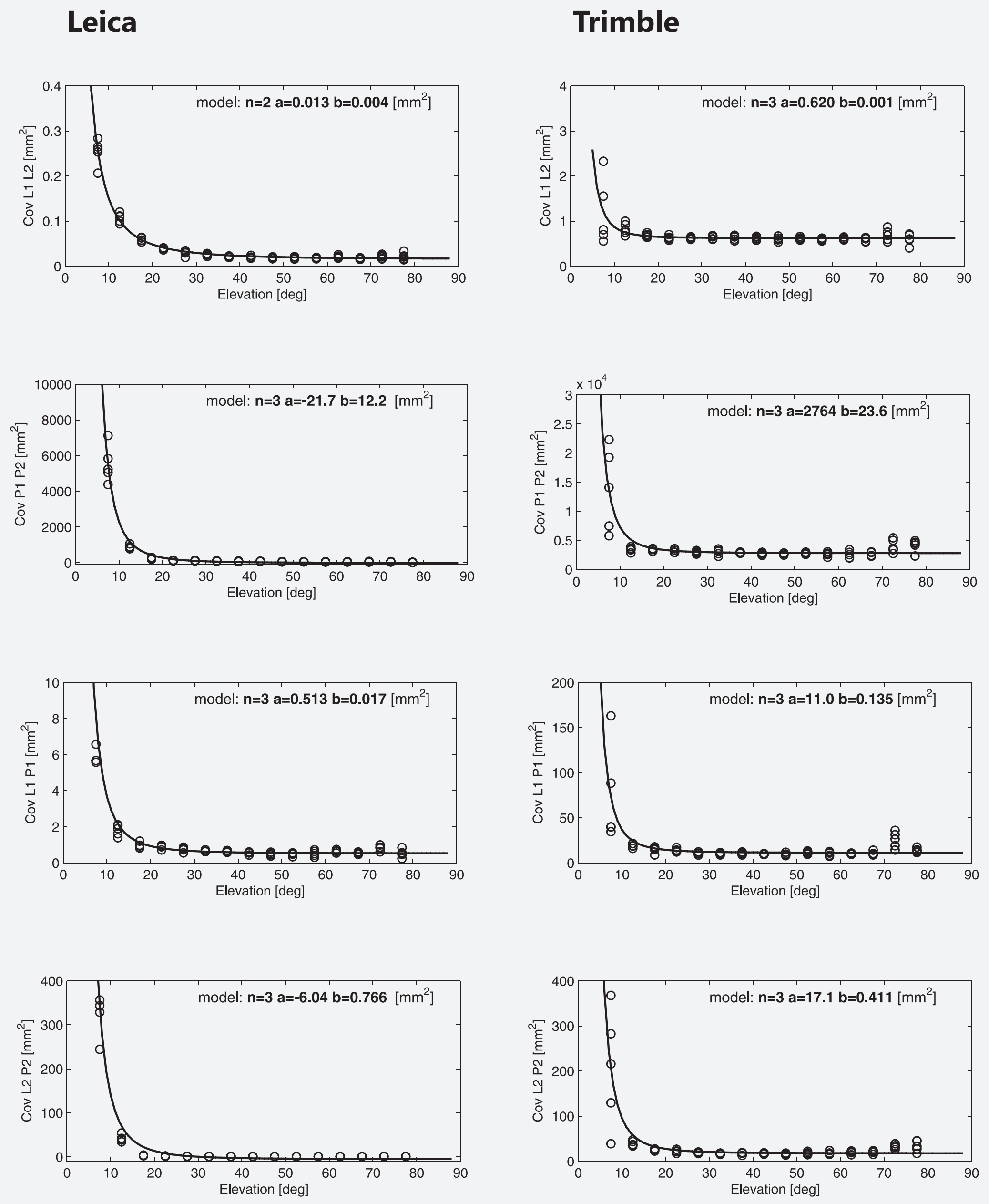
4A. ELEVATION DEPENDENCY



Parameters of **variance** model for undifferenced observations

	Leica			**Trimble**		
Std.	n	a	b	n	a	b
L1	2	0.21mm	0.01mm	1	0.77mm	0.07mm
L2	3	0.39mm	0.01mm	1	0.70mm	0.28mm
P1	3	0.03m	0.01m	1	0.07m	0.02m
P2	3	0.02m	0.01m	1	0.13m	0.01m

