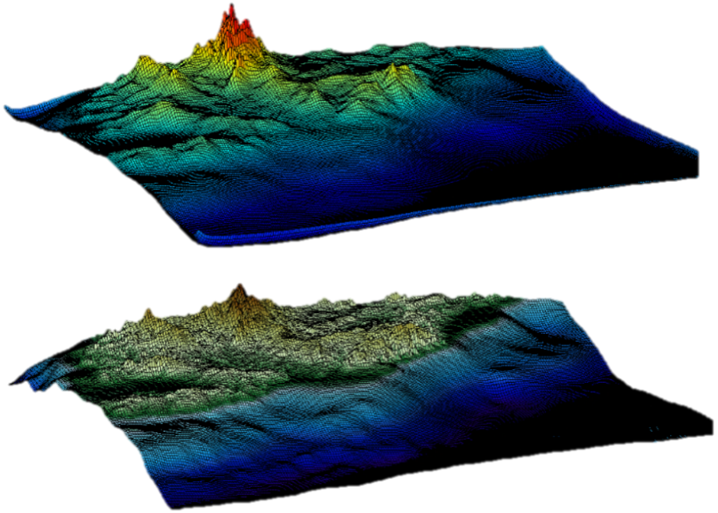
GTE: a new FFT based software to compute terrain correction on airborne gravity surveys in spherical approximation



M. Capponi[1,2], D. Sampietro[3], F. Sansò[3]

[1] DICA, Politecnico di Milano, Milano, Italy

[2] DICEA, Università di Roma La Sapienza, Roma, Italy

[3] Geomatics Research & Development, Lomazzo, Italy

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Introduction

The computation of the vertical attraction due to topographic masses is still a matter of study both in geodetic as well as in geophysical applications

Major issues for the terrain correction computation

- The **huge number of observation points** → e.g from airborne/shipborne gravimetry measurements (~ 1 milion points)
- The **high resolution of Digital Terrain Models** → e.g SRTM globally available at 90m (regionally DTM up to 5m are available)
- The **high accuracy of gravity data** → better than 1mGal also for airborne data

ONE HAS TO QUICKLY COMPUTE THE TC WITH HIGH ACCURACY FOR LARGE GEOGRAPHIC REGIONS AND ON MANY POINTS

Gravity Terrain Effect (GTE) software

The GTE software is based on a **new combined algorithm** thought with the aim to **maximize the result accuracy minimizing the computational time**

- quickly compute the **terrain correction at airborne level**
- a new algorithm which **combines the classical prisms and FFT methods**
- work in **spherical approximation**
- **multiresolution approach**
- thought for **geophysical applications** it allows to compute the **effects of the oceanic and topographic masses but also those due to sediments and Moho undulation**

GTE theoretical aspects

$$\delta g_t(P) = -\frac{\partial}{\partial r}\mu \int dr \int r_Q^2 \frac{dh_Q}{l_{PQ}} d\sigma$$ disregarding terms below 0.1 mGal (*) $$\Rightarrow \delta g_t(P) \cong \delta g_t^P(P) + \delta g_t^{SC}(P)$$

So to compute the gravity effect, the calculation can be split between planar and spherical correction terms

Planar correction

$$\delta g_t^P(P) = -\frac{\partial}{\partial h_p}\mu \int d_2x \int_0^{H_Q} \frac{dh_Q}{L_{PQ}} = \\ = \mu \int d_2x \left\{ \frac{1}{L_{PQ}} - \frac{1}{L_{PQO}} \right\}$$

COMBINED ALGORITHM PRISMS+FOURIER

Spherical correction

$$\delta g_t^{sc}(P) = 2\mu \int d_2x \int_0^{H_Q} dh_Q \varepsilon_Q \frac{\partial}{\partial h_Q}\frac{1}{L_{PQ}} + \mu \int d_2x \frac{s_Q^2}{2R^2}\int_0^{H_Q} dh_Q \frac{\partial}{\partial h_Q}\frac{1}{L_{PQ}} \\ - \frac{\mu}{2R}\int d_2x \int_0^{H_Q} \frac{s_Q^2}{L_{PQ}^3} dh_Q - \frac{\mu}{2}\int d_2x \int_0^{H_Q} s_{PQ}^2 \varepsilon_{PQ} \frac{\partial}{\partial h_Q}\frac{1}{L_{PQ}^3} dh_Q$$

NUMERICALLY COMPUTED

(*) see Sampietro, D., et al. "GTE: a new software for gravitational terrain effect computation: theory and performances." *Pure and Applied Geophysics* (2016):1-19

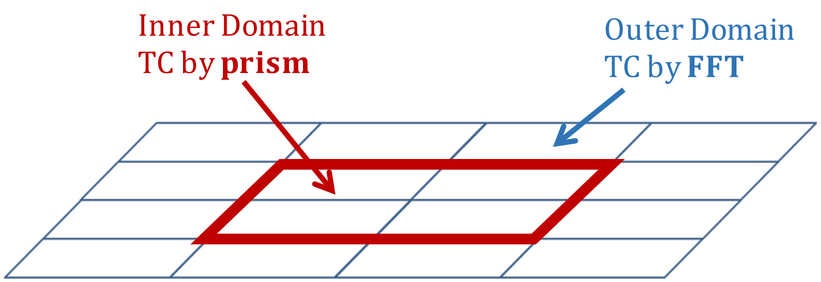
GTE algorithms

When the number of grid points and of computation points goes **up to 10^6** (i.e. an airborne dataset) the classical prism approach can become **very time consuming and not efficient**

PROBLEMS with Fourier approaches:

observation points close to the DTM
- **reduced accuracy**
- **non convergence of convolution integrals**

