Unsupervised Feature Selection Based on the Morisita Index

Jean Golay and Mikhail Kanevski

Institute of Earth Surface Dynamics (IDYST)
Faculty of Geosciences and Environment (FGSE)
University of Lausanne, Switzerland

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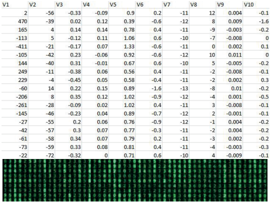
Introduction

High-Dimensional Data Sets

←Variables / Features→

Instances



V1	V2	V3	V4	V5	V6	V7	V8	V9	V10
2	-56	-0.33	-0.09	0.9	0.2	-11	12	0.004	-0.1
470	-39	0.02	0.12	0.39	-0.6	-12	8	0.009	-1.6
165	4	0.14	0.14	0.78	0.4	-11	-9	-0.003	-0.2
-113	5	-0.12	0.11	1.06	0.6	-10	-7	-0.008	0
-411	-21	-0.17	0.07	1.33	-0.6	-11	0	0.002	0.1
-105	-42	0.23	-0.06	0.92	-0.6	-12	10	0.011	0
144	-40	0.31	-0.01	0.67	0.6	-10	5	-0.005	-0.2
249	-11	-0.38	0.06	0.56	0.4	-11	-2	-0.008	-0.1
229	-4	-0.45	0.05	0.58	-0.4	-11	-2	0.002	0.3
-60	14	0.22	0.15	0.89	-1.6	-13	-8	0.01	-0.2
-206	8	0.35	0.12	1.02	-0.9	-12	-4	0.001	-0.5
-261	-28	-0.09	0.02	1.02	0.4	-11	3	-0.008	-0.1
-145	-46	-0.23	0.04	0.89	-0.7	-12	2	-0.001	-0.1
-27	-55	0.2	0.06	0.76	-0.9	-12	-1	0.004	-0.2
-42	-57	0.3	0.07	0.77	-0.3	-11	-2	0.004	-0.2
-61	-58	0.34	0.07	0.79	0.2	-11	-3	0.002	-0.2
-73	-59	0.33	0.08	0.81	0.4	-11	-4	-0.003	-0.3
-22	-72	-0.32	0	0.71	0.6	-10	4	-0.009	-0.1

Issues

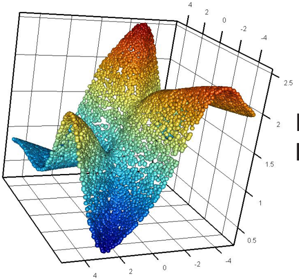
1. Curse of dimensionality
2. Computer performance
3. Data visualization
4. Interpretability of the results

Solutions

1. PCA
2. MDS
3. etc.

A New Solution

1. The concept of Intrinsic Dimension (ID)
2. The Morisita estimator of ID
3. An ID-based algorithm for selecting the smallest subset of features conveying all the information content of a data set



ID=2
E=3

Outline

1. The Morisita index
   - Calculation
2. The Morisita Estimator of ID
   - Calculation
3. ID-based Feature Selection
   - ID and Redundancy
   - The Proposed Algorithm
   - A Simulated Case Study
   - Real Case Studies
4. Conclusion

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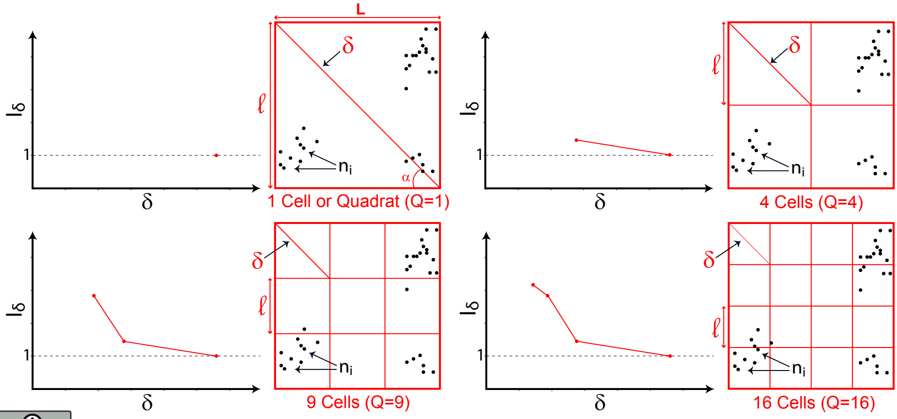
Calculation

