Nonstationary frequency analysis for the trivariate flood series of the Weihe River

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I. Introduction

Background

➢Hydrological systems are changing (**Panta Rhei-Everything Flows, *Montanari et al.*, 2013**)

➢Under changing environments, stationarity assumption for hydrological series may be invalid (**Stationarity is dead, *Milly et al.*, 2008**)

➢Nonstationarity in multivariate hydrological series has not been widely and deeply investigated (***Jiang et al.*, 2015**)

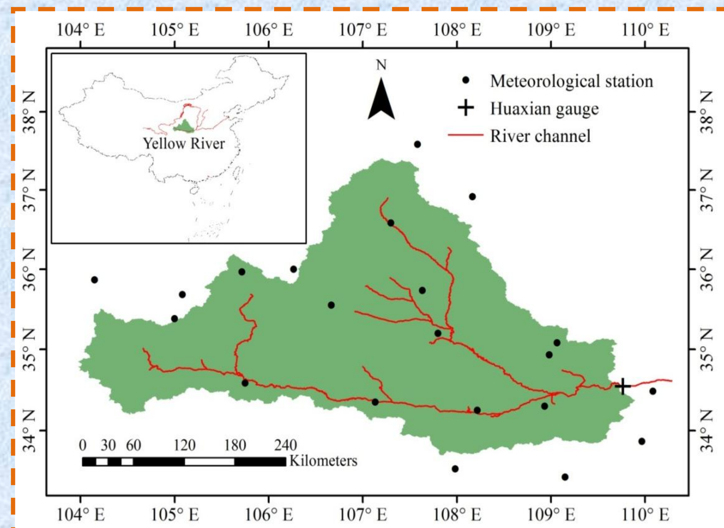
Objectives

➢Detecting and attributing the nonstationarities in both **marginal distributions** and **dependence structure of multivariate flood series**

Case study

➢Trivariate flood series of the Weihe River, China.

II. Study area and data



Weihe River

✓A Catchment of 100,800 km²

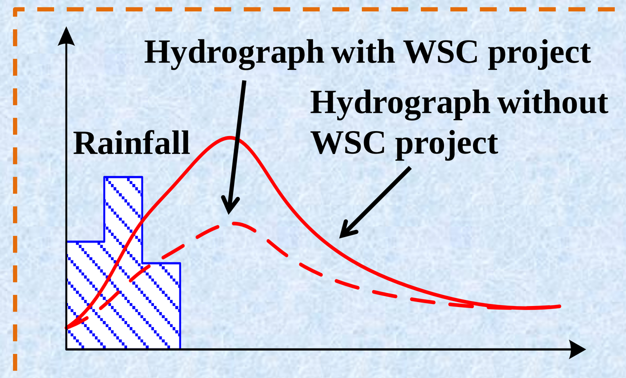
✓The largest tributary of the Yellow River

✓Human effect: **Water and Soil Conversation Projects (WSC)**

Effects of WSC project

➢Altering conditions of vegetation cover and topographical feature

➢Affecting the nature hydrological processes



Rainfall-runoff relationships with and without WSC project

Study data

➢Trivariate flood series (1951-2011)

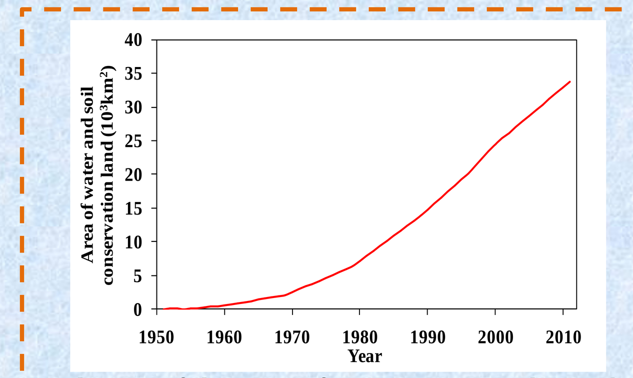
✓Annual maxima daily discharge (Q_1)

✓Annual maxima 3-day flood volume (V_3)

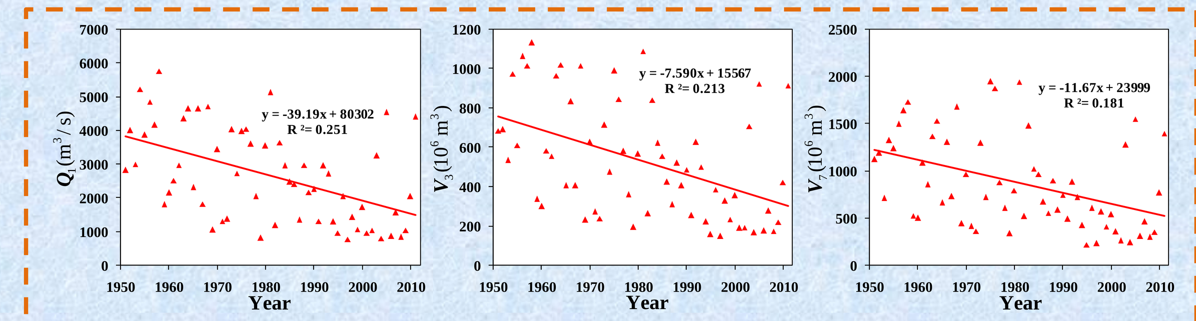
✓Annual maxima 7-day flood volume (V_7)