Inner and Outer domain



$$\delta g_t^+(P) = \delta g_{ou}^+(P) + \delta g_{in^+}(P)$$

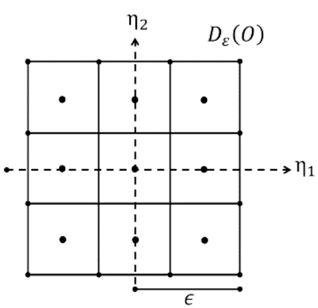
computed by the **Fourier formula**

$$\delta g_{ou}^+(P) = \mu \int_D \frac{[1 - I_\varepsilon(\xi - \eta)]}{L_{PQ}} d_2\eta$$

$$\delta g_{ou}^+(P) = \sum_{k=0}^{N} \sum_{j=0}^{2k} C_{kj} H_\xi^{2k-1} \left\{ K_k * H_\eta^j \right\}$$

$$K_k * H_\eta^j = F^{-1}\left\{ F\{K_k\} \cdot F\{H_\eta^j\} \right\}$$

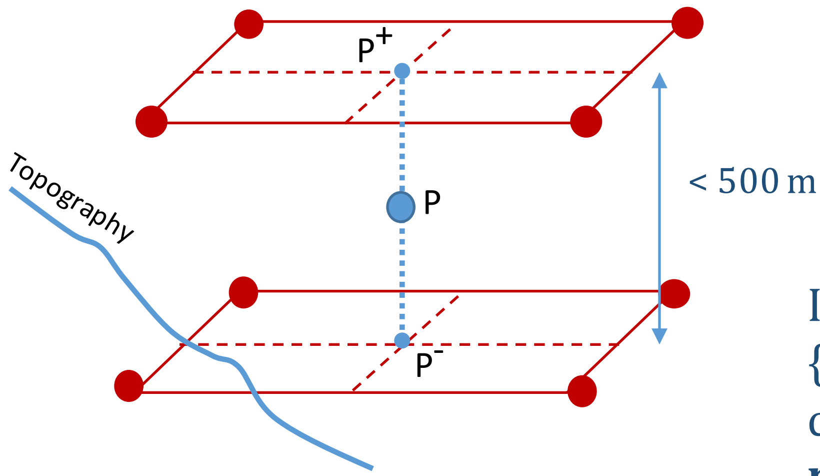
easily computed by discretization in **prisms**



$$I_\varepsilon(\xi - \eta) = \begin{cases} 1 & \text{for } \xi - \eta \in D_\varepsilon(0) \\ 0 & \text{elsewhere.} \end{cases}$$

$$K_k(\xi - \eta) = \frac{1 - I_\varepsilon(\xi - \eta)}{|\xi - \eta|^{2k+1}}$$

GTE for sparse points



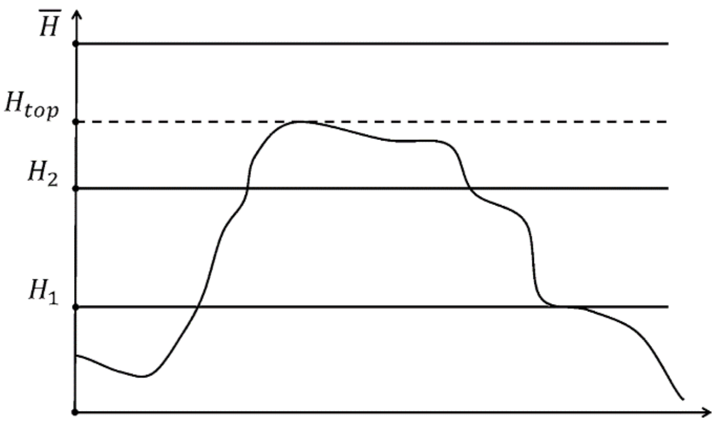
To exploit the advantages of the FFT method the main algorithm of GTE is computing **terrain corrections on grids**

In case of an airborne dataset of sparse points $\{P_k\}$ above the topography, the software computes **two grids** in correspondence of the **minimum and the maximum heights of $\{P_k\}$**

Two horizontal interpolations are performed by bilinear functions on the two grids and finally a linear interpolation is performed from P^-, P^+ to P

Slicing

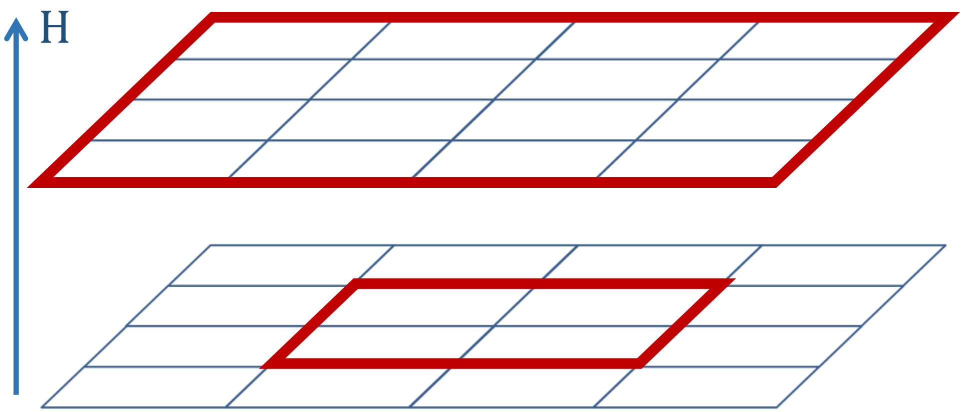
The **slicing** is a particular algorithm implemented in GTE



The **TC** of the topographic body is computed at height $\overline{H}$ slice by slice and then all the terms are summed up

