Inversion of P-wave traveltimes from a VSP experiment in a homogeneous anisotropic medium

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Resumé

Determination of seismic anisotropy plays an important role both in structural and exploration seismology. Knowledge of the orientation and strength of anisotropy has important geological implications as, e.g., estimation of the orientation of structural elements (layering, dikes, fissures) or of the orientation of the tectonic stress. We perform a sequence of synthetic tests in a homogeneous model, on the P-wave traveltime inversion based on weak-anisotropy (WA) approximation. A typical VSP (vertical seismic profiling) configuration is considered. Results of the inversion are estimates of 15 P-wave WA parameters and corresponding resolution and covariance matrices. A number of synthetic tests for varying source-receiver configurations, varying noise types/levels, etc. are performed and selected examples are discussed below.

Governing equation — Box 1

Commonly the 6x6 symmetric Voigt matrix ***A*** is used as an equivalent of the density normalized elastic tensor a_{ijkl}. Conveniently, alternate parameterization introduces background isotropic velocity α_0 and a set of 15 WA parameters, rearranged in the vector ***m***, which equivalently describe the model:

$$\boldsymbol{m}=\left(\epsilon_x,\epsilon_y,\epsilon_z,\delta_x,\delta_y,\delta_z,\epsilon_{15},\epsilon_{16},\epsilon_{24},\epsilon_{26},\epsilon_{34},\epsilon_{35},\chi_x,\chi_y,\chi_z\right)^T$$

The relation between ***m*** and ***A*** is simply

$$\epsilon_x=\frac{A_{11}-\alpha_0^2}{2\alpha_0^2},\ \epsilon_y=\frac{A_{22}-\alpha_0^2}{2\alpha_0^2},\ \epsilon_z=\frac{A_{33}-\alpha_0^2}{2\alpha_0^2},\ \delta_x=\frac{A_{23}+2A_{44}-\alpha_0^2}{\alpha_0^2},\ \delta_x=\frac{A_{13}+2A_{55}-\alpha_0^2}{\alpha_0^2},\ \delta_x=\frac{A_{12}+2A_{66}-\alpha_0^2}{\alpha_0^2},$$

$$\epsilon_{15}=\frac{A_{15}}{\alpha_0^2},\ \epsilon_{16}=\frac{A_{16}}{\alpha_0^2},\ \epsilon_{24}=\frac{A_{24}}{\alpha_0^2},\ \epsilon_{26}=\frac{A_{26}}{\alpha_0^2},\ \epsilon_{34}=\frac{A_{34}}{\alpha_0^2},\ \epsilon_{35}=\frac{A_{35}}{\alpha_0^2},\ \chi_x=\frac{A_{14}+2A_{56}}{\alpha_0^2},\ \chi_y=\frac{A_{25}+2A_{46}}{\alpha_0^2},\ \chi_z=\frac{A_{36}+2A_{45}}{\alpha_0^2}$$

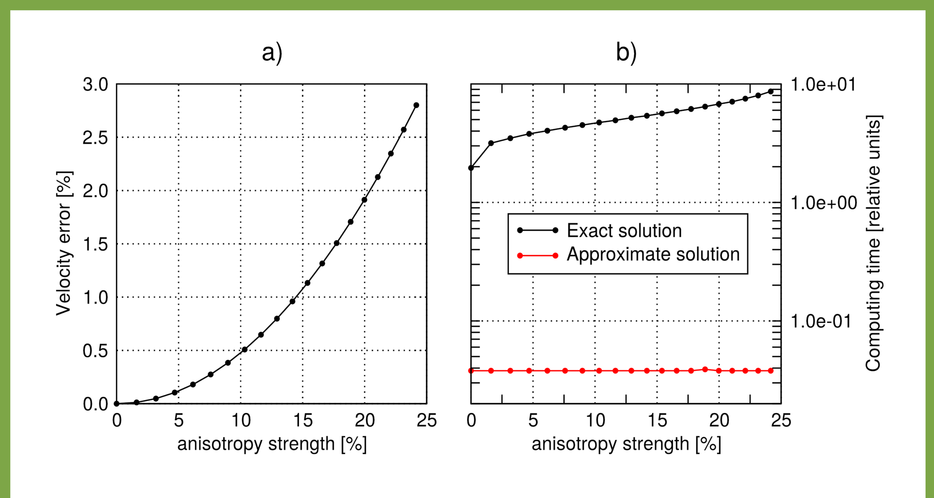
But the most important formula relates linearly WA parameters and squared direction-dependent phase P-wave velocity *v*. However, the following formula is approximate.

