4 Example of Tensor Parameter

As an example, we chose site A09 of the Dublin Secret Model One (DSM1, Miensopust et al., 2013), because the data are: (i) noise-free, (ii) distortion-free and (iii) truly 2D with only a few oriented 2D structures at different depths. Visual inspection of the impedance data (panels 1a and 1b) shows equal off-diagonal amplitude and very small diagonal elements in the time domain. After the fourth period off-diagonal elements differ and main diagonal elements begin to be significant after the sixth period (around 10 s), translating into the observation that the data is roughly 2D for the first three periods, roughly 2D till the sixth and increasingly 3D thereafter. Examination of the phase tensor skew (panel 1c) of the data reveals a clear orientation of the plane (see (12)) concluding that distortion analysis cannot be uniquely resolved without assuming some initial model. Given the phase tensor [Cawthra et al., 2004], we propose a complementary Amplitude Tensor to represent the impedance. Both tensors are independent and exhibit significant structural similarity corresponding to the present subsurface, even if Amplitude Tensor properties are described in its similarity to the Phase Tensor is demonstrated in terms of regional subsurface strike angle, dimensionality and anisotropy. Since the Amplitude Tensor combines galvanic distortion and regional angle, we postulate that the latter can be approximated by Phase Tensor properties, allowing to estimate distortion up to a constant without regional dimensionality assumptions or explicit constraints on the distortion matrix.

3 Amplitude Phase Decomposition

Let \( Z = \begin{pmatrix} \rho & i \phi \\ i \phi & \rho \end{pmatrix} \) be represented in analogy to the polar form of complex numbers \( a + bi \) with \( a = \rho \cos \phi \) and \( b = \rho \sin \phi \). Hence, a unique, real-valued Amplitude Tensor can be defined as:

\[
\begin{align*}
\rho &= \sqrt{\rho_x^2 + \rho_y^2} \\
\phi &= \arctan\left(\frac{\rho_y}{\rho_x}\right)
\end{align*}
\]

In order to compare Amplitude and Phase, it is imperative that the scales of both are comparable (see figure 1g and h). Since the real and imaginary parts of the logarithm of complex numbers \( z = \rho \exp(i \phi) \) relate to the logarithm of the amplitude \( \rho \) and phase \( \phi \), we argue that assumptions are meaningful as real and imaginary parts of the logarithm of complex numbers:

\[
\begin{align*}
\text{Real & Imaginary part} \\
\text{Amplitude & Phase}
\end{align*}
\]

7 Estimating Inductive and Galvanic Amplitude From the Phase Tensor

Consider a 2D impedance tensor in regional strike coordinates, with the two principal components:

\[
Z_{12} = \exp(\rho_{12} + i \phi_{12})
\]

with the singular values \( \rho_{12} \) and \( \rho_{22} \) of Phase and Amplitude Tensor, respectively. Let

\[
\rho_{21} = \exp(\rho_{12} + i \phi_{12})
\]

then we obtain:

\[
\begin{align*}
Z_{12} &= \exp(\rho_{12} + i \phi_{12}) \\
Z_{21} &= \exp(\rho_{21} + i \phi_{21})
\end{align*}
\]

In a general 3D situation, we can follow the same procedure, allowing us to assume similar geometric and (subdimensional) sensitivities for inductive amplitude and phase. Thereby, by substituting phase stress (skew) as inductive amplitude stress (skew) and the singular values as in the 2D example, we obtain an estimate for the galvanic Amplitude Tensor, which represents purely galvanic subsurface information:

\[
\text{Galvanic stress} = \exp(\rho_{21} + i \phi_{21})
\]