The reference plane is brought up at the base of the slice in such a way that

$$\frac{H_i - H_{i-1}}{H - \overline{H}_{i-1}} < 1$$

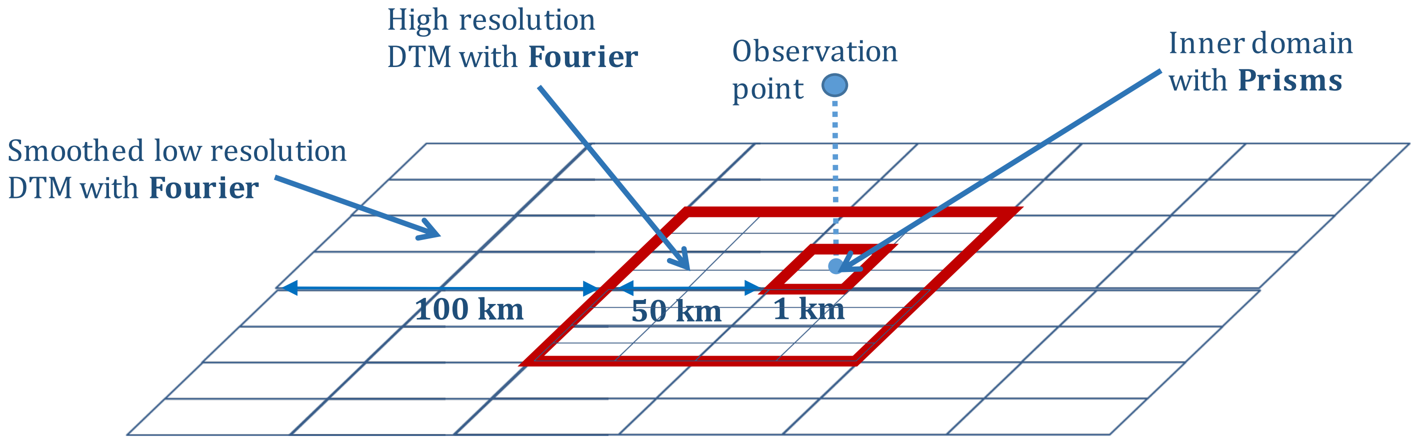


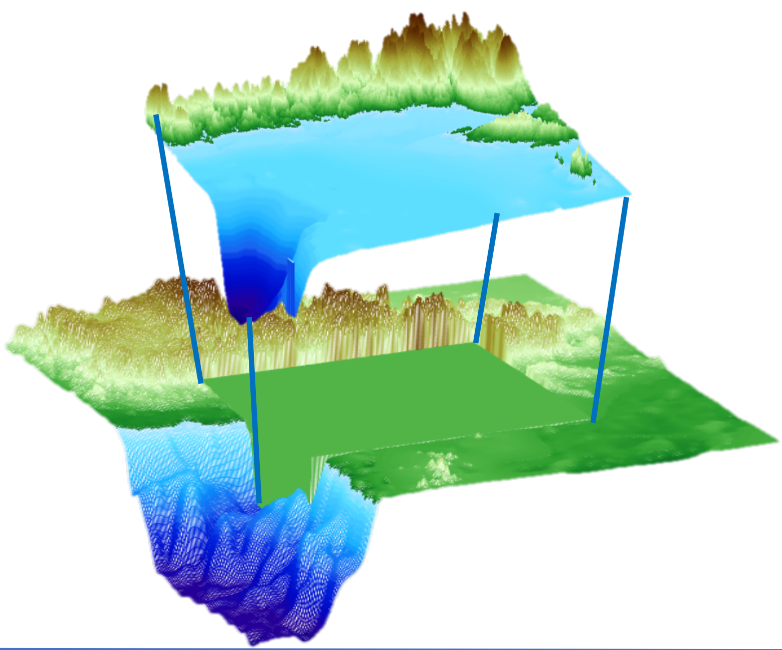
The slicing allow to increase the size of the inner domain with the height

IMPROVEMENT OF THE CONVERGENCE OF CONVOLUTION INTEGRALS

OPTIMIZATION OF COMPUTATIONAL TIMES

Multiresolution

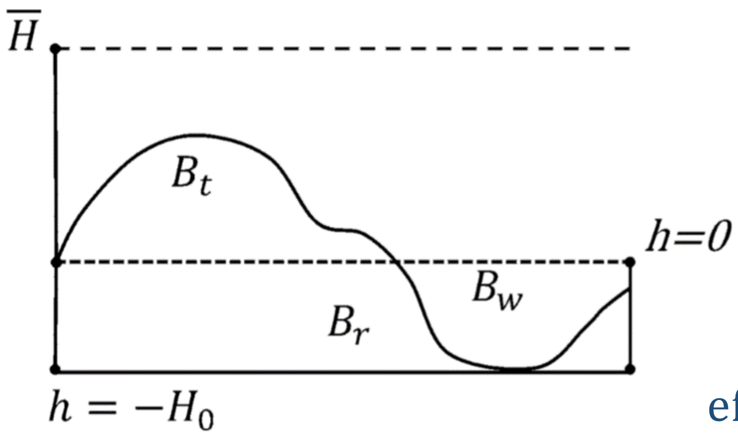




With the multiresolution approach the TC is first computed for the high resolution DTM and then for the smoothed DTM with a lower resolution and finally summed up

GTE for bathymetry, sediments and Moho

Bathymetry



$$B_0 = B_r \cup B_w$$

GTE is capable of computing the gravity effect of $\mathbf{B_t}$

How to use the same software to compute the gravity effect of $\mathbf{B_w}$?

