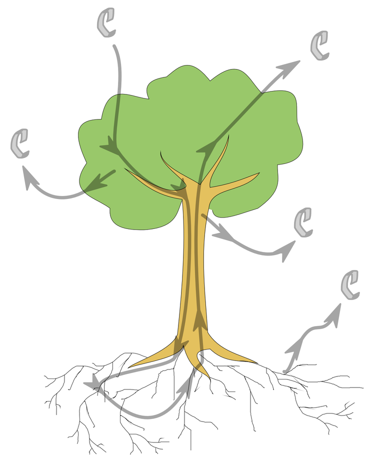

Nonlinearities in Carbon Allocation and Vegetation Functioning: *The Model Database*

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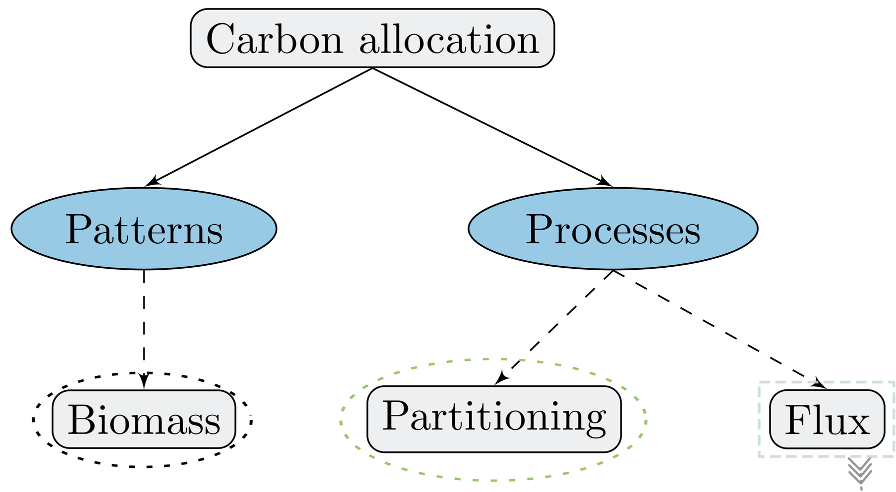
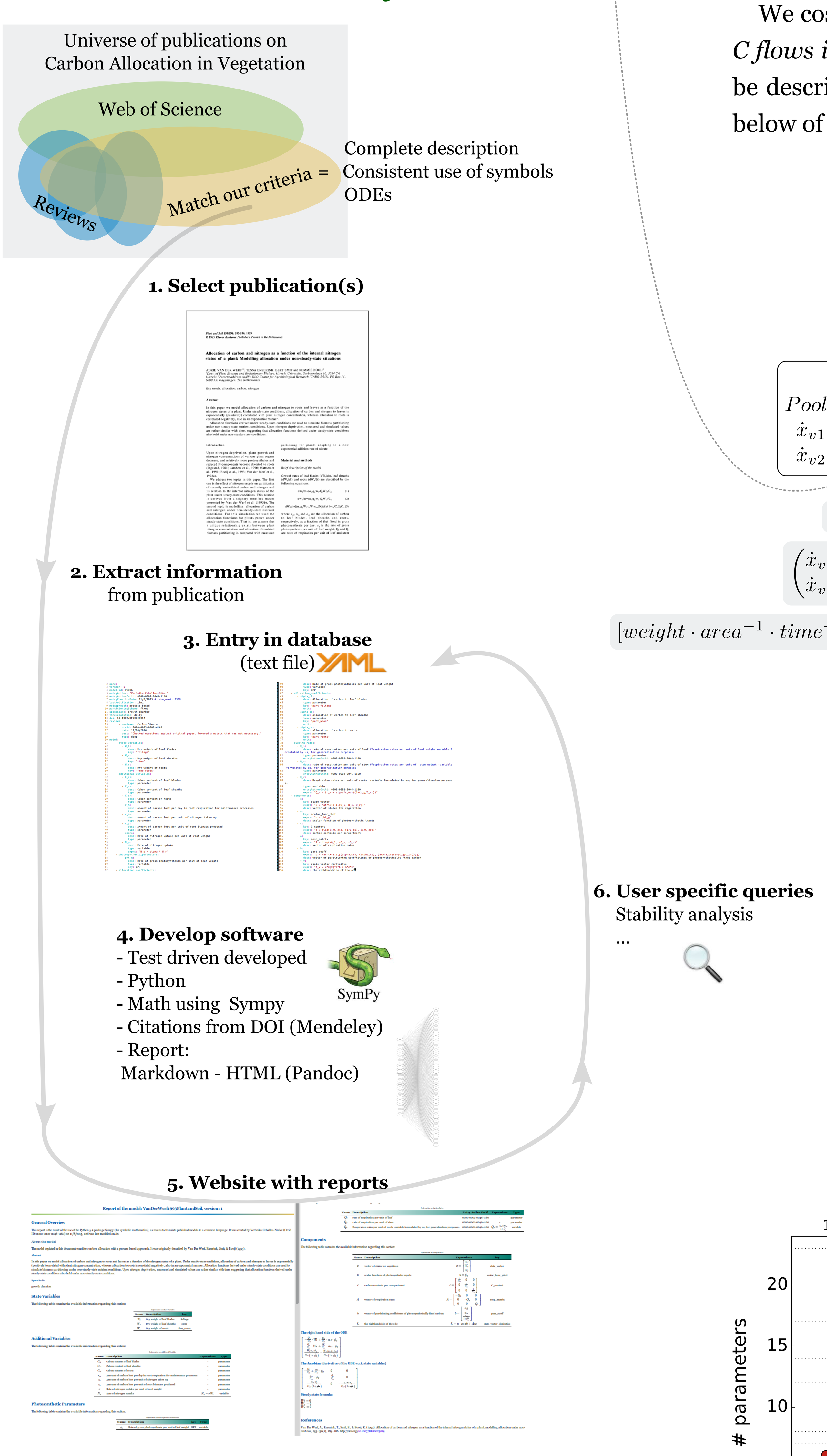