$$\left\{\begin{array}{l} N_1^4\epsilon_x+N_2^4\epsilon_y+N_3^4\epsilon_z+N_2^2N_3^2\delta_x+N_1^2N_3^2\delta_y+N_1^2N_2^2\delta_z \\ +2N_1^3N_3\epsilon_{15}+2N_1^3N_2\epsilon_{16}+2N_2^3N_3\epsilon_{24}+2N_2^3N_1\epsilon_{26}+2N_3^3N_2\epsilon_{34}+2N_3^3N_1\epsilon_{35} \\ +2N_1^2N_2N_3\chi_x+2N_1N_2^2N_3\chi_y+2N_1N_2N_3^2\chi_z \end{array}\right\}=\frac{\left(\frac{v}{\alpha_0}\right)^2-1}{2}=\frac{\left(\frac{r}{\alpha_0 t}\right)^2-1}{2}$$

where the coefficients N_i are components of the ray direction vector ***N***, *r* is distance and *t* is traveltime. Application of this formula enables easy and fast calculation in arbitrary homogeneous anisotropic medium.

Forward traveltime calculations — Box 2

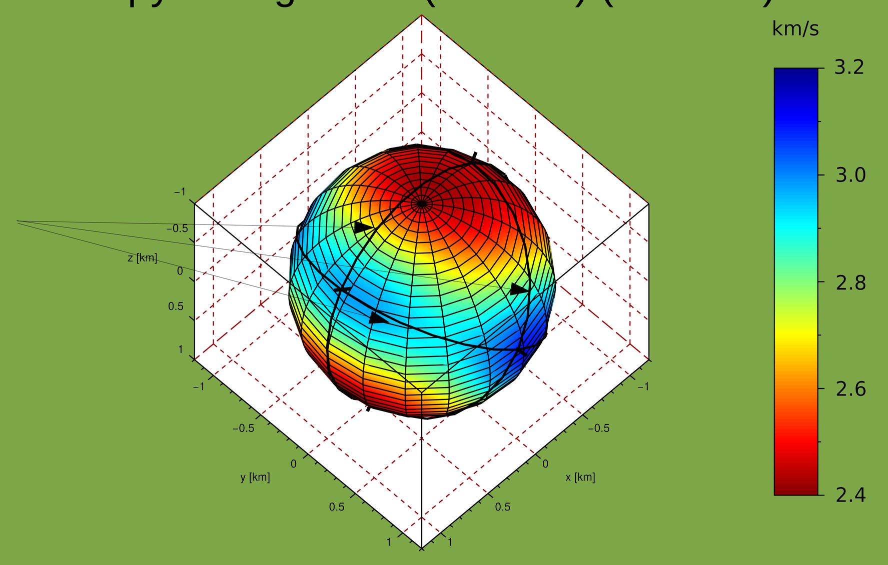
Exact determination of traveltime between a source and a receiver requires iterative solution of the Christoffel equation.
Approximate determination of traveltime between a source and a receiver requires evaluation of a simple explicit formula linearly relating WA parameters and squared velocity (Box 1).
Accuracy of the approximate solution is satisfactory and also computationally efficient.