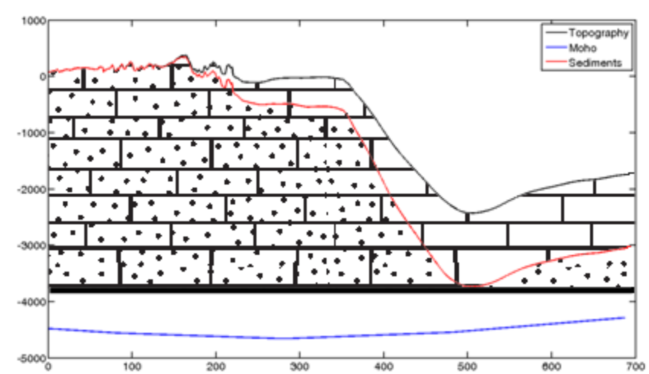
$$\delta g(\rho_w | B_w) = \delta g(\rho_w | B_O) - \delta g(\rho_w | B_r)$$

effect of the big prism $\mathbf{B_0}$

effect of the body $\mathbf{B_r}$ computed at a grid on a plane at height $H = H_0 + \overline{H}$

Sediments and Moho

To handle the **Moho effects** can be used the same reordering, by suitably changing the density constant. For **sediments** the algorithm will have to be applied once for the lower surface and once for the upper surface of the sediments

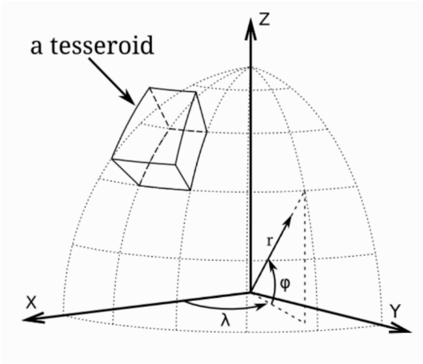


Numerical tests

Some numerical **tests** have been performed mainly **focused on the computation of terrain correction for airborne gravimetry**

AIM

To compare the **accuracy** and the **computational times** of GTE algorithms and software with respect to those implemented in **Tesseroids** (*) which is one of the standard scientific software



Tesseroids is a collection of command-line C programs to model the gravitational potential, acceleration and gradient tensor of topographic masses.

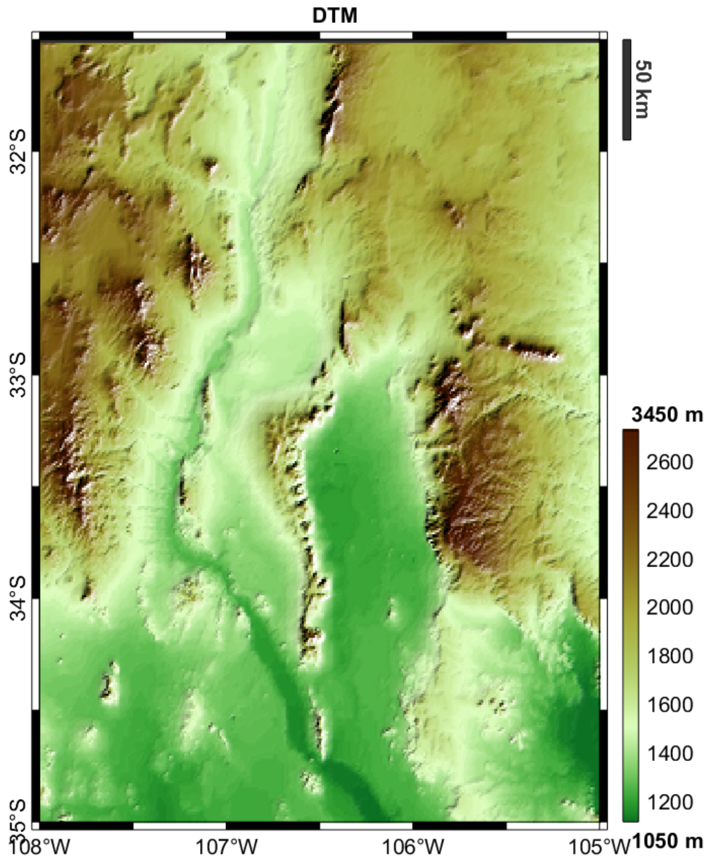
The software computes the gravitational effect of each tesseroid by summing up the effect of a number of point masses optimally distributed and weighted inside the tesseroid

All the tests have been performed on a single node of a HPC equipped with two 8-cores Intel Haswell 2.40 GHz processors (for a total of 16 cores) with 128 GB RAM

(*) see Uieda, L., Ussami, N., & Braitenberg, C. F. (2010). Computation of the gravity gradient tensor due to topographic masses using tesseroids. EOS, Trans Am geophys Un, 91, 26.

Test 1: Dataset

The first test performed consists in comparing, in terms of accuracy and computational time, the results computed on a regular grid

