Using tesseroid mascons to improve the estimations of water-mass variations with GRACE

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Introduction

Space-borne geodetic sensors offer an opportunity for monitoring the terrestrial water-storage (TWS). The TWS fields, inverted from the temporal variations of the Earth's gravity field, are generally computed based on spherical harmonic coefficients, which have global carriers. Results of TWS can also be inverted from a regional recovery approach considering the *in-situ* measurements of GRACE (Gravity Recovery and Climate Experiment). An approach based on the **tesseroids** was proposed by Grombein et al. (2012) to compute the TWS where their results showed improvements in comparison with the **point-mass** solution. The mass concentration (**mascon**) solution (e.g., Rowlands et al. 2005), generally based on **surface layer**, can be modelled in terms of tesseroids. Thus, the mascon parameters (heights of the tesseroids) can be determined given the gravitational potential (or its functional) at the altitude of the spacecraft. The mascon approaches are illustrated in Fig. 1 as suggested by Watkins et al. (2015).

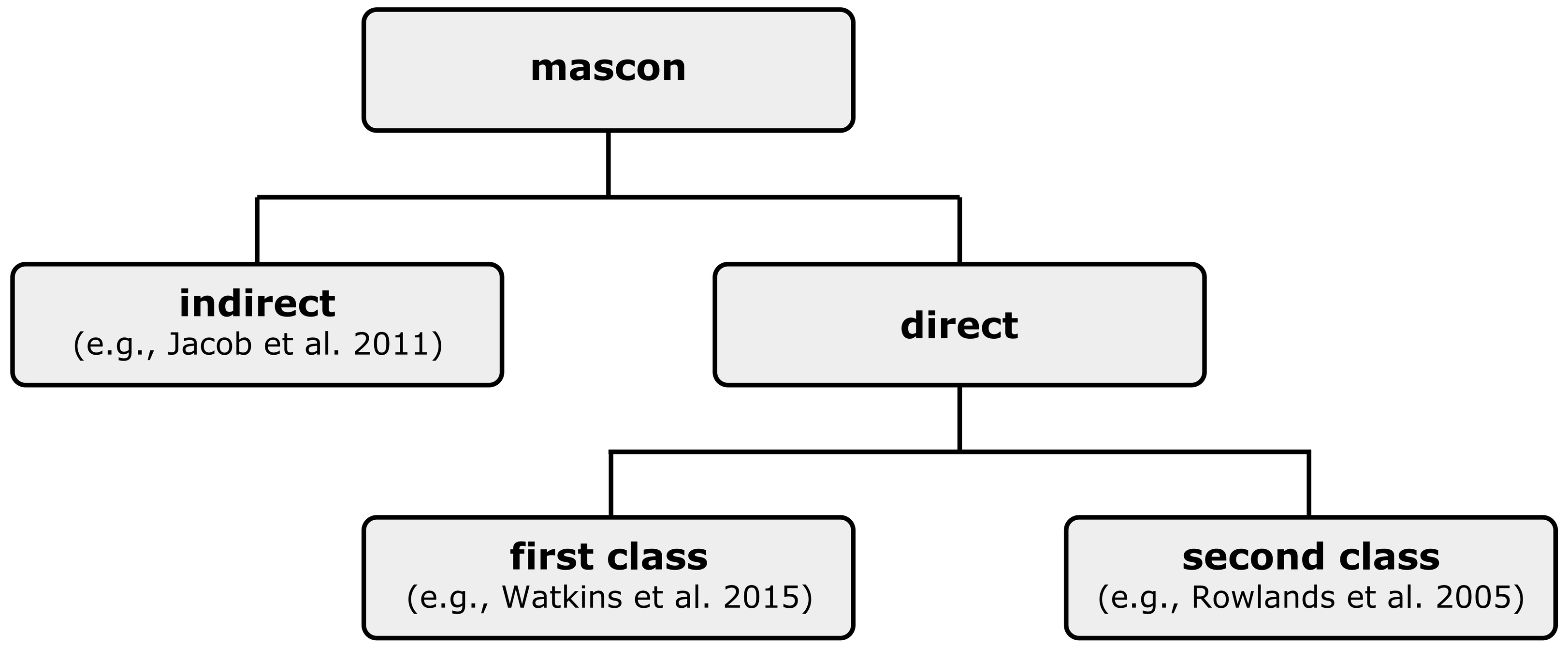

Figure 1: Classification of mascon solutions.