Phase velocity model — Box 3

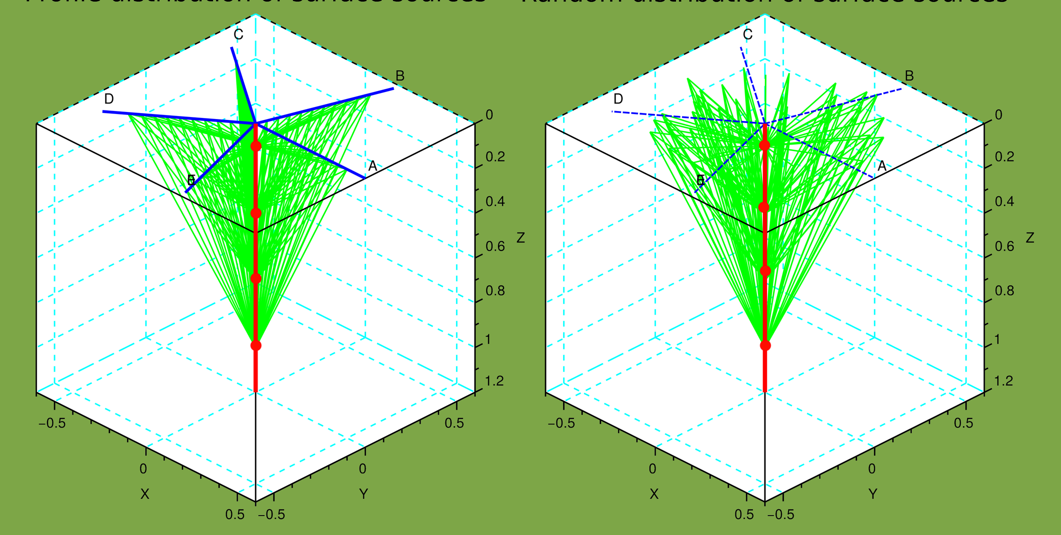
We consider a homogeneous anisotropic model with tilted orthorhombic symmetry. The P-wave phase velocity ranges between ~2.4 - ~3.1 km/s, the anisotropy strength is $2^*(c^{max}-c^{min})/(c^{max}+c^{min})$ ~25%.

planes of symmetry



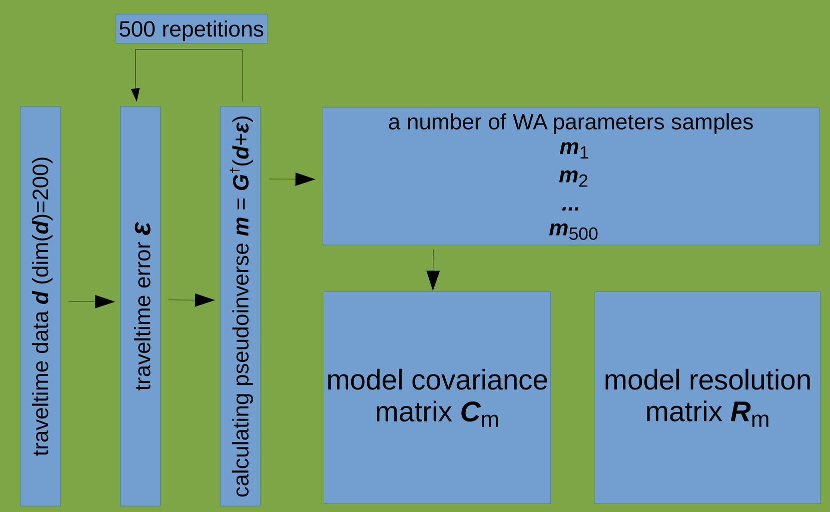
Measurement setup — Box 4

Four receivers are situated in a borehole, 50 sources are located either along 5 radial profiles or randomly.