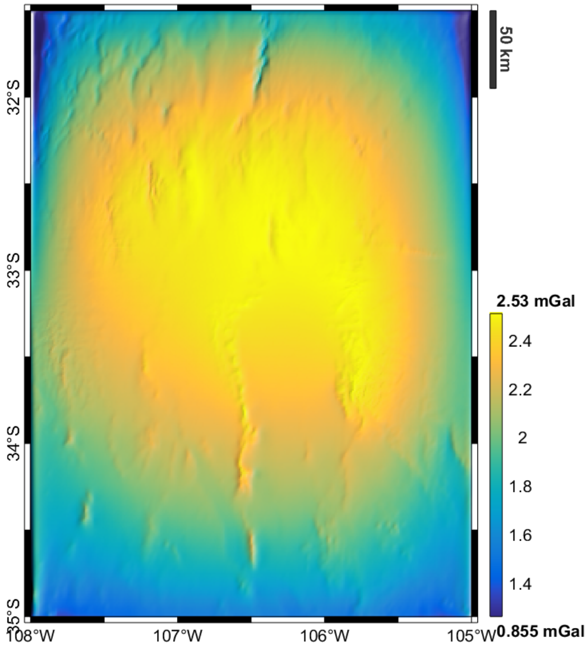


This dataset is located in the south part of New Mexico

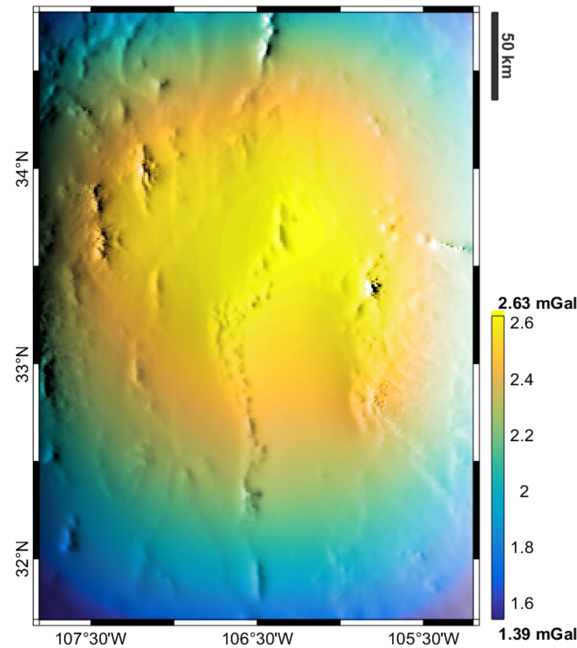
- DTM **spatial resolution 36 arcsecond**
- Region between **31.5° and 35° S** and **105° and 108° W** (**351x301 grid cells**)
- DTM height range between **1050 m** and **3445 m**
- **Computation grid height 3500 m**

Test 1: GTE performances

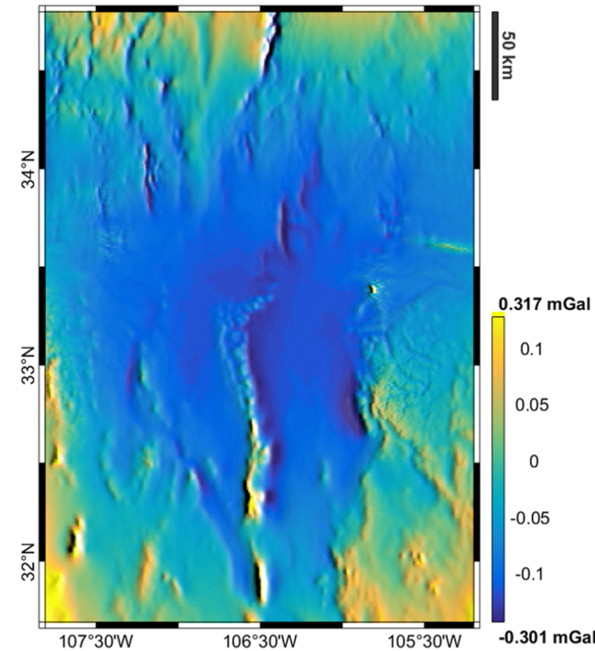
TESSEROIDS - PRISMS



SPHERICAL CORRECTION



GTE+SPHERICAL CORRECTION - TESSEROIDS

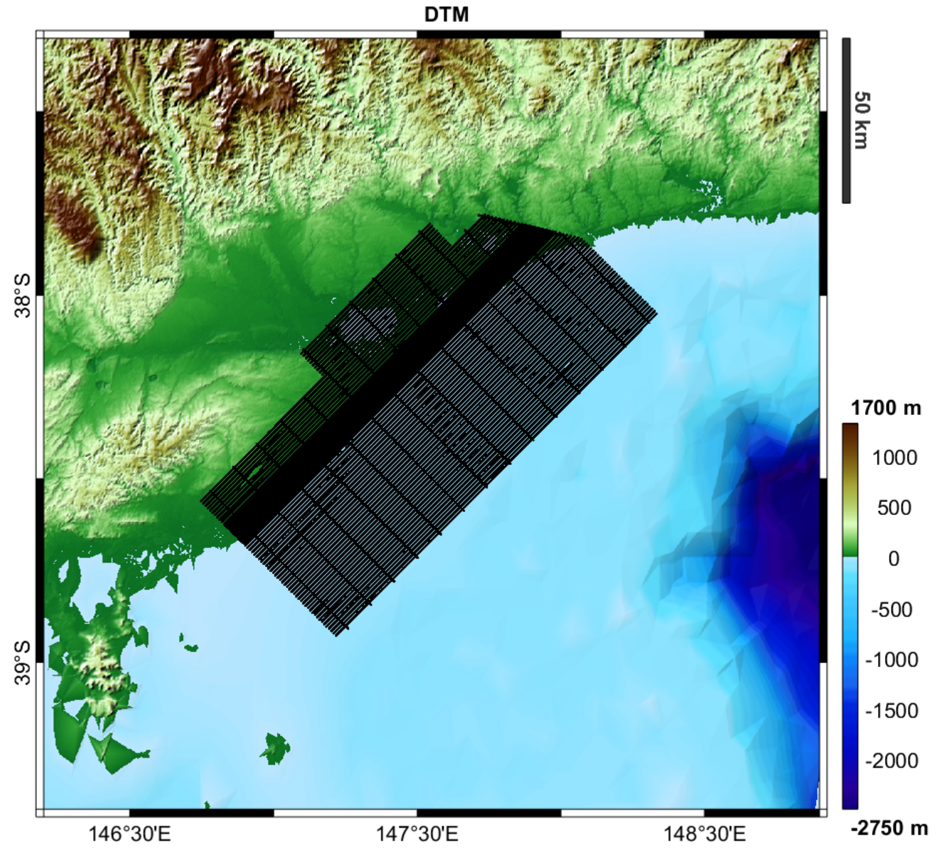


SW	Time [s]	Mean [mGal]	Std [mGal]	Min[mGal]	Max[mGal]
Tesseroids	1937(*)	178.53	40.4	32	349
GTE (no slicing)	11.9	-0.32	0.1	-0.6	-0.07
GTE 3 slices	29.4	-0.09	0.05	-0.3	0.3