Here, a **flat tesseroid** is proposed for recovering the water-mass variations and, opposed to Grombein et al. (2012), the inversion is formulated as a **linear inverse problem**.

Methodology

1 Gravitational potential by tesseroids

Spherical approximation (Fig. 2):

- Height of water column $\delta h(\varphi, \lambda; t)$
- Condensation of water on sphere of radius R
- Surface density $\mu(\varphi, \lambda; t) = \varrho_w \cdot \delta h(\varphi, \lambda; t)$
- Gravitational potential:

$$\delta V_P(t) = G \iint_\sigma \frac{\mu(\varphi', \lambda'; t)}{\ell(r, \varphi, \lambda; R, \varphi', \lambda')} \mathrm{d}\sigma(\varphi', \lambda') \quad (1)$$

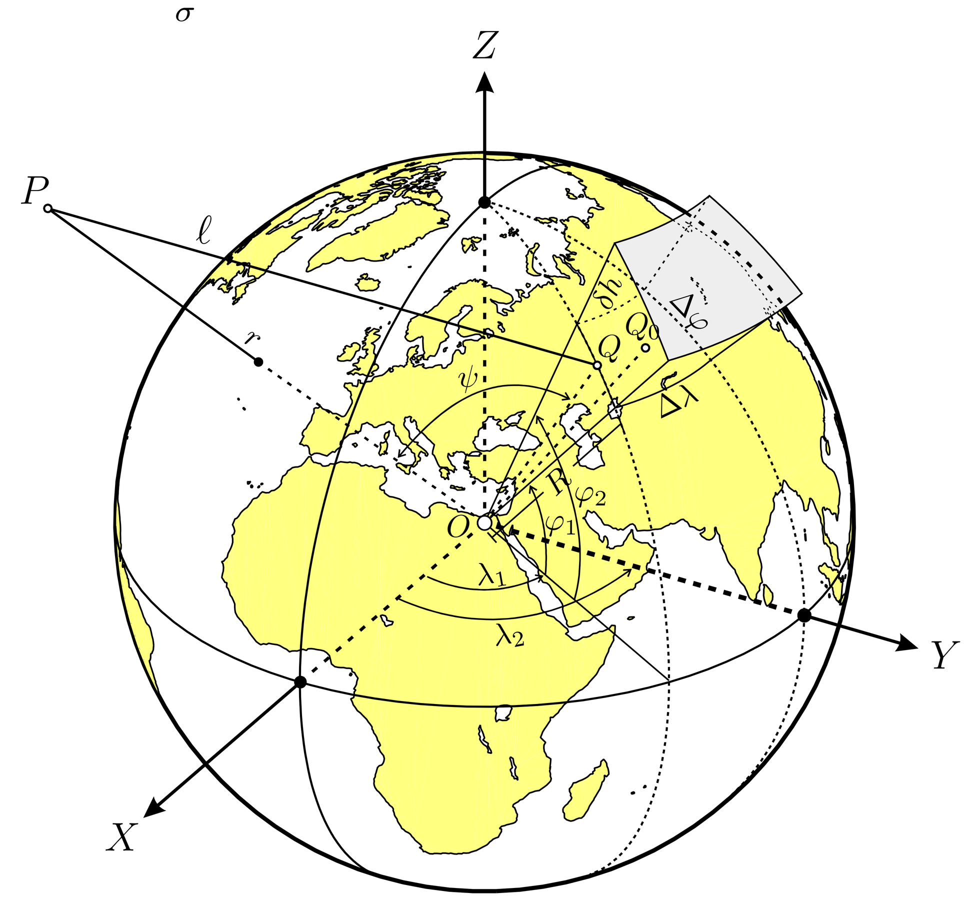

Figure 2: Geometry of a spherical tesseroid.