Overarching concept of carbon allocation, based on Litton et al. (2007)

Our workflow



Motivation

Given the current drastic changes in climate and atmospheric CO_2 concentrations, and the role of vegetation in the global carbon cycle, there is an increasing attention to the carbon allocation component in *biosphere terrestrial models*. Improving the representation of carbon (C) allocation in models could be the key to having *better predictions of the fate of C* once it enters the vegetation and is partitioned to C pools of different residence times. Although efforts have been made to compare different models, the outcome of these qualitative assessments has been a conceptual categorization of them. In this contribution, we introduce a new effort to *identify the main properties* of groups of models by studying their *mathematical structure*.

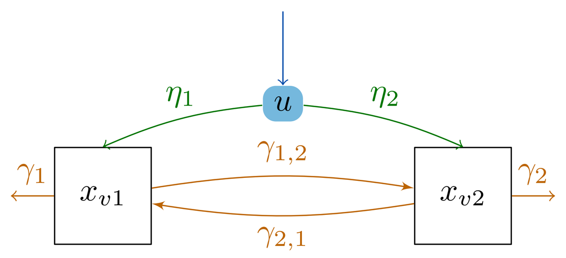
Outstanding Results

Most, if not all, the models included in the database were formulated under the assumption that C in vegetation can be subcategorized in pools (compartments).

We cosider that ***C allocation*** *is the balance (net flux) of the C flows into, between, and out of these pools.* This concept can be described using the following system of ODEs (see example below of 2 veg. pools).

This mathematical abstraction may vary among models, depending on how their variables and parameters interact with each other (i.e. whether they depend on each other or not).

Both, linear and non-linear structures have been proposed (see fig. below). The most common non-linearity is $u(\mathbf{x}_v,t)*\mathbf{b}(\mathbf{x}_v,t)$. This is a positive feedback loop that assures that the more C in foliage, the more photosynthesis can be performed.