4B. CROSS-CORRELATION



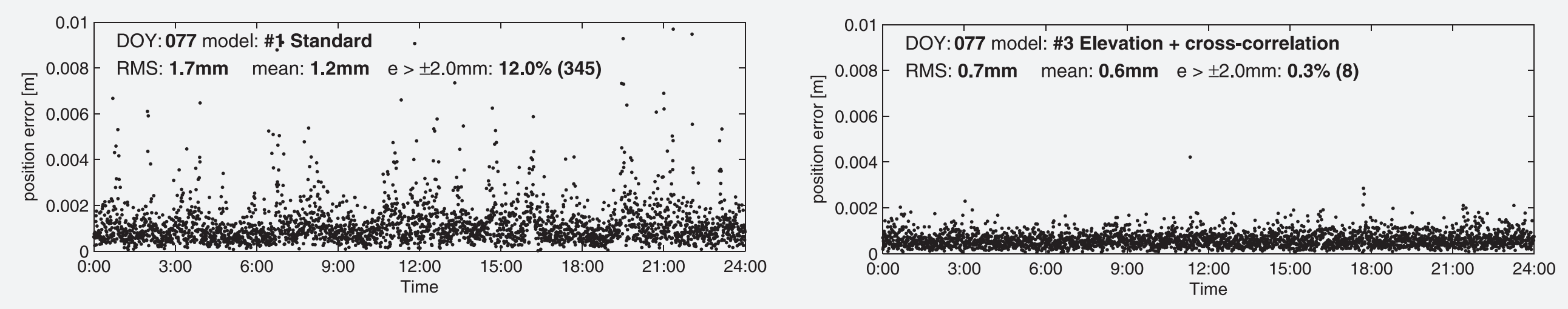
Parameters of **covariance** model for undifferenced observations

	Leica			**Trimble**		
Cov.	n	a	b	n	a	b
L1 L2	2	$0.013mm^2$	$0.004mm^2$	3	$0.620mm^2$	$0.001mm^2$
P1 P2	3	$-21.7mm^2$	$12.2mm^2$	3	$2764mm^2$	$23.6mm^2$
L1 P1	3	$0.513mm^2$	$0.017mm^2$	3	$11.0mm^2$	$0.135mm^2$
L2 P2	3	$-6.04mm^2$	$0.766mm^2$	3	$17.1mm^2$	$0.411mm^2$

4C. POSITIONING PERFORMANCE

Results of **instantaneous** positioning performance (cut-off angle: 5 deg., int. 30 sec., **5 days**)

	Leica (no. solutions: 2621-2880)			**Trimble** (no. solutions: 2760-2879)		
	#1**Standard**	#2**Elev.**	#3**C.-corr.**	#1**Standard**	#2**Elev.**	#3**C.-corr.**
failure AR [no. solutions]	1-2	0	0	0	0	0
RMS [mm]	1.62-1.72	0.69-0.70	0.68-0.70	3.21-3.75	2.85-3.38	2.82-3.41
mean position error [mm]	1.2	0.6	0.6	2.8-3.0	2.4-2.7	2.4-2.6
max position error [mm]	14.8-22.6	2.6-4.1	2.8-4.2	17.6-37.7	14.6-40.7	14.3-45.5
pos. error > 5mm [rate]	1.4-1.8%	0.0%	0.0%	8.4-10.8%	5.0-7.9%	5.0-7.9%
pos. error > 2mm [rate]	10.3-12.0%	0.3-0.4%	0.2-0.3%	62.8-65.8%	54.9-57.9%	53.9-56.9%
pos. error > 1mm [rate]	41.1-44.9%	11.2-11.8%	10.5-11.7%	92.1-93.0%	89.8-90.4%	88.7-90.4%



5. CONCLUSIONS

- The stochastic modelling of individual components of the variance-covariance matrix of observation noise allows for **increased reliability of solution** in both the **ambiguity resolution** and **solution accuracy** aspects;
- The use of individual determined models of observations noise and cross-correlation is especially important for **kinematic application** based on a **single observational epoch** where instantaneous stochastic properties of observations could be significantly differ from parameters of standard stochastic model;
- The different types of GNSS receivers are characterized by **significantly different** stochastic parameters;
- The **elevation dependency model** shows good capability to model the variance as well as the cross-correlation functions;
- Presented approach of stochastic modelling of GNSS observation can be important part in a comprehensive **calibration procedure** of GNSS equipment.

European Geosciences Union General Assembly 2015
Vienna | Austria | 12-17 April 2015

This research is supported by **Polish National Science Centre** (No 2012/07/N/ST10/03728)
and **Faculty of Geodesy and Cartography** Warsaw University of Technology