2 Taylor expansion of the integral kernel

For each compartment σ_k, the surface densities are replaced by their average value in this compartment, thus (1) becomes:

$$\delta V_P = G \sum_k \mu_k \iint_{\sigma_k} \frac{R^2 \cos\varphi'}{\sqrt{r^2 + R^2 - 2rR\cos\psi}} \mathrm{d}\varphi' \, \mathrm{d}\lambda' \quad (2)$$

Taylor point $Q_0(R, \varphi_0, \lambda_0)$: Central point of panel

$$\begin{bmatrix} \varphi_0 \\ \lambda_0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \varphi_1 + \varphi_2 \\ \lambda_1 + \lambda_2 \end{bmatrix} \quad (3)$$

Approximate solution by Taylor expansion of the integrand in (2):

$$K(R, \varphi', \lambda') = \sum_{i,j=0}^{\infty} K_{ij} \frac{(\varphi' - \varphi_0)^i}{i!} \frac{(\lambda' - \lambda_0)^j}{j!} \quad (4)$$

$$K_{ij} = \left. \frac{\partial^{i+j}}{\partial^i \varphi' \partial^j \lambda'} \right|_{\varphi' = \varphi_0, \lambda' = \lambda_0} \quad (5)$$

3rd order series expansion, the gravitational potential turns

$$\delta V_k = G \mu_k \Delta\varphi \Delta\lambda \times \left[K_{00} + \frac{1}{24} \left(K_{20} \Delta\varphi^2 + K_{02} \Delta\lambda^2 \right) + \mathcal{O}(\Delta^4) \right]_k \quad (6)$$

3 GRACE *in-situ* measurements and tesseroids

- The partial derivatives of the gravitational potential (or its functional) w.r.t. the satellite state and model parameters can be applied. For example, the gravitational acceleration, computed in the local coordinate system, can be used for orbit integration.
- Also the potential difference approach (Wolff 1969) can be used, for example:

$$\delta V_{AB}^{\nabla} = G \Delta\varphi \Delta\lambda \sum_k \mu_k \Big\{ \left(K_{00}^{B} - K_{00}^{A} \right) + \frac{1}{24} \times \left[\Delta\varphi^2 \left(K_{20}^{B} - K_{20}^{A} \right) + \Delta\lambda^2 \left(K_{02}^{B} - K_{02}^{A} \right) \right] \Big\}_k \quad (7)$$

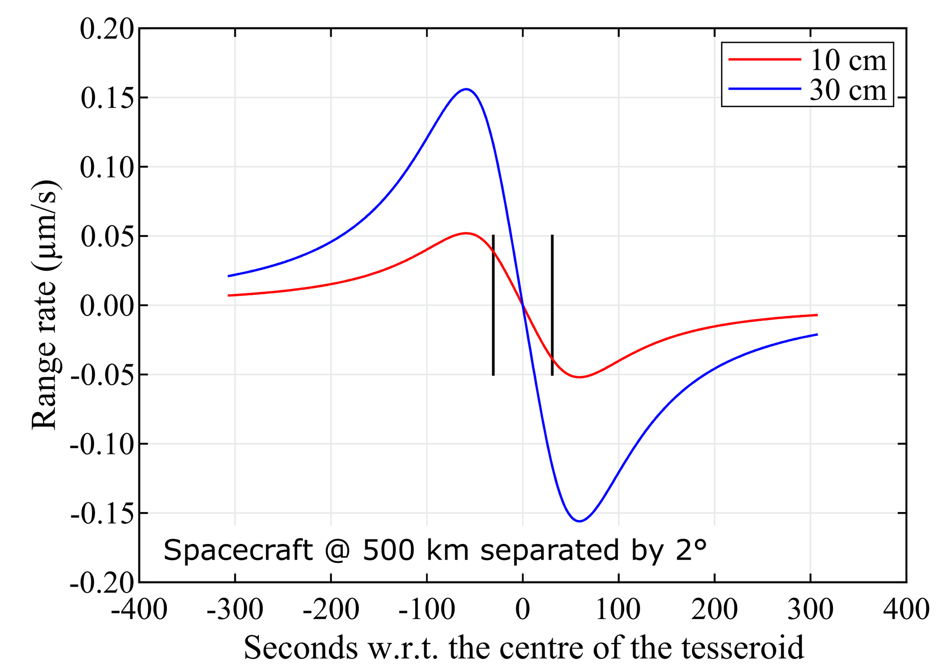

Figure 3: Simulation of GRACE KBRR signal for a direct overflight of 4°×4° mass surplus considering tesseroids of 10 and 30 cm of water column. The orbital period of GRACE satellites is approximately 90 minutes and ±10° corresponds to approximately ±154 seconds (~0.067°/sec of latitude rate).