Pools		Inflows		Outflows
$\dot{x}_{v1}$	$=$	$u\cdot\eta_1+\gamma_{2,1}\cdot x_{v2}$	$-$	$(\gamma_1+\gamma_{1,2})\cdot x_{v1}$
$\dot{x}_{v2}$	$=$	$u\cdot\eta_2+\gamma_{1,2}\cdot x_{v1}$	$-$	$(\gamma_2+\gamma_{2,1})\cdot x_{v2}$

Fluxes

$$\boldsymbol{f}_v = u \cdot \boldsymbol{b} + \mathbf{A} \cdot \boldsymbol{x}_v$$

$$\begin{pmatrix}\dot{x}_{v1}\\ \dot{x}_{v2}\end{pmatrix} = u \cdot \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} + \begin{pmatrix}-(\gamma_1+\gamma_{1,2}) & \gamma_{2,1}\\ \gamma_{1,2} & -(\gamma_2+\gamma_{2,1})\end{pmatrix} \cdot \begin{pmatrix}x_{v1}\\ x_{v2}\end{pmatrix}$$

$$[weight\cdot area^{-1}\cdot time^{-1}] = [weight\cdot area^{-1}\cdot time^{-1}]\cdot \square + [time^{-1}]\cdot[weight\cdot area^{-1}]$$

Mathematical formulation

Difference eq.

$$\boldsymbol{x}_v(t_{n+1}) = \boldsymbol{x}_v(t_n) + \boldsymbol{f}_v(\boldsymbol{x}_v,\boldsymbol{g}(t_n))\cdot(\Delta t)$$

Linear ODEs

$$\dot{\boldsymbol{x}}_v = \boldsymbol{f}_v(\boldsymbol{x}_v,t) = u(t)\cdot\boldsymbol{b}(t) + \mathbf{A}(t)\cdot\boldsymbol{x}_v$$

$$\dot{\boldsymbol{x}}_v = \boldsymbol{f}_v(\boldsymbol{x}_v,t) = u(t)\cdot\boldsymbol{b} + \mathbf{A}\cdot\boldsymbol{x}_v$$

