Response of the Ice Sheets to Fluctuating Temperatures

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Objectives & Conclusion

We find that **temperature fluctuations *by themselves* lower the equilibrium state volume** of an ice sheet model.
For long term ice sheet simulations **this effect is significant** and **necessary to include to correctly model the ice sheets**.

Minimal ice sheet model

We use a minimal ice sheet model by Oerlemans (2003) that specifies

$$\dot{V} = f(T, V) \tag{1}$$

where T is temperature, V is total ice volume and $\dot{V} := dV/dt$.
$f = 0$ represents a steady state.

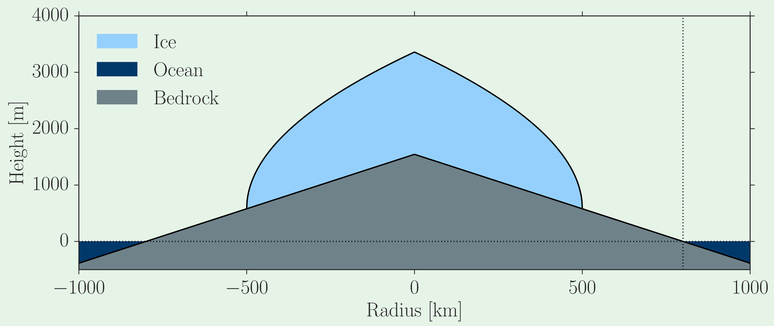

Figure 1: Sketch of the Oerlemans (2003)-model.

Fluctuating temperatures

Next we simulate temperature time series T_t with an AR(1)-process

$$T_{t+1} - \overline{T} = \phi(T_t - \overline{T}) + \sigma_{AR} W_t \tag{2}$$

for some mean temperature $\overline{T}$. W_t is white noise with unit variance and the parameters (ϕ, σ_{AR}) are obtained by fitting Equation (2) to observed temperatures over Greenland between 1851 and 2011. T_t and V_t are now stochastic processes and T_t is stationary:

$$\langle T_t \rangle = \overline{T}. \tag{3}$$

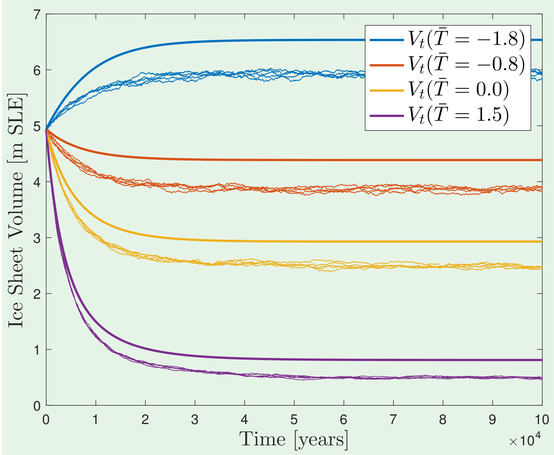

Figure 2: Simulations of Equation (1) with constant and fluctuating temperatures T_t.

At the equilibrium states shown in Figure 2

$$\begin{aligned} \langle V_t \rangle &= \overline{V}, \\ \langle f(T_t, V_t) \rangle &= 0. \end{aligned} \tag{4}$$

Analytical approximation – results

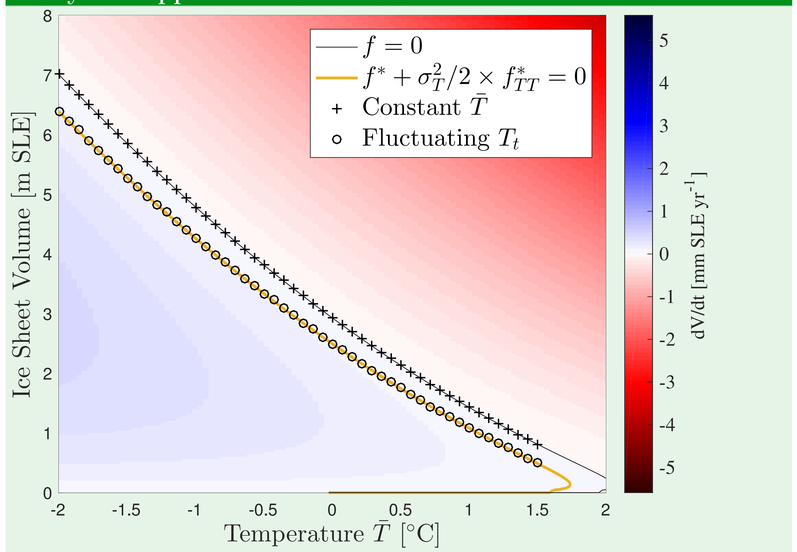

Figure 3: Approximation of the equilibrium states (as in Figure 2) using Equation (8) (yellow line) and numerical simulations, for constant (+) and fluctuating (∘) temperatures T_t.