$$I_\delta = Q \frac{\sum_{i=1}^{Q} n_i(n_i-1)}{N(N-1)}$$



1 Cell or Quadrat (Q=1)

4 Cells (Q=4)

9 Cells (Q=9)

16 Cells (Q=16)

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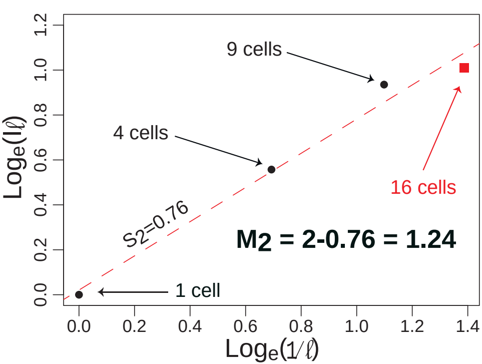
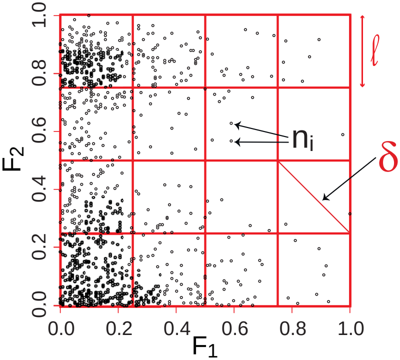
Calculation

$$M_2 = E - S_2$$

The concept of ID is extended to non-integer (fractal) dimensions.





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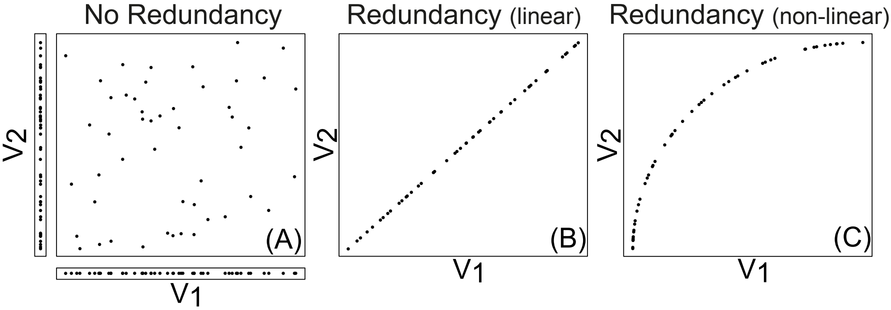
ID and Redundancy

V_1 and V_2 are two uniformly distributed variables and one has that:
$ID(V_1, V_2) \approx ID(V_1) + ID(V_2) \approx 1 + 1 = 2$ (see (A))
$ID(V_1, V_2) \approx ID(V_1) \approx ID(V_2) \approx 1$ (see (B) and (C))

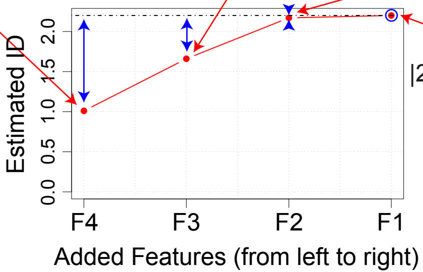


Redundant features (variables) do not contribute to the data ID
Idea (Traina's work): select the features which increase the data ID.

The Proposed Algorithm

A = {F1,F2,F3,F4} and M_2(A) = 2.20

Step 1	Step 2	Step 3
|2.20 - M_2(F1)| = 1.20	|2.20 - M_2(F4,F1)| = 1.14	|2.20 - M_2(F4,F3,F1)| = 0.52
|2.20 - M_2(F2)| = 1.34	|2.20 - M_2(F4,F2)| = 0.59	|2.20 - M_2(F4,F3,F2)| = 0.03
|2.20 - M_2(F3)| = 1.30	|2.20 - M_2(F4,F3)| = 0.54	
|2.20 - M_2(F4)| = 1.19		