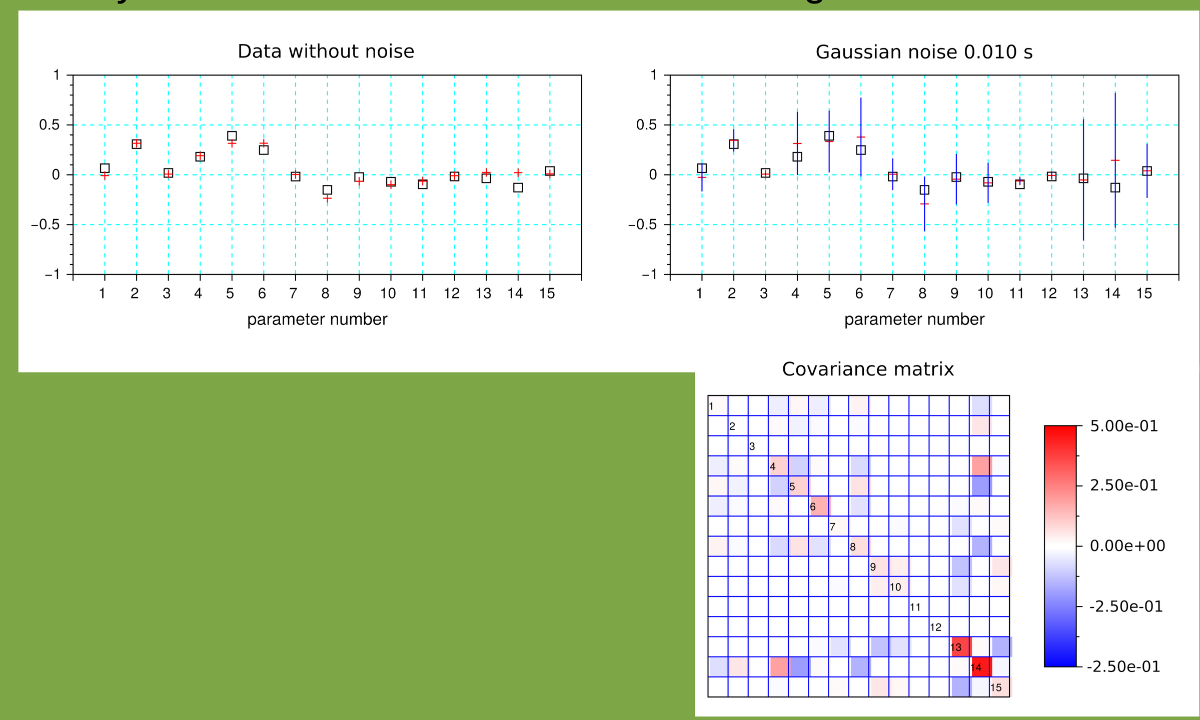
Testing scheme — Box 5

The tested problem is linear with respect to v^2 but not to traveltimes. For that reason and also for bigger flexibility, we perform Monte Carlo sampling technique to estimate the robustness and accuracy of determination of the WA parameters.



Inverting traveltimes from sources along profiles — Box 6a

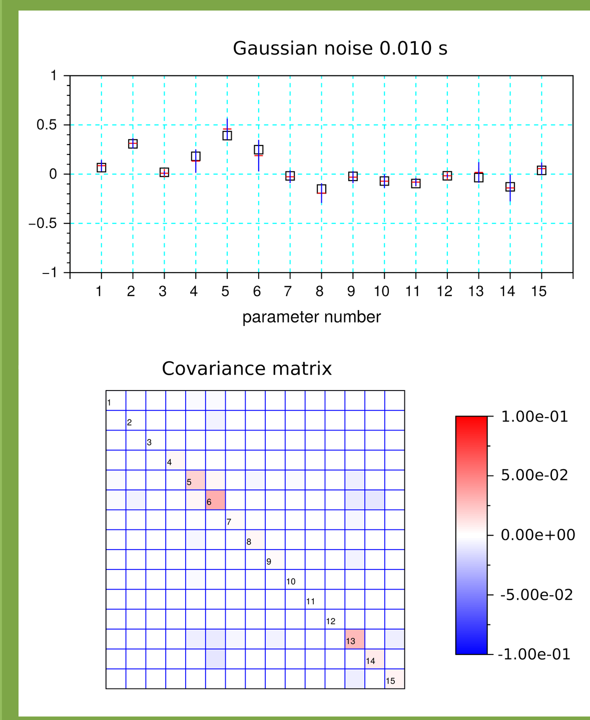
Inversion of noise-less data is straightforward. True values of the WA parameters are indicated by blue squares, inverted parameters are indicated by red crosses. Small imperfection is only due to inconsistent forward modelling.