Notation

$$\begin{aligned} f_T &:= \frac{\partial f}{\partial T}, \quad f_{TV} := \frac{\partial^2 f}{\partial T \partial V}, \quad \dots \\ f^* &:= f(\overline{T}, \overline{V}), \quad f^*_T := \left.\frac{\partial f}{\partial T}\right|_{(\overline{T},\overline{V})}, \quad \dots . \end{aligned}$$

Taylor expansion around equilibrium state

To calculate $\overline{V}$ (see Equation (4)) we expand $\langle f(T_t, V_t) \rangle$ around $(\overline{T}, \overline{V})$:

$$\begin{aligned} \langle f(T_t, V_t) \rangle \approx f^* &+ \langle T_t - \overline{T} \rangle f^*_T + \langle V_t - \overline{V} \rangle f^*_V + \frac{1}{2}\langle (T_t - \overline{T})^2 \rangle f^*_{TT} \\ &+ \frac{1}{2}\langle (V_t - \overline{V})^2 \rangle f^*_{VV} + \langle (T_t - \overline{T})(V_t - \overline{V}) \rangle f^*_{TV}. \end{aligned} \tag{5}$$

We use the relations:

$$\begin{aligned} \langle T_t - \overline{T} \rangle = 0, \quad \langle V_t - \overline{V} \rangle = 0, \\ \langle (T_t - \overline{T})^2 \rangle = \sigma_T^2 = \frac{\sigma_{AR}^2}{1 - \phi^2}. \end{aligned} \tag{6}$$

After combining Equations (5) and (6) we get:

$$\begin{aligned} \langle f(T_t, V_t) \rangle \approx f^* &+ \frac{\sigma_T^2}{2} f^*_{TT} \\ &+ \frac{1}{2}\langle (V_t - \overline{V})^2 \rangle f^*_{VV} + \langle (T_t - \overline{T})(V_t - \overline{V}) \rangle f^*_{TV}. \end{aligned} \tag{7}$$

In an equilibrium state V_t fluctuates very slowly around $\overline{V}$, ie. $V_t \approx \overline{V}$. We take this into account and arrive at

$$\langle f(T_t, V_t) \rangle \approx f^* + \frac{\sigma_T^2}{2} f^*_{TT}. \tag{8}$$

The zero contour of Equation (8) is shown in Figure 3 along with numerical simulations.

Consequences for long term ice sheet simulations

We use data from long term Greenland ice sheet simulations (Robinson et al. 2012) and – following Fettweis et al. (2012) – fit 3rd degree polynomials to the surface mass balance SMB(T). This way we obtain maximum likelihood estimates (MLE) and credible intervals of ΔT and ΔSMB (definition in Figure 4) from 99 different simulations.

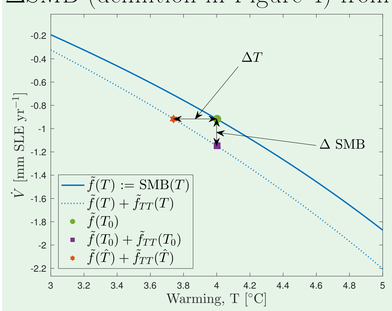

Figure 4: Example of SMB(T) from Robinson et al. (2012) and adjustment.

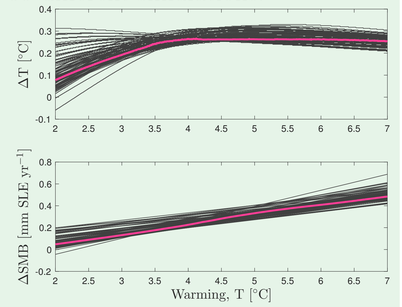

Figure 5: MLE of ΔT and ΔSMB (see Figure 4) from 99 simulations.

T[°C]	ΔT[°C]	Lower	Upper
2	0.08	0.00	0.29
3	0.19	0.16	0.29
4	0.26	0.24	0.31
5	0.26	0.24	0.33

Table 1: MLE of $\Delta T$and 95% credible intervals.

T[°C]	ΔSMB	Lower	Upper
2	0.05	0.00	0.19
3	0.13	0.10	0.26
4	0.23	0.19	0.33
5	0.33	0.27	0.41

Table 2: MLE of ΔSMB [mm SLE/yr] and 95% credible intervals.

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