

Variational Data Assimilation for the Optimized Ozone Initial State and the Short-Time Forecasting

Soon-Young Park¹, Dong-Hyeok Kim¹, Soon-Hwan Lee², and Hwa Woon Lee³

¹Institute of Environmental Studies, Pusan National University, Busan, Republic of Korea ²Department of Earth Science Education, Pusan National University, Busan, Republic of Korea ³Division of Earth Environmental System, Pusan National University, Busan, Republic of Korea

1. Introduction

Data assimilation (DA) provides a consistent representation of the physical state by blending imperfect model predictions and noisy observations. With more chemical observations available in recent years data assimilation is expected to improve the predictability of air quality.

- One of the important elements affecting results of data assimilation in the 4D-Var process is the background error covariance (BEC) of the model.
- In this study, the previously developed 4D-Var code has been modified to treat background errors in matrix forms, and various numerical tests have been conducted.
- Characteristics of the background errors obtained through long-term modeling results are analyzed. Also, the predictability of high ozone concentration was investigated.

2. 4D-Var Data Assimilation

In the maximum likelihood approach, the 4D-Var data assimilation gives the maximum a posteriori estimator of the true initial concentration distribution, which is obtained by minimizing the cost function:

$$\mathcal{J}(\boldsymbol{c}_0) = \frac{1}{2} \left(\boldsymbol{c}_0 - \boldsymbol{c}_0^b \right)^{\mathrm{T}} \mathbf{B}_0^{-1} \left(\boldsymbol{c}_0 - \boldsymbol{c}_0^b \right) + \frac{1}{2} \sum_{k=1}^{F} \left(\mathcal{H}(\boldsymbol{c}_k) - \boldsymbol{c}_k^{obs} \right)^{\mathrm{T}} \mathbf{R}_k^{-1} \left(\mathcal{H}(\boldsymbol{c}_k) - \boldsymbol{c}_k$$

The 4D-Var analysis can be obtained by the initial concentration that minimizes (1) with respect to the model equation. Formally, a gradient-based optimization procedure is used to obtain minimum value. Assuming a linear observation operator $\mathbf{H}_k = \mathcal{H}'(\boldsymbol{c}_t)$, the gradient of (1) with respect to \boldsymbol{c}_0 is

$$\nabla_{\boldsymbol{c}_0} \mathcal{J}(\boldsymbol{c}_0) = \mathbf{B}_0^{-1} (\boldsymbol{c}_0 - \boldsymbol{c}_0^b) + \sum_{k=1}^F \left(\frac{\partial \boldsymbol{c}_k}{\partial \boldsymbol{c}_0} \right)^T \mathbf{H}_k^T \mathbf{R}_k^{-1} (\mathbf{H}_k \boldsymbol{c}_k - \boldsymbol{c}_k^{obs}).$$
(2)

The gradient for the 4D-Var cost function can be effectively obtained by forcing the adjoint model with observation increments and calculating it backwards. When the forward and reverse adjoint models are performed, it results in the problem of solving the following equation:

 $\nabla_{\boldsymbol{c}_0} \mathcal{J}(\boldsymbol{c}_0) = \mathbf{B}_0^{-1} (\boldsymbol{c}_0 - \boldsymbol{c}_0^b) + \boldsymbol{\lambda}_0 = 0.$

 λ_0 is the sensitivity of the cost function (1) with respect to the initial concentration c_0 . Since B_0^{-1} , c_0^b , and λ_0 values are known, if the value of c_0 that satisfies equation (3) is found, it becomes the analysis field c_0^a .

The adjoint code for CMAQ (CMAQ-ADJ) model was implemented from the project H98 (University of Huston, 2009) by Huston Advanced Research Center / Texas Environmental Research Consortium (HARC/TERC). The validation and several numerical tests of this code are well described in Hakami et al. (2007). This model only considers the model and observation errors as its variance, i.e. a constant value of σ_0^B and σ_k^{obs} with Gaussian distribution.

$$\mathcal{J}(\boldsymbol{c}_{0}) = \frac{1}{2(\sigma_{0}^{b})^{2}} (\boldsymbol{c}_{0} - \boldsymbol{c}_{0}^{b})^{\mathrm{T}} (\boldsymbol{c}_{0} - \boldsymbol{c}_{0}^{b}) + \frac{1}{2(\sigma_{k}^{obs})^{2}} \sum_{k=1}^{N} (\mathbf{H}_{k} \boldsymbol{c}_{k} - \boldsymbol{c}_{k}^{obs})^{\mathrm{T}} (\mathbf{H}_{k} \boldsymbol{c}_{k} - \boldsymbol{c}_{k}^{obs})$$
(4)

4D-Var

periods

 $(k_k) - \boldsymbol{c}_k^{obs}$). (1)

(2)

3. Experimental Design

The capital region of South Korea is selected for the present data assimilation study because measurement sites are relatively evenly distributed.