Inverting traveltimes from randomly distributed sources — Box 6b

Note on numbering the WA parameters: Parameters are sequentially numbered in accordance with Box 1:

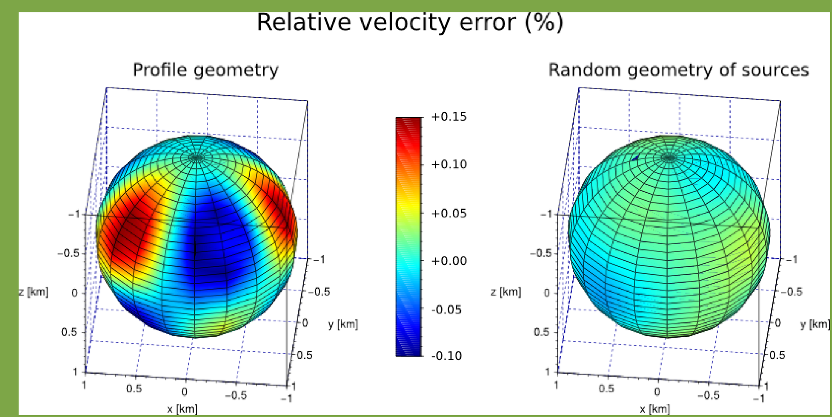
parameter No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Eq. Box 1	ϵ_x	ϵ_y	ϵ_z	δ_x	δ_y	δ_z	ϵ_{15}	ϵ_{16}	ϵ_{24}	ϵ_{26}	ϵ_{34}	ϵ_{35}	χ_x	χ_y	χ_z



Inversion results depend, among others, on the measurement geometry. Results for random distribution of sources are better than those achieved by using profiles. Errors of individual parameters are smaller, and correlation between parameters is lower (see the off-diagonal elements of the covariance matrix). Note also different color scales of the two covariance matrices in Boxes 6a,b.

Relative phase velocity errors — Box 7

Estimated WA parameters are used for reconstructing phase velocity surface. Random distribution of sources outperforms profile arrangement. In profile setup, the error pattern is segmented according the azimuths of the five profiles.

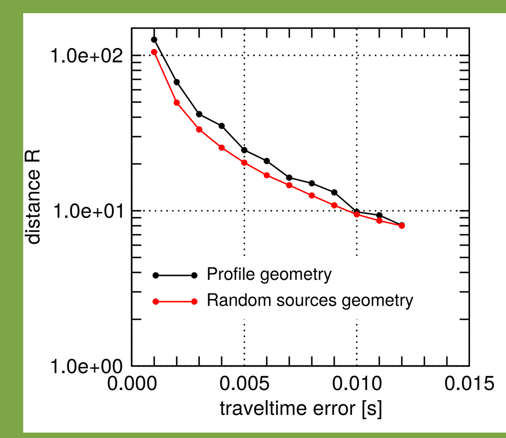


Distinguishing anisotropy/isotropy — Box 8

The distance *R* between vectors ***m*** (WA parameters) and $\boldsymbol{m}_0$ (closest isotropic medium) is

$$R = (\boldsymbol{m}-\boldsymbol{m}_0)^T\boldsymbol{H}(\boldsymbol{m}-\boldsymbol{m}_0),$$

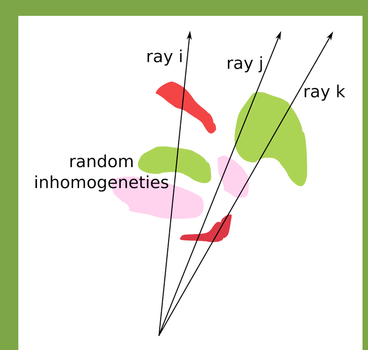
where the metric tensor is set to $\boldsymbol{H} = (\boldsymbol{C}_m)^{-1}$. If $R < 1$, anisotropy is not distinguished from isotropy, if $R >> 1$ anisotropy is surely distinguished.