Results

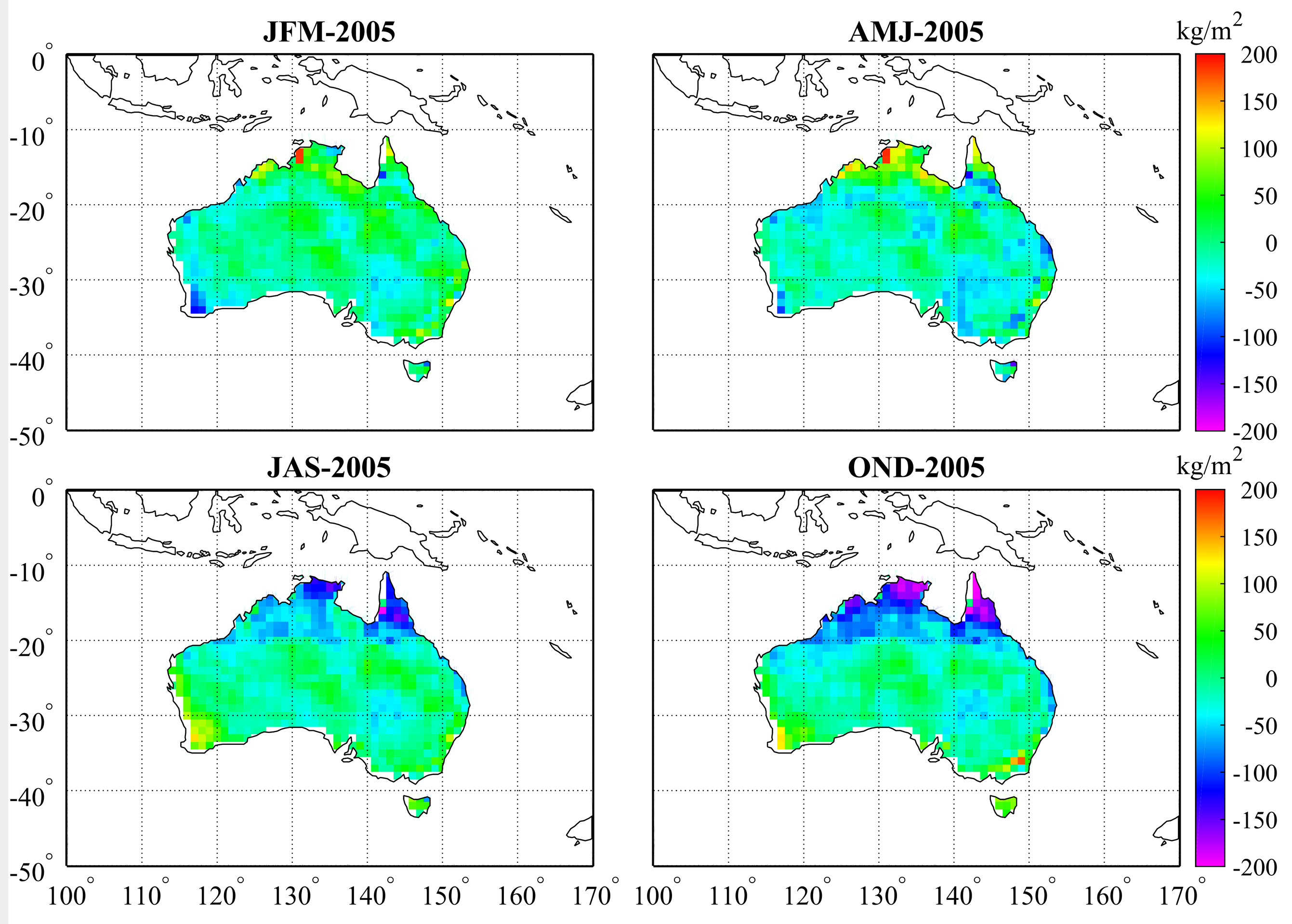

Figure 4: Simulation: GLDAS-Noah water-mass (1°×1°) averaged over the months of Jan-Mar (JFM), Apr-Jun (AMJ), Jul-Sep (JAS), and Oct-Dec (OND) for 2005.