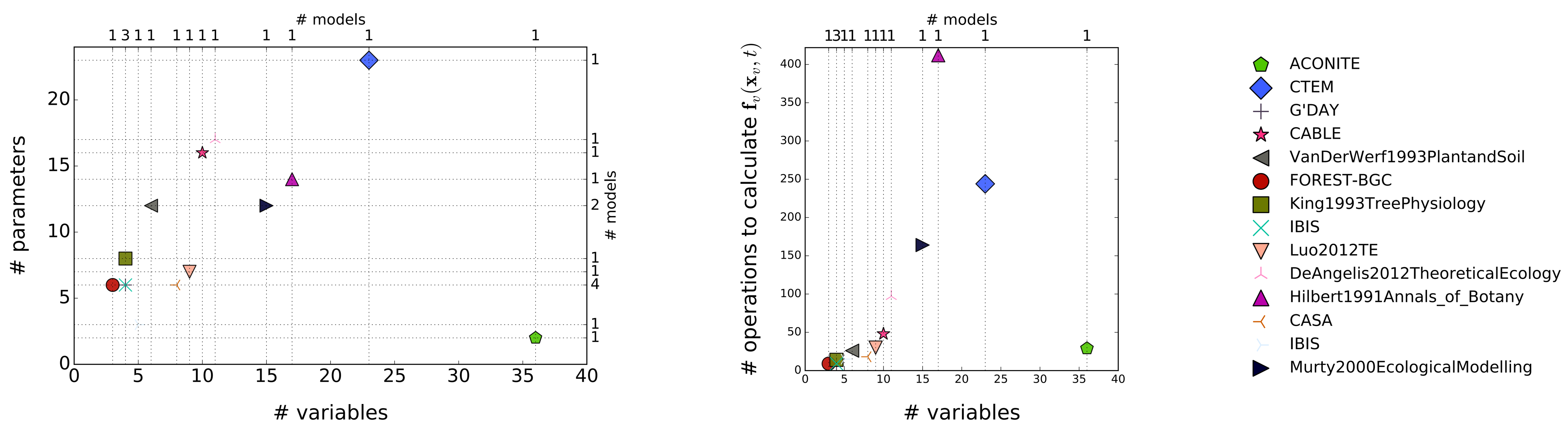
Nonlinear ODEs

$$\dot{\boldsymbol{x}}_v = \boldsymbol{f}_v(\boldsymbol{x}_v,t) = u(t)\cdot\boldsymbol{b}(\boldsymbol{x}_v,t) + \mathbf{A}(t)\cdot\boldsymbol{x}_v$$

$$\dot{\boldsymbol{x}}_v = \boldsymbol{f}_v(\boldsymbol{x}_v,t) = u(\boldsymbol{x}_v,t)\cdot\boldsymbol{b}(\boldsymbol{x}_v,t) + \mathbf{A}(t)\cdot\boldsymbol{x}_v$$

Glossary:
ODE: Ordinary differential equation
Variable(1): model atribute that changes during the model simulation
State variable: "dynamically (i.e.time) varying characteristic of the system/model that represents the storage of mass/volume of the time varying quantity of interest within the system/model." (1)
Parameter(1): model atribute that does not change during the model simulation
Difference equations(1): are obtained when dependent variables are assumed to take discrete (not continuous) values
Legend:
$\boldsymbol{x}_v$: vector of states for vegetation (state variables).
$\dot{\boldsymbol{x}}_v = d\boldsymbol{x}_v/dt$: Changing quantity of vegetation compartment x_v with respect to time (state variables).
$\boldsymbol{f}_v(...)$: Function valued vector that relates the changing quantities that take place between the vegetation compartments. $\boldsymbol{f}_v(\boldsymbol{x}_v,\boldsymbol{g}(t_n))$ represents its dependency on state variables ($\boldsymbol{x}_v$) and time-dependent functions ($\boldsymbol{g}(t_n)$). $\boldsymbol{f}_v(\boldsymbol{x}_v,t)$ represents its dependency on state variables and time.
$\Delta t = t_{n+1}-t_n$: Time step size.
$u(t)$: Scalar function of photosynthetic inputs. $u(\boldsymbol{x}_v,t)$ represents its time dependency.
$\boldsymbol{b}$: Vector containing the partitioning coefficients of photosynthetic input. ($\boldsymbol{b}(t)$ represents its time dependency. May also depend on vegetation compartments (allowing for nonlinearities) as described in the partitioning function valued vector: ($\boldsymbol{b}(\boldsymbol{x}_v,t)$).
$\mathbf{A}$: Carbon transfer coefficients between plant, litter, and soil pools. $\mathbf{A}(t)$ represents its time dependency.

Model complexity

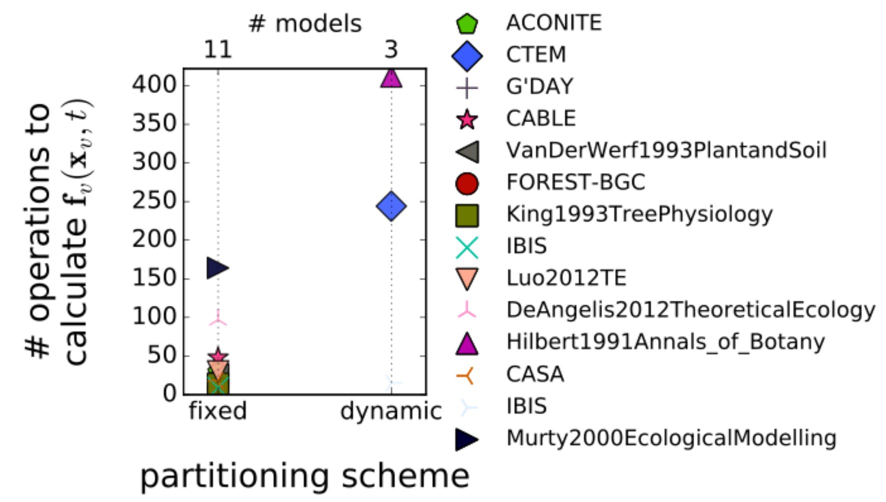


Preliminary results

What are the main groups in which the models can be classified?