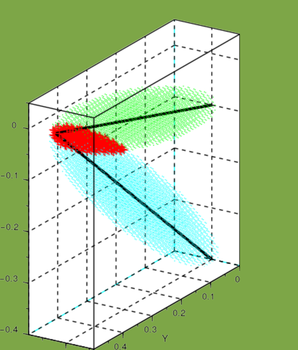
Anisotropy / random inhomogeneity — Box 9

Random inhomogeneous medium causes random traveltime perturbations. However these perturbations are correlated since close rays transect nearly the same geological structure (**a**).
Inversion artifacts due to random inhomogeneity can be discussed similarly as shown in Box 5. Since traveltime errors are correlated, full data covariance matrix $\boldsymbol{C}_d$ must be used. $\boldsymbol{C}_d$ can be approximately constructed from Fresnel volumes (**b**): $C_{ii} = \varepsilon^2$, $C_{ij} = V_{ij}/(V_i + V_j).\varepsilon^2$, where V_i is Fresnel volume of the *i*-th ray (green), V_j is Fresnel volume of *j*-th ray (blue) and V_{ij} (red) is intersection of V_i and V_j.
Resulting $\boldsymbol{C}_d$ (**c**) is used for Monte Carlo sampling method.

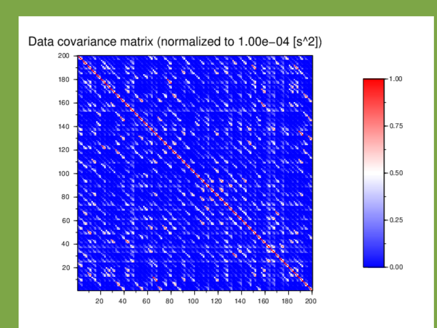
a)



b)

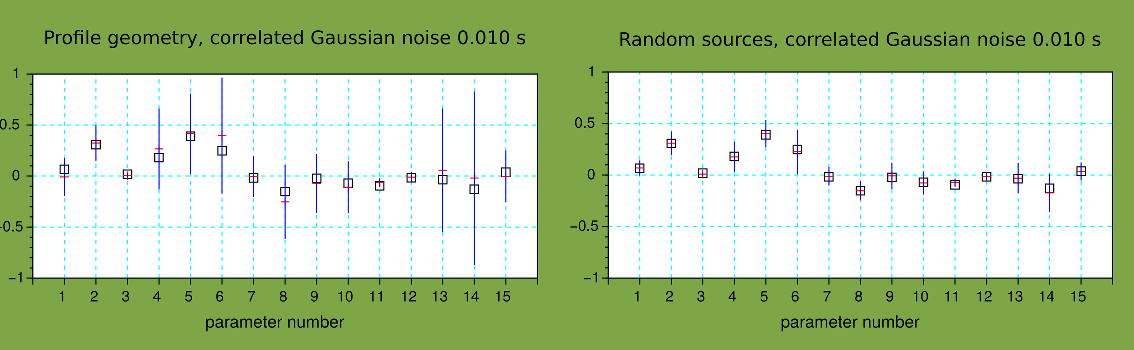


c)



Simulation of random inhomogeneity — Box 10

Correlated noise in data results in slightly larger uncertainty of inverted WA parameters. Again, randomly located sources provide much better estimates than traditional profile measurements (compare this Box with Box 6).



Conclusions

- 15 P-wave WA parameters are suitable representation of a general anisotropy
- approximate linear WA parameters↔v^2 relation (Box 1) enables efficient traveltime inversion of arbitrary anisotropic medium
- VSP measurement geometry provides sufficient angular illumination to assess all WA parameters uniquely
- accuracy of WA parameters depends strongly on noise in data and also on measurement geometry
- profile arrangement of sources provides much worse angular illumination than random distribution of sources
- random inhomogeneity causes similar effects as random noise in traveltime data

Acknowledgements

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