Alternative method for estimating the cross-sectional interpolation errors of discharge measurements using the velocity-area method

INTRODUCTION AND OBJECTIVE

Discharge measurements are used for multiple purposes including hydrological forecasting. **Uncertainties have to be estimated** to provide information about the quality of the measurement to the data user.

The velocity-area method is one measurement technique typically used for gaugings. It consists in integrating depths and point velocities through the cross-section. Due to the limited number of point measurements, the quality of the measurement depends mainly on the sampling strategy.

Different methods of uncertainty estimation are available in the literature (ISO 748, Q+ and IVE). The main uncertainty component, noted \( u_m \), is related to the cross-sectional interpolation errors (limited number of subsections). However, the computation of this term according to these approaches does not evaluate the sampling strategy and the complexity of the cross-section. We propose a new computation of \( u_m \) component based on the FLAURE method (FLow Analog UnceRtainty Estimation).

FLAURE METHOD OVERVIEW (Despax, 2016)

Fifty-three high-resolution stream-gaugings (made with a high number of subsections) are subsampled by reducing the number of subsections to generate a sample of realistic stream-gaugings. A statistical analysis is performed to estimate the \( u_m \) component as a function of the sampling quality index (SQI).

- **High-resolution stream-gaugings**: 53 stream-gaugings (with more than 30 subsections).
- **Subsampled stream-gaugings**: The subsampling method simulates the behavior of stream gaugers by pseudo-random sampling.
- **\( u_m \) as a function of the sampling quality index**: Statistical analysis based on relative errors between subsampled and high-resolution gaugings.

The velocity-area method (Herschy 1993):

Measuring the velocities of an entire cross section is impossible, so the cross-section is divided into a number \( m \) of subsections. The discharge is the product of velocity \( v_i \), depth \( D_i \) and width \( B_i \), of each subsection \( i \):

\[
Q = \sum_{i=1}^{m} B_i D_i V_i
\]
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**HS2.3.4 : River flow monitoring:** Innovative methods and uncertainties

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**U_m as a function of the sampling quality index**

The **relative errors** can be computed for each subsampled gaugings (stream-gauging number k where k = 1; ...; 1000 with j verticals) as: 

\[ E_{j,k} = \frac{Q_{j,k} - Q_m}{Q_m} \]

A **sampling quality index** is defined as: 

\[ SQI = \frac{\sigma(\Delta q)}{\sigma(q)} \sum_{i=1}^{m} (\Delta x_i \Delta q_i) \]

where \( q_i \) is the product of velocity \( V_i \) and depth \( D_i \).

This **dimensionless** criterion has the advantage to take into account:

- The spacing between verticals (\( \Delta x_i \)) which will increase SQI if they are widely spaced,
- The variation of unit flow between two adjacent verticals (\( \Delta q_i \)) which will also increase SQI if changes in flow distribution are not enough sampled,
- A ratio between the standard deviation of adjacent measurement (\( \sigma(\Delta q) \)) and the standard deviation of unit flow (\( \sigma(q) \)). The denominator of the ratio is quite stable (intrinsic variability of the cross-section) while the numerator must reduce when the number of verticals increases and variations between adjacent verticals are low.

There is a link between the **uncertainty (taken as the standard deviation of relative errors)** and the SQI.

For any stream-gauging, the \( u_m \) component is computed depending on the SQI. \( u_m \) is finally injected in the Q+ equation (**Le Coz, 2012**)

**Results and conclusions**

This new method was applied to 3185 stream-gaugings with various flow conditions and compared with the other methods (**click here to see all results**). Results show that FLAURE method is overall consistent with the Q+ method but not with ISO 748 and IVE methods, which produce clearly overestimated uncertainties for discharge measurements with less than 15 verticals.

The FLAURE approach therefore appears to be a consistent method.
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**EXISTING METHODS**

The following methods are based on the Guide to the Expression of Uncertainty in Measurement (JCGM 2008).

The $u_m$ component computation only is presented below.