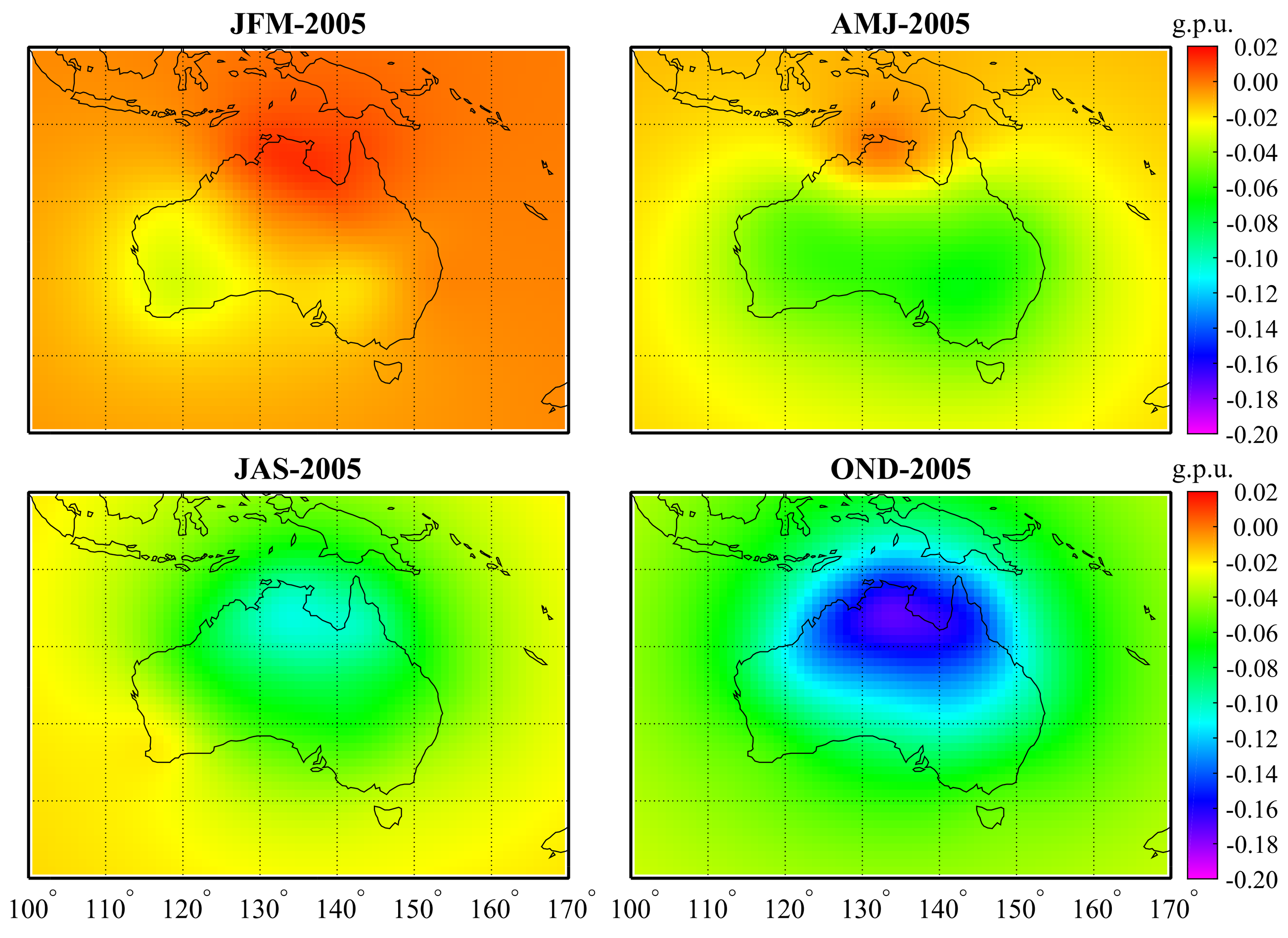

Figure 5: Synthesis: Computed gravitational potential at satellite altitude (500 km) using Eq. (6) in the units of g.p.u. (geopotential units).