(*)Time has been divided by a factor 16 to account for parallel computiong in GTE

Test 2: Dataset

Tests have been performed on a **real airborne dataset** acquired in the framework of the CarbonNet project (DOPI 2012)

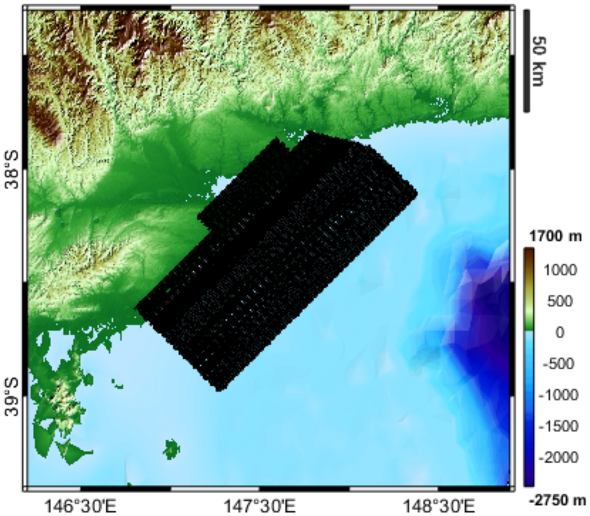


404.384 real airborne observations acquired in 2011 by Sanders Geophysics Ltd to provide a better understanding of the onshore and immediate offshore geology of the Gippsland Basin (200 km east of Melbourne)

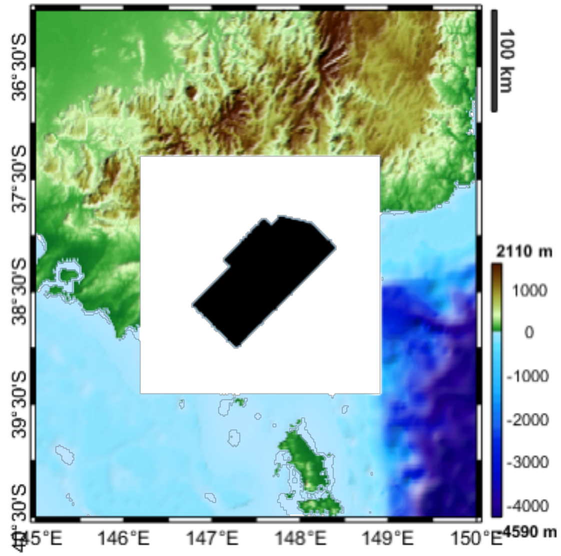
- DTM **spatial resolution 250 m**
- Region between **37.3° and 39.4° S** and **146.2° and 148.9° E (819x1093 grid cells)**
- DTM height range between **1700 m** and **-2754 m**
- The aeroplane is flying **165 m above the ocean** and follows the topography on shore (**max altitude 369 m**)

Test 2: GTE performances

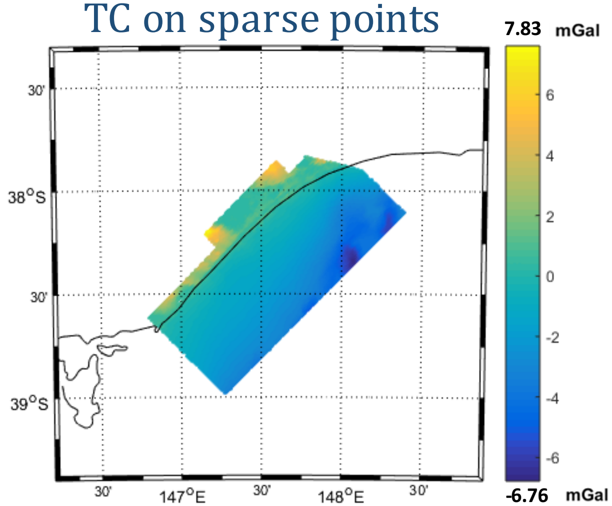
High resolution DTM (250m)



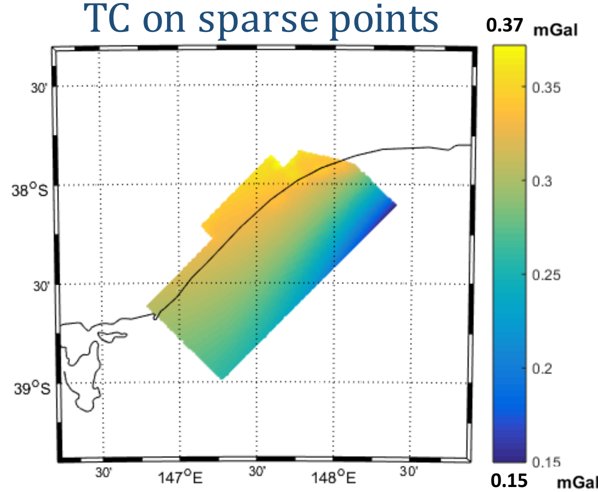
Low resolution DTM (2km)



