Ginzburg-Landau equation as a heuristic model for generating rogue waves

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1. Introduction

Envelope equations have many applications in the study of physical systems. Particularly interesting is the case of surface water waves. In steady conditions, laboratory experiments are carried out for multiple purposes either for researches or for practical problems. In both cases envelope equations are useful for understanding qualitative and quantitative results. The Ginzburg-Landau equation provides an excellent model for systems of that kind with remarkable patterns. Taking into account the above paragraph the main aim of our work is to generate waves in a water tank with almost a symmetric spectrum according to Akhmediev (2011) and thus, to produce a succession of rogue waves. The envelope of these waves gives us some patterns whose model is a type of Ginzburg-Landau equation, Danilov et al.(1988). From a heuristic point of view the link between the experiment and the model is achieved. Further, the next step consists of changing generating parameters on the water tank and also the coefficients of the Ginzburg-Landau equation, Lechuga (2013), in order to reach a sufficient good approach.

2. Ginzburg-Landau equation

Ginzburg-Landau equation has proved very useful to model some regular structures on the surface of a liquid , particularly the kind shows below. The two parameters provide the envelope amplitude and the period.

$$\frac{\partial u}{\partial t} + i\varepsilon^2 \frac{\partial^2 u}{\partial x^2} + u - iu^* - iu|u|^2 + i\sigma u = 0$$

$$u = n_1 + in_2$$

$$u^* = n_1 - in_2$$

$$\varepsilon^2 \frac{d^2 n_2}{dx^2} + n_1 - \sigma n_1 - n_2 + n_2(n_1^2 + n_2^2) = 0$$

$$-\varepsilon^2 \frac{d^2 n_1}{dx^2} + n_2 - n_1 + \sigma n_1 - n_1(n_1^2 + n_2^2) = 0$$

If $n_1 = n_2 = n$, *then*

$$-\varepsilon^2 \frac{d^2 n}{dx^2} + \sigma n - 2n^3 = 0$$

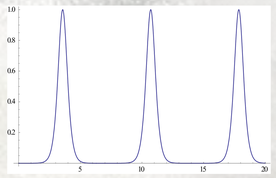

Figure 1. Ginzburg-Landau solution

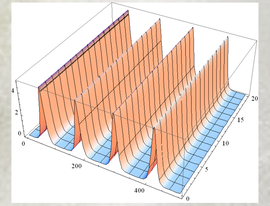

Figure 2. Three-Dimensional view

3. Water Wave experiment

In order to reproduce waves, in the maritime practice, we use, normally, Jonswap spectrum. However, when we use a more symmetric spectrum, either on shallow or in deep water the energy concentrates itself and surface waves with characteristics of rogue waves are generated. Controlling wave maker parameters we can generate this kind of breathers (Akhmediev breathers). See Figure 3.

Figure 3. Generated waves

$$H(\eta) = \int_0^T \frac{\eta(x+t) - \eta(x-t)}{\pi t} dt$$

$$E = \left(\eta^2 + H(\eta)^2 \right)^{\frac{1}{2}}$$

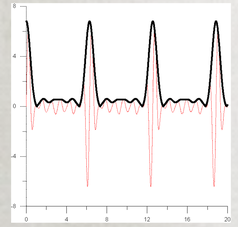

Figure 4. Wave envelope

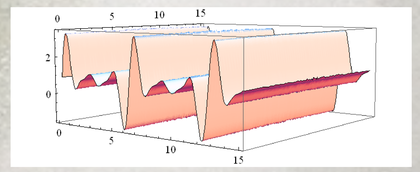

Figure 5. Wave train

Figure 6. Wave Crest

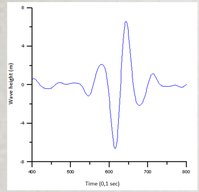

Figure 7. Zoom of the wave

4. Conclusions

We generated rogue waves in controlled laboratory experiment in a deterministic way using a kind of triangular spectra in order to get a greater concentration of the energy with higher and more separated waves. The nonlinear structure of such waves and their regularity make the use of the Ginzburg-Landau equation suitable for modeling the amplitude envelope of the generated waves from a heuristic point of view giving us a way of controlling to the results of our experiment.