Step 4

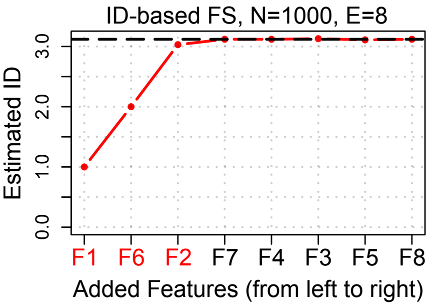
|2.20 - M_2(F4,F3,F2,F1)| = 0.00

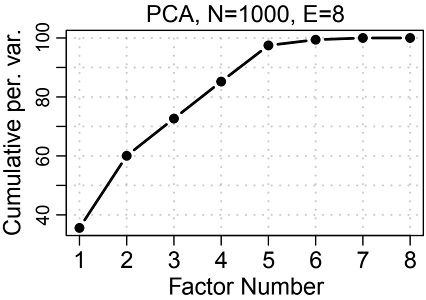
F1 is redundant, since it hardly contributes to increasing the data ID

A Simulated Case Study: the Input Space of the Butterfly Data Set

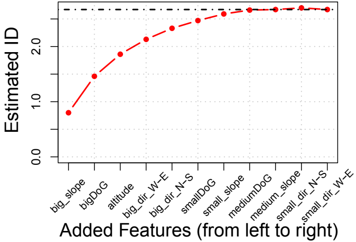
$F_1, F_2, F_6 \in]-5, 5[$ $\quad F_3 = \log_{10}(F_1 + 5)$ $\quad F_4 = F_1^2 - F_2^2$

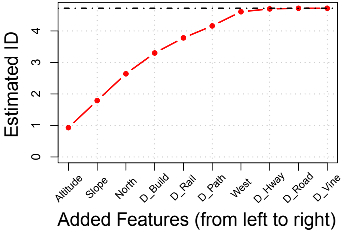
$F_5 = F_1^4 - F_2^4$ $\quad F_7 = \log_{10}(F_6 + 5)$ $\quad F_8 = F_6 + F_7$





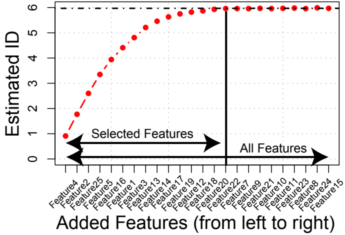
Real Case Studies I


Windspeed: N=127, E=11


Forest Fires: N=2275, E=10

Permafrost: N=54674, E=24

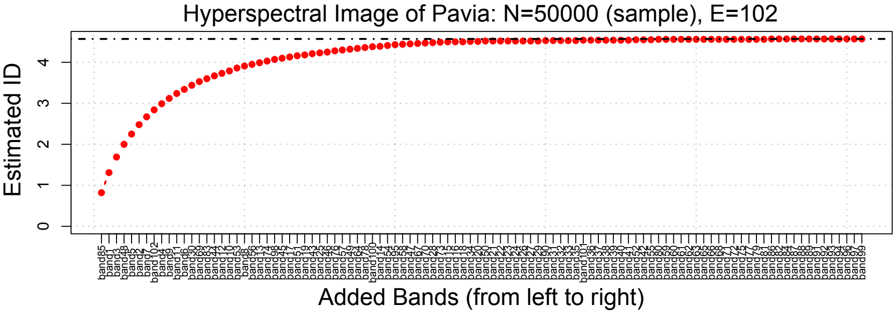
The data set was kindly provided by Nicola Deluigi



The information content was assessed by means of random forest

	All Features	Selected Features
Error rate (%)	14.93 (3.13)	14.40 (2.63)

Real Case Studies II



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Take-Home Message

The concept of Intrinsic Dimension (ID) can help find solutions to the issues raised by large data sets.

- C. Traina Jr., A. J. M. Traina, L. Wu, C. Faloutsos, Fast feature selection using fractal dimension, *Proceedings of the XV Brazilian Symposium on Databases (SBBD)*, pp. 158-171, 2000.
- J. Golay, M. Leuenberger, M. Kanevski, Feature Selection for Regression Problems Based on the Morisita Estimator of Intrinsic Dimension, *arXiv:1602.00216*, 2016.
- J. Golay and M. Kanevski, A new estimator of intrinsic dimension based on the multipoint Morisita index, *Pattern Recognition*, 48(12):4070-4081, 2015.
- J. Golay, M. Kanevski, C. D. Vega Orozco and M. Leuenberger, The multipoint Morisita index for the analysis of spatial patterns, *Physica A*, 406:191-202, 2014.