Some theoretical and applied aspects of sequential variational data assimilation (in Russian)


Introduction

- Atmospheric chemistry dynamics is studied with connection-diffusion-reaction model. Measurement data is provided as the truth, whereas concentration measurements as a test can be done by an automated city monitoring network.
- The numerical Data Assimilation (DA) algorithm is presented on the additive-splitting and variational data assimilation schemes applied to separate splitting stages.
- This design provides for significant implementation due to the data assimilation scheme applied to the separate transport process on each coordinate line containing in situ measurement points.
- The goal of the work is to investigate the performance of the algorithm.

Data assimilation problem for transport and transformation model

- 4D models describing the processes of heat, moisture, radiation, and pollutants transport and transformation in the atmosphere have the generic structure:
  \[ \frac{\partial P}{\partial t} + \frac{\partial}{\partial x} (v P) + \alpha \frac{\partial}{\partial y} (v P) + \beta \frac{\partial}{\partial z} (w P) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Here \( \phi, \alpha, \beta, \gamma, \delta \) are model parameters that can be calculated with a hydrodynamics model. \( \phi, \alpha, \beta, \gamma, \delta \) are model parameters that can be calculated with a hydrodynamics model. \( \phi, \alpha, \beta, \gamma, \delta \) are model parameters that can be calculated with a hydrodynamics model.

Inexact measurement data is connected with the state function by means of observation operator \( M \) resulting in measurement error:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Sensitivity conditions of the augmented functional are independent for different dimensions:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

The problem is to solve the system:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Split model

- Consider an additive-splitting scheme (5) or time interval (6) is divided as follows:

Transformation process:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Transport process:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Next step approximation:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Splitting and data assimilation

One-dimensional direct data assimilation algorithm for convection-diffusion

In case of orthogonal model parameters:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

and in situ concentration measurements the solution of the minimization problem

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Explicit chemistry data assimilation algorithm

Chemical kinetic equation defining transformation operator have the following structure:

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

where \( \phi, \alpha, \beta, \gamma, \delta \) are model parameters that can be calculated with a hydrodynamics model. \( \phi, \alpha, \beta, \gamma, \delta \) are model parameters that can be calculated with a hydrodynamics model. \( \phi, \alpha, \beta, \gamma, \delta \) are model parameters that can be calculated with a hydrodynamics model.

Data assimilation result is sought as the minimum of

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

\[ \frac{\partial}{\partial t} (P + M) + \frac{\partial}{\partial x} (v (P + M)) + \alpha \frac{\partial}{\partial y} (v (P + M)) + \beta \frac{\partial}{\partial z} (w (P + M)) = \phi \frac{\partial c}{\partial x} + \gamma \frac{\partial c}{\partial y} + \delta \frac{\partial c}{\partial z}. \]

Vertical composition data assimilation scenario

Scenario description: Synthetic data case. Gaussian noise is added. Double in 24 hours. In the middle of domain there is an additional TV source (X, Y, Z) and are measured every 2 hours in every 10 meters. Measurement parameters are taken from real measurements.

Conclusions

- Combined splitting schemes and data assimilation schemes allow to construct computationally effective algorithms without iterations for data assimilation of in situ measurements to connection-diffusion-reaction models.
- In numerical experiments fine-grained data assimilation scheme for split model showed almost the same precision as the conventional scheme being more computationally effective. Both schemes showed better performance than direct data assimilation methods.
- Fine-grained scheme was less effective as a source reconstruction method.
- In the considered real measurement data case the DA algorithm showed better performance than direct model (X, Y, Z) concentrations and less improvement for (X, Y, Z).
- In a synthetic data case with the "untraceable" X, Y, Z source DA has improved the result significantly.

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