* Modeling approach * Mathematical formulation * **Conceptual design of C partitioning**

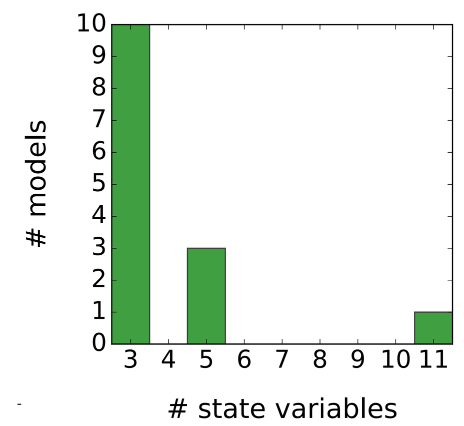
Models with dynamic partitioning ($\boldsymbol{b}(\boldsymbol{x}_v,t)$) *of photosynthetically fixed C require more mathematical operations than the ones with fixed coef.*



What are the main model characteristics (driving variables, parameters)?

a) 3 state variables

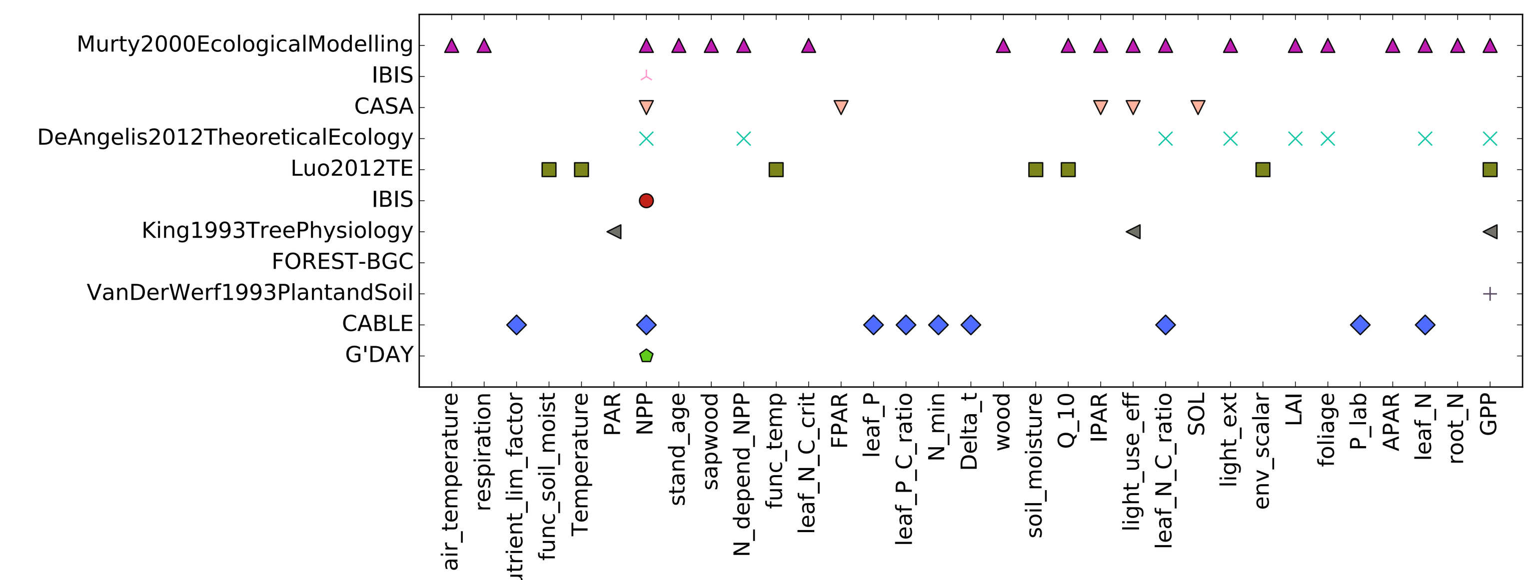
Most common pools:
* Foliage
* Wood
* Roots