- **ISO 748 (ISO, 2009)**

$$u^2(Q) = u_s^2 + u_m^2$$

$$u_m = \sum_{i=1}^{m} Q_i \left[ a_i^2(B_i) + a_i^2(D_i) + a_i^2(V_i) + \frac{1}{n_i} \left\{ a_i^2(V_i) + u_m^2(V_i) \right\} \right]$$

- **Q+ (Le Coz, 2012)**

$$u^2(Q) = u_s^2 + \frac{\sum_{i=1}^{m} Q_i^2 \left\{ a_i^2(B_i) + a_i^2(D_i) + a_i^2(V_i) + u_m^2(D_i) + u_m^2(V_i) \right\}}{\left( \sum_{i=1}^{m} Q_i \right)^2}$$

- **IVE (Kiang 2009, Cohn 2013)**

$$u^2(Q) = u_s^2 + \frac{\sum_{i=1}^{m} Q_i^2 \left\{ a_i^2(B_i) + a_i^2(V_i) + u_{IVG}(D_i) \right\}}{\left( \sum_{i=1}^{m} Q_i \right)^2}$$

$$u_{IVG}(D_i) = \sqrt{\frac{\sum_{i=1}^{m-2} \Delta_i^2 D}{\sum_{i=1}^{m-1} \Delta_i^2}}$$

$$\Delta_i^2 D = D_i - D_{i+1}$$

$$\Delta_i = D_i - D_{i-1}$$

$$\Delta_i = \frac{x_{i+1} - x_i}{x_{i+1} - x_{i-1}}$$

$$x_i = \frac{d_i + d_{i+1}}{2}$$

The $u_m$ term accounts for an average 84% of the final expanded uncertainties. This component depends only on the number $m$ of verticals (or subsections). Their spatial distribution, the complexity of the riverbed shape and flow distribution are not taken into account. These empirical values are based on non-traceable experiments.

It suggests dividing $u_m$ into uncertainties due to transverse integration of depths $u_m(D_i)$ and averaged velocities $u_m(V_i)$ while including discharge extrapolations to the edges. The authors defined an angle $\alpha$ as the maximum transverse slope in order to quantify riverbed irregularity. It is limited by the difficulty of choosing the value of $\alpha$ which has a large influence on the final uncertainty.

It estimates separately the deviation of depth and velocity measurements from the linear interpolation of adjacent measurements. It is unclear how departure from linearity can capture all the sources of error on depth or velocity. This assumption may be false when few verticals are made.
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HS2.3.4 : River flow monitoring:
Innovative methods and uncertainties

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HIGH RESOLUTION STREAM-GAUGINGS

A set of 53 high-resolution gaugings (each based on a number of verticals between 31 and 95) with various site and flow conditions is used as reference gaugings.

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**SUBSAMPLING METHOD**

The subsampling method consists in generating several streamgaugings with diverse spatial distribution of verticals while trying to simulate as well as possible the behavior of a hydrometric technician. For each number of verticals, 1000 stream-gaugings are generated and associated discharges are calculated.

A probability is affected to vertical selection depending on:
- The proximity with a vertical already selected,
- the vicinity with the shore,
- the gain of information provided by the vertical.

**Probability affected to each vertical**

**Partial discharge [m³/s] and already selected verticals**

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U\textsubscript{M} AS A FUNCTION OF THE SAMPLING QUALITY INDEX

The relative errors can be computed for each subsampled gaugings (stream-gauging number k where k = 1; . . . ; 1000 with j verticals) as:

\[ E_{j,k} = \frac{Q_{j,k} - Q_m}{Q_m} \]

A sampling quality index is defined as:

\[ SQI = \frac{\sigma(\Delta q)}{\sigma(q)} \sum_{i=1}^{m} (\Delta x_i \Delta q_i) \]

where \( q_i \) is the product of velocity \( V_i \) and depth \( D_i \).

This dimensionless criterion has the advantage to take into account:

- The spacing between verticals (\( \Delta x_i \)) which will increase SQI if they are widely spaced,
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There is a link between the uncertainty (taken as the standard deviation of relative errors) and the SQI (see Buech river gauging example below).

For any stream-gauging, the \( u_m \) component is computed depending on the SQI. \( u_m \) is finally injected in the Q+ equation (Le Coz, 2012)

See an example:
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**COMPUTATION EXAMPLE**

Altier @ La Goulette (L = 2,1 m / Q = 0,21 m³/s)

Bathymetric profile

Unit flow profile

1) The SQI is computed
   \[
   SQI = \frac{\sigma(\Delta q)}{\sigma(q)} \times \frac{\sum_i (\Delta x_i \times \Delta q_i)}{Q_{total}} = 0,15
   \]

2) The \( u_m \) component is computed depending on the SQI
   \( u_m(SQI) = 2,61\% \)

3) \( u_m \) is finally injected in the Q+ equation (Le Coz, 2012)

   \( IC (Flaure) = 7,2\% \)

   \( IC (ISO 748) = 13\% \)
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VELOCITY-AREA METHOD – SOME PICTURES
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Uncertainties computed with the four methods

Uncertainties depending on the number of verticals (at 95% confidence level) computed with ISO, Q+, IVE and Flaure methods using EDF-DTG dataset.
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References