TC on sparse points



TC on sparse points



Tesseroids	GTE (no slicing)	GTE (3 slice)
Time [s]		
3.8e4$^{(*)}$	388	1000
Mean [mGal]		
-0.97	-0.019	-2e-3
Std [mGal]		
1.87	0.11	0.016
Min [mGal]		
-6.59	-0.95	-0.082
Max [mGal]		
7.96	0.26	1e-3

$^{(*)}$Time has been divided by a factor 16 to account for parallel computing in GTE

Conclusions

GTE is a new software for the computation of the gravitational terrain effect

- The GTE solution is a **combination of FFT techniques and classical prism modelling** aiming to keep **errors lower than 0.1 mGal**

- It has been developed addressing two major issues required by modern geodetic and geophysical applications, **high accuracy and high computational performances**

- the **prism-FFT mixed algorithm** and the **slicing** provide a solution to the problem of convergence of the series and of its singularity

- It works in **spherical approximation**

- The **multiresolution approach** allows to enlarge the region in which the TC is computed without heavily influences computational times

- the comparison performed have shown that GTE gives **results very close to those obtained by Tesseroids** (differences smaller than 0.1 mGal) but reducing the computational time (few hours vs. few minutes)

Future developments

Looking forward the future developments, the next activities will be concerned with

- Implementation of **gravitational potential** and **gravity gradients computation** (second radial derivative of the gravitational field)
- Implementation of the computation of **TC on the DTM surface** (this requires to apply different FFT algorithms, a prototypal SW have been already developed and tested in Matlab)
- for observation points close to the topography the actual slope of the terrain should be considered. A solution for this problem (e.g implementation of **triangulated polyhedrons**(*)) should be studied and implemented

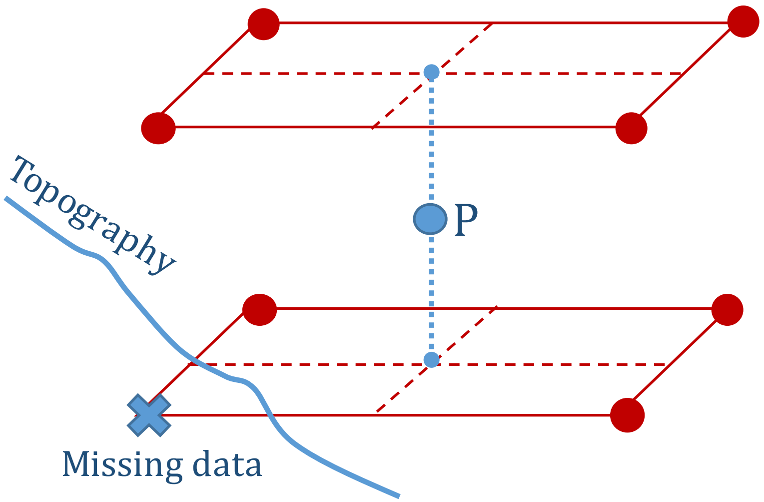
(*) see Götze H-J., and Lahmeyer B., Application of three-dimensional interactive modeling in gravity and magnetics, Geophysics, 53, 1096–1108 (1988).