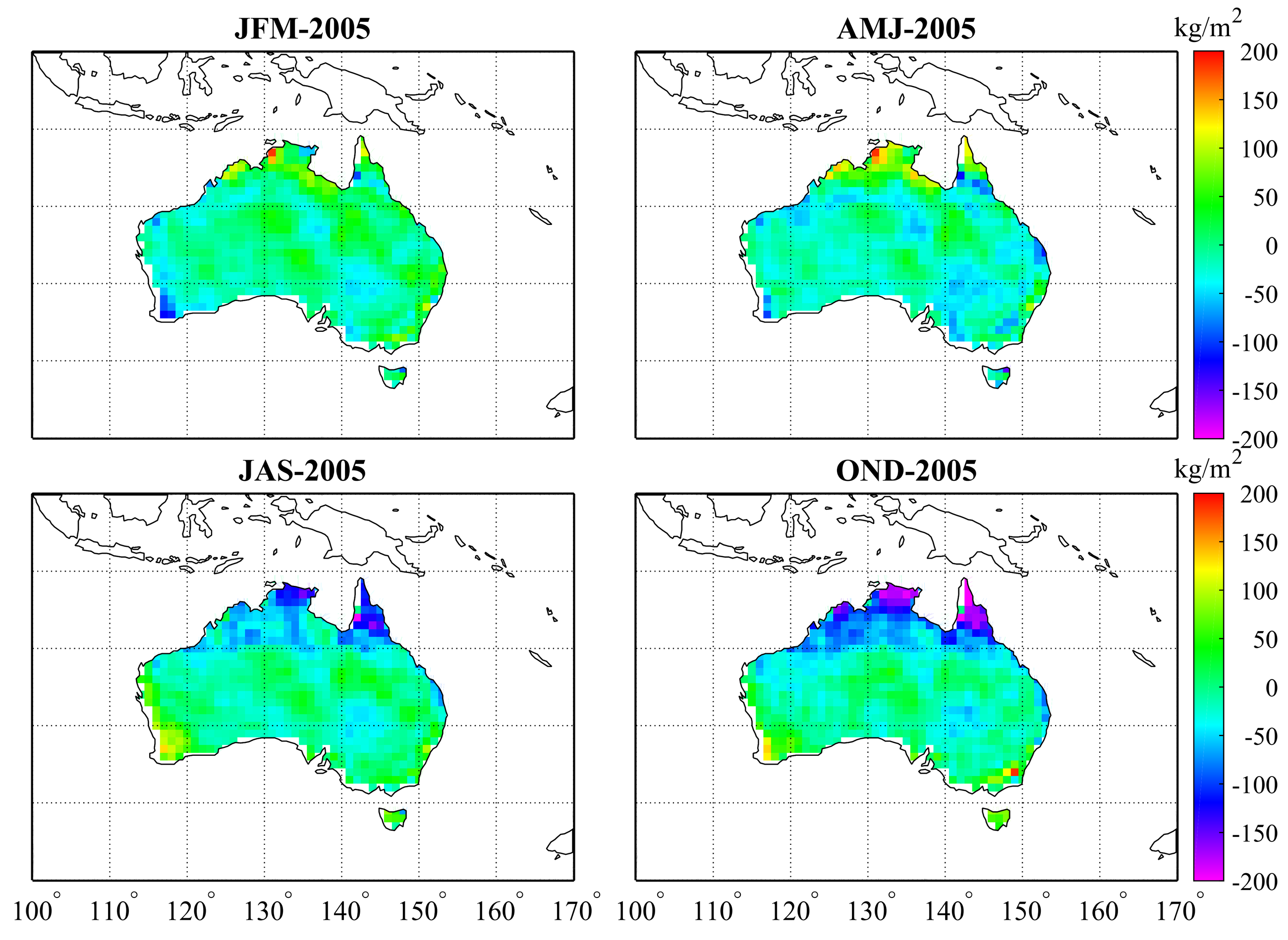

Figure 6: Analysis: inverted water-mass using generalized inverse (Moore-Penrose).