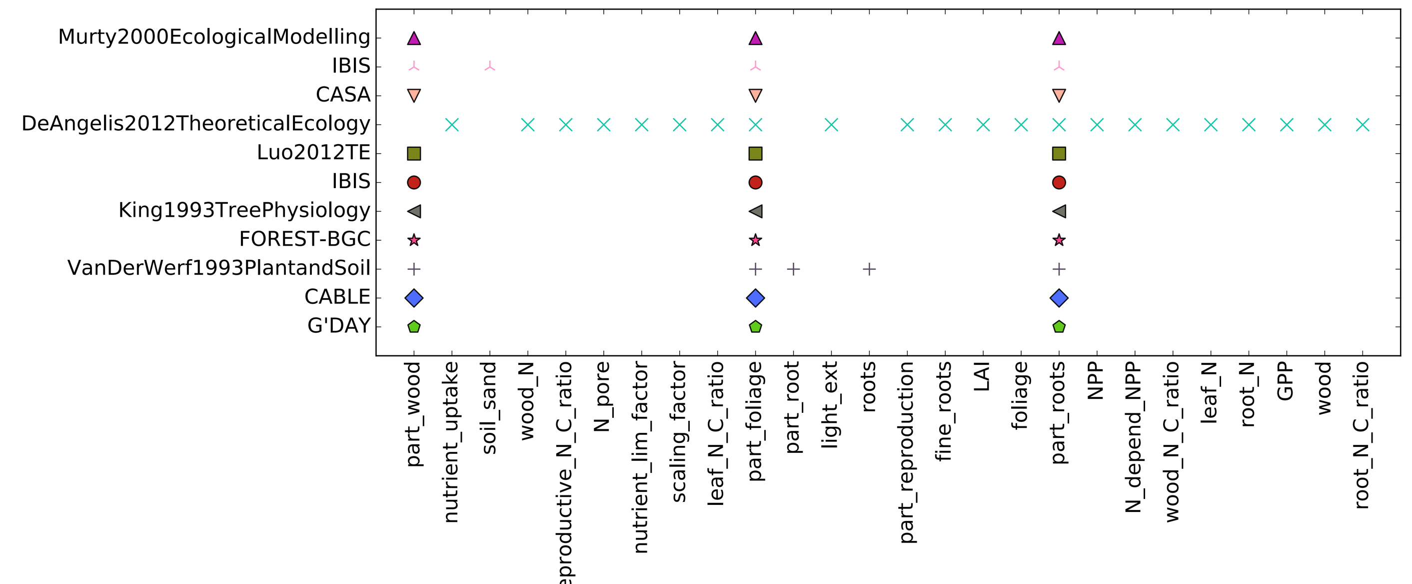
b) When dynamic, partitioning depends on water, and nutrient and light availability

c) Partitioning of photosynt. fixed C is not directly regulated by temperature, nor [CO2]
* Water content, LAI. . .

d) Principal constraints of allocation:
– Ratios (foliage:roots, etc. . .) – Sum of allocation coef. = 1



Dependencies of scalar function of photosynthetic input



Dependencies of vector of partitioning coeff. of photo. input

Cited literature

Litton, C. M., Raich, J. W., and Ryan, M. G. (2007). Carbon allocation in forest ecosystems. Global Change Biology, 13(10):2089–2109.
Luo, Y., Weng, E., and Yang, Y. (2012). Ecosystem ecology. In Hastings, A. and Gross, L., editors, Encyclopedia of Theoretical Ecology, pages 219–229. University of California Press, Berkeley.

(1) http://web.hwr.arizona.edu/hwr642/Generic/Content/Definitions/DefinitionsText.html also http://www.math.tamu.edu/~phoward/m647/modode.pdf

Max Planck Institute for Biogeochemistry