Thanks for your kind attention

Backup slides

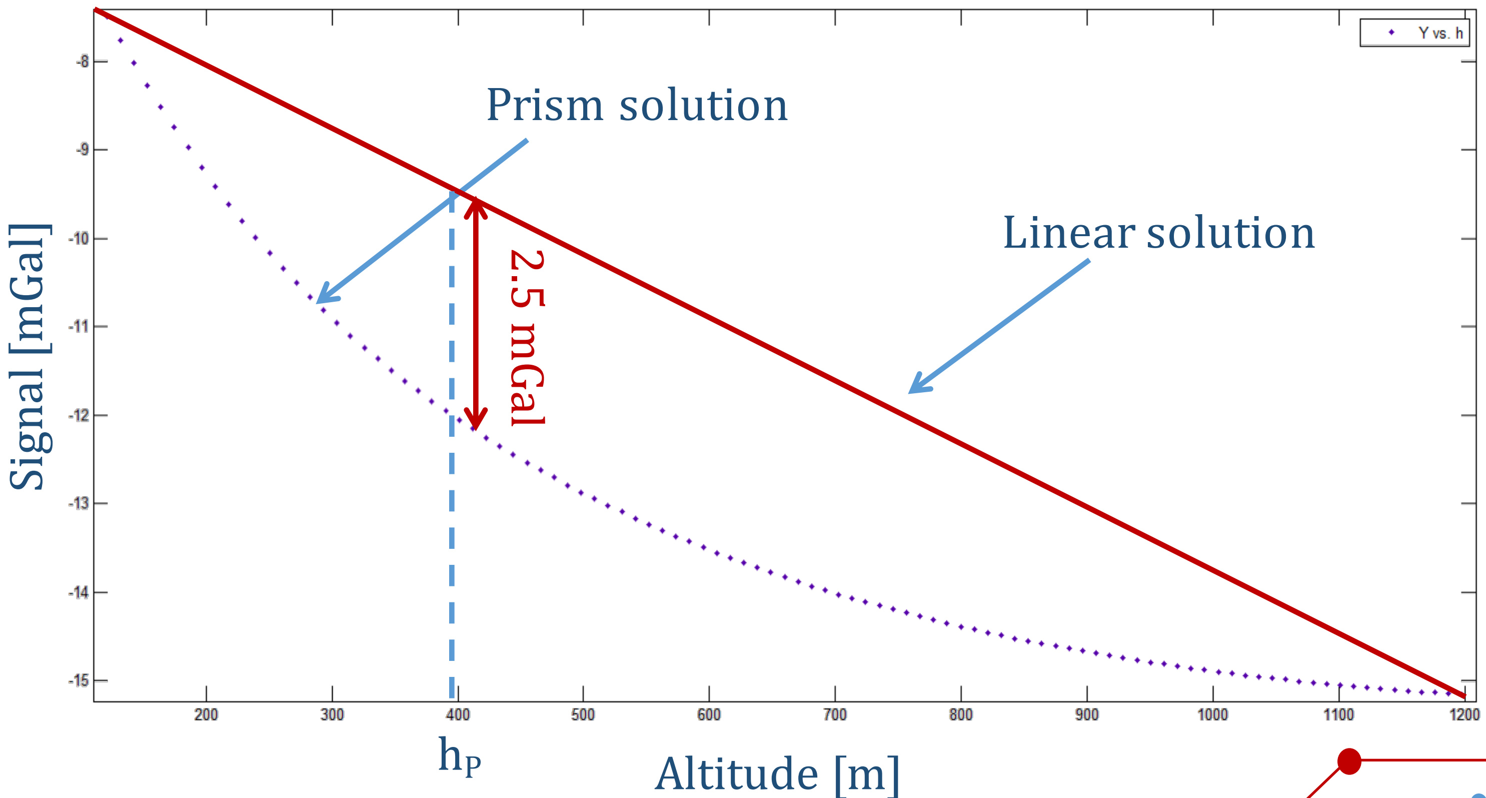
GTE for sparse points

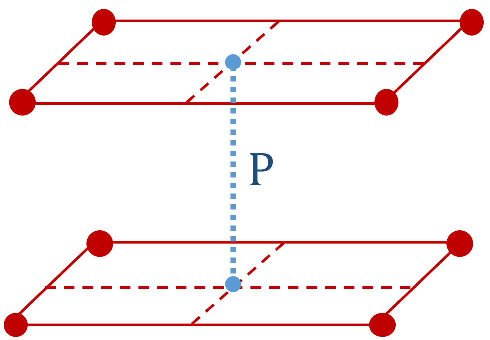
Problem



When we are **close to the topography** it can happen that **a knot of the grid is not computed because it is below the topography.**

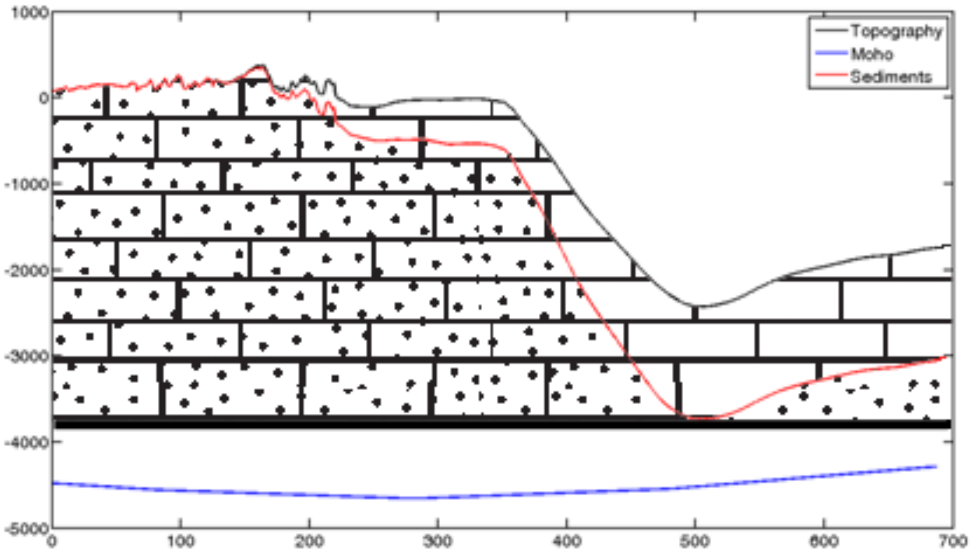
“Problematic” Points





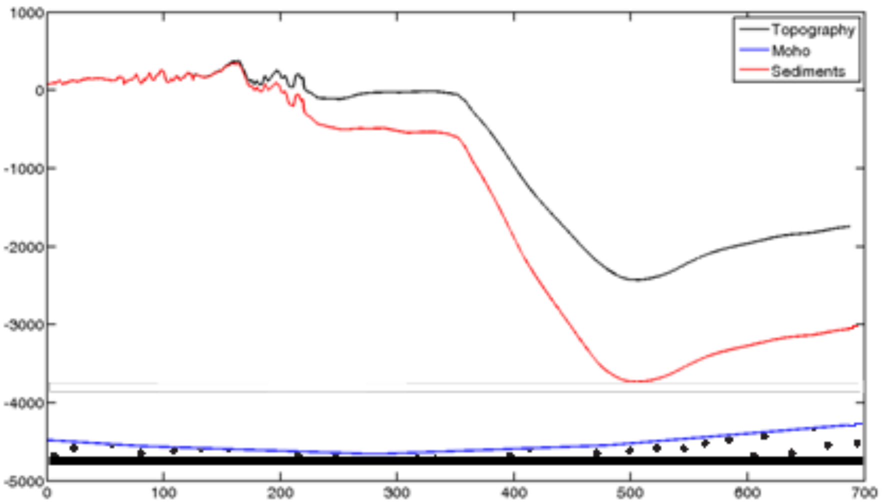
We compute the TC on a column in correspondence of P every 10 meters by prism and with the linear interpolation.

Terrain Correction



The gravitational effect of sediments is brought back to the computation of two terrain corrections: one for the top and one for the bottom.

(The two grids defining sediments should be properly shifted)



The gravitational effect of the crust-mantle discontinuity is brought back to the computation of one terrain correction