Figure 1. The model domains for WRF and CMAQ. The air quality monitoring sites at ground level are marked by green blank circles.

4. Idealized BEC Results



Figure 2. Covariance distribution for Balgovind functions with respect to the grid distance (r) and the values of radius of influence (L).

This experiment is used to evaluate the characteristics of the BEC based on a single observation experiment. In this experiment, 100 ppb of O_3 was incorporated as an arbitrary value rather than actual observation data at the initial time at the center of the model domain.



Figure 3. Horizontal distribution of analysis increments at surface resulted from the single observation experiment with respect to radius of influence (L).

At the model grid point (29, 31), where arbitrary observation data were applied, all three tests showed an O_3 increment of about 50.0 ppb. The background concentration of O_3 at the grid was 40.1 ppb, but the value was up to about 90 ppb in the analysis when the synthetic observation of 100 ppb was applied. However, as the value of L increased, the O_3 increment in the analysis occurs at more surrounding grids. These results indicate that the idealized BEC performs well in the revised codes, and proper analysis increments can be achieved when the spatial correlation is taken into account.

CONTACT: soon@pusan.ac.kr

uration of WRF modeling system							
	d27	d09	d03				
d	123 × 130	72 × 84	65 × 68				
n	27 km	9 km	3 km				
S	33 la	33 layers (top: 50 hPa)					
	WSM5 scheme, Kain-Fritsch scheme,						
	Noah LSM, Yonsei University PBL,						
	RRTM Longwave, Dudhia Shortwave						
NCEP FNL data							
00 UTC 03 August ~ 00 UTC 07 August, 2008							
uration of CMAQ 4D-Var modeling system							
	d27	d09	d03				
l input	nput correspond to each WRF domain						
grid	118×125	5 67 × 79	60 × 63				
ion	27 km	9 km	3 km				
ers	1	15 layers (top: 20 km)					
ata	INTEX-B	CAPSS	CAPSS				
d (FWI	WD) 00 UTC 03 ~ 00 UTC 07 August, 2008 (4 days)						

12 UTC 05 ~ 12 UTC 06 August, 2008 (forecast)

12 UTC 05 ~ 00 UTC 06 August, 2008 (analysis)

00 UTC 06 ~ 00 UTC 07 August, 2008 (forecast)

Table 3. Experimental design for the idealized background error covariance test.

	Obs. Data	Radius of Influence	σ_0^B	σ_k^{obs}
	100 ppb at (29,31)	L=02	BEC	8.00
		L=05	BEC	8.00
		L=10	BEC	8.00

Figure 4. Cross-section of analysis increments along the blue line in Figure 6. (b) as the radius of influence (L) values are increase.



5. Development of Realistic BEC

The BEC is obtained using the NMC approach, which is based on a real simulation for the realistic 4D-Var data assimilation study. The error statistics for the CMAQ model is defined by the differences between +48 hours and +24 hours forecast, $\epsilon^i = c^i_{+48h} - c^i_{+24h}$. We assume **B** can be written as **B** = $X \otimes Y \otimes Z \otimes C$ where, X, Y, and Z representing the error correlation in the three directions. C is the error covariance matrix at a single grid point that refers to the error variances and correlation between different species. In this study, C is considered to be no correlation between the species. It seems to be error-prone to invert ill-conditioned matrices. Based on the property of Kronecker product, B^{-1} can be expressed as $B^{-1} = (X \otimes Y \otimes Z)^{-1} = X^{-1} \otimes X^{-1}$ $Y^{-1} \otimes Z^{-1}$. Singular Value Decomposition (SVD) is applied to **B** matrix.



Figure 5. Model error correlation coefficients (a) between vertical levels and (b) between two layers as a function of Δz . The fitted line is $R = exp(-\Delta z^{1.2}/l_z^{1.2})$, where $l_z=300$ m.

The correlation coefficient for the east-west direction is somewhat higher than that for the south-north direction.

6. Validation and Forecasting Results



and its time variations.

The 4DV experiment shows a diurnal variation of O_3 concentration that conforms well to the observation. RMSE decreases by about 49.4%, and the IOA increases by 59.9%, suggesting that the initial conditions of ozone concentration are successfully improved by application of DA.



Figure 9. Time variations of observed and forecast ozone Figure 10. Time variations of RMSE and IOA for 24 h forecast concentration after (a) daytime and (b) nighttime assimilation. after (a) daytime and (b) nighttime assimilation.

A potential improvement for the ozone predictability is presented using the optimized initial condition after the time-window. In particular, a larger improvement in the predictability of daytime ozone concentration is expected if DA is performed over the nighttime than in the daytime.







corresponds to $R = exp(-\Delta h^{1.0}/l_h^{1.0})$ where $l_h = 10$ km.

selected sites.