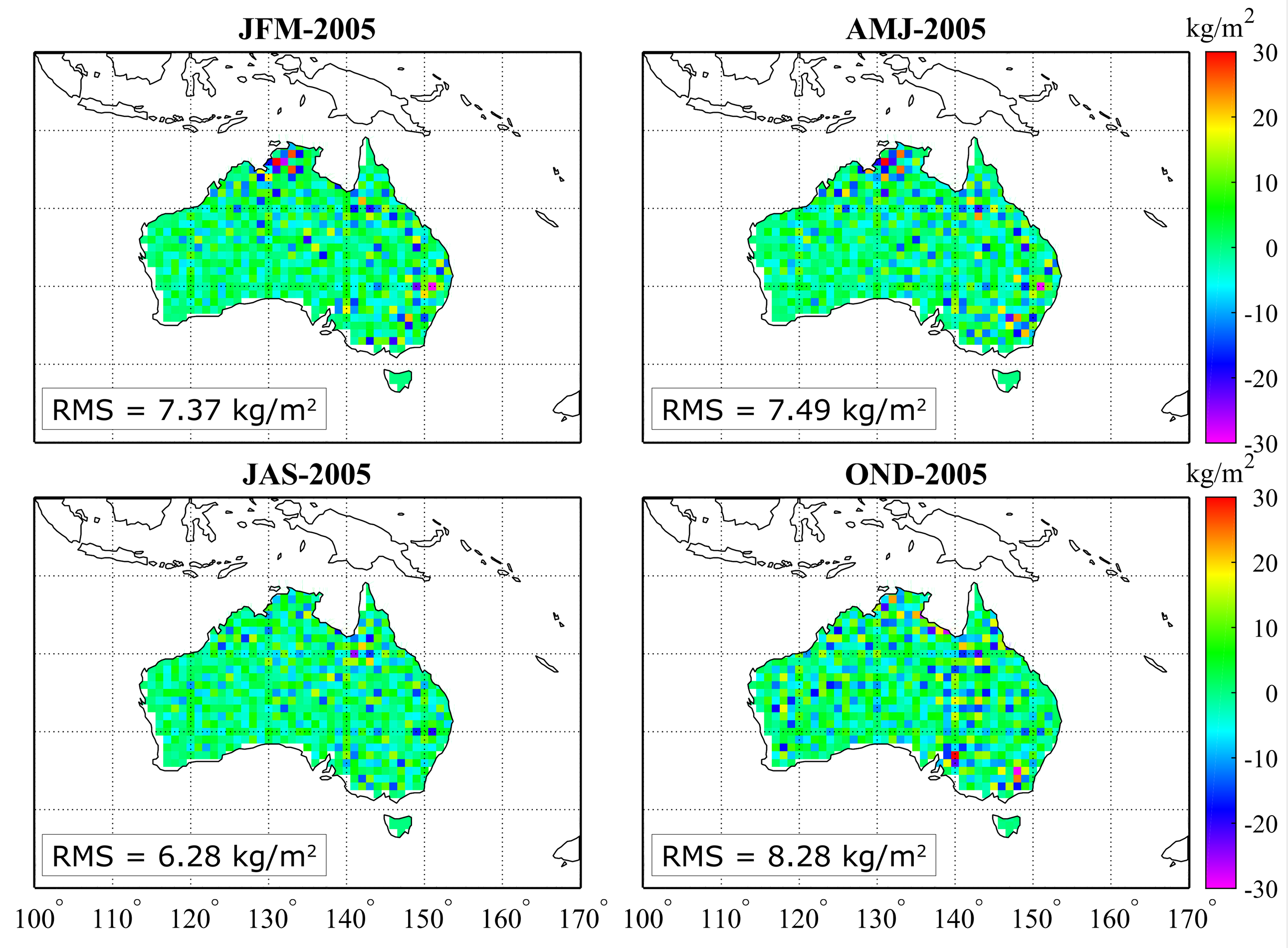

Figure 7: Residuals between simulated (Fig. 4) and inverted (Fig. 6) water-masses.

Summary

- Tesseroids can be used for inverting the water-mass variations and can be formulated as **first class mascons** (Fig. 1) considering the GRACE observations.
- Improvements w.r.t. the **point-mass** solution is shown in Eq. (6), where its zeroth order approximation is equivalent to a point-mass at central point.
- The closed-loop simulation (Figs. 4-7) shows the feasibility of the tesseroid for regional water-mass recovery using the inverse flat tesseroid approach; here as a linear inverse problem, in Grombein et al. (2012) as non-linear problem.

Sources

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