➢Human effect variable

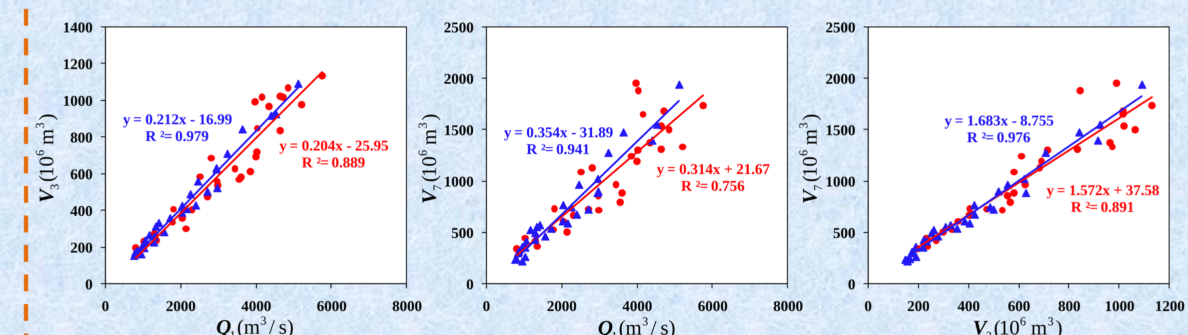
✓Area of **WSC** project



Evolution of the area of WSC project in the Weihe basin



Linear Trend of the three individual flood series of the Weihe River



Comparison between the linear correction of the bivariate flood variables in different periods. The red scatter represents the data from 1951 to 1980, while the blue scatter the data from 1981 to 2011

➢All individual flood series present significant decreasing trend

➢Dependence of the flood series might be nonstationary

III. Methods

Methodological Framework

The joint distribution of the trivariate flood variables at time t is constructed by **time-varying copula model**

$$H\left(Q_1^t,V_3^t,V_7^t\right)=C\left(F_{Q_1}\left(Q_1^t\middle|\boldsymbol{\theta}_{Q_1}^t\right),F_{V_3}\left(V_3^t\middle|\boldsymbol{\theta}_{V_3}^t\right),F_{V_7}\left(V_7^t\middle|\boldsymbol{\theta}_{V_7}^t\right)\middle|\boldsymbol{\theta}_c^t\right)$$

The dependence structure of trivariate flood series is described by **pair-copula** (***Xiong et al.*, 2015**), and expressed as

$$f\left\{F_{Q_1}\left(Q_1^t\middle|\boldsymbol{\theta}_{Q_1}^t\right),F_{V_3}\left(V_3^t\middle|\boldsymbol{\theta}_{V_3}^t\right),F_{V_7}\left(V_7^t\middle|\boldsymbol{\theta}_{V_7}^t\right)\middle|\boldsymbol{\theta}_c^t\right\}=f\left(u_{Q_1}^t,u_{V_3}^t,u_{V_7}^t\middle|\boldsymbol{\theta}_c^t\right)$$
$$=c_{12}\left(u_{Q_1}^t,u_{V_3}^t\middle|\theta_{c12}^t\right)\cdot c_{13}\left(u_{Q_1}^t,u_{V_7}^t\middle|\theta_{c13}^t\right)\cdot c_{23|1}\left\{F\left(u_{V_3}^t\middle|u_{Q_1}^t\right),F\left(u_{V_7}^t\middle|u_{Q_1}^t\right)\middle|\theta_{c23|1}^t\right\}$$

Application Steps

Step 1: Detecting and modeling nonstationarities in marginal distributions of (Q_1, V_3, V_7)

Using **Pearson Type III distribution** to fit the marginal distribution, where only the location parameter is time-varying and expressed as a function of WSC

$$f_Y\left(y^t\mid\mu^t,\sigma,\nu\right)=\frac{1}{\sigma\left|\mu^t\nu\right|\Gamma\left(1/\nu^2\right)}\left(\frac{y^t-\mu^t}{\mu^t\sigma\nu}+\frac{1}{\nu^2}\right)^{\frac{1}{\nu^2}-1}e^{-\left(\frac{y^t-\mu^t}{\mu^t\sigma\nu}+\frac{1}{\nu^2}\right)}$$

$$\mu^t=e^{a_0+a_1\cdot WSC^t}$$

Step 2: Detecting and modeling nonstationarity in dependence structure

Using **Gumbel Copula** to construct the pair-copula, where each copula parameter is expressed as a function of WSC

$$C\left\{F_{Y_1}\left(y_1^t\right),F_{Y_2}\left(y_2^t\right)\mid\theta_c^t\right\}=C\left\{u_1^t,u_2^t\mid\theta_c^t\right\}=\exp\left\{-\left[\left(-\ln u_1^t\right)^{\theta_c^t}+\left(-\ln u_2^t\right)^{\theta_c^t}\right]^{1/\theta_c^t}\right\}$$

$$\theta_c^t=e^{b_0+b_1\cdot WSC^t}$$

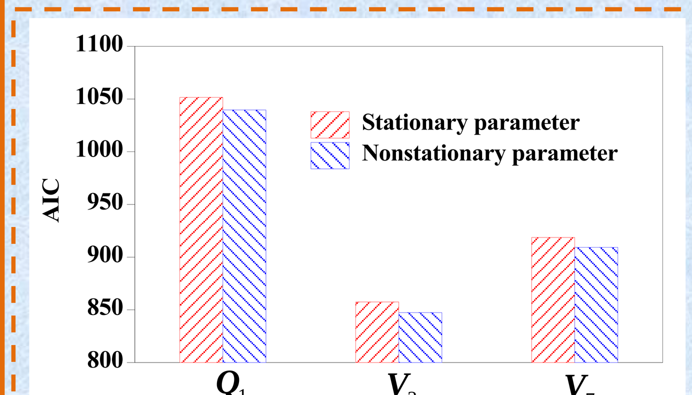
According to the hierarchical levels of pair-copula, nonstationarities in θ_{c12} and θ_{c13} are examined firstly, and then $\theta_{c23|1}$.

Model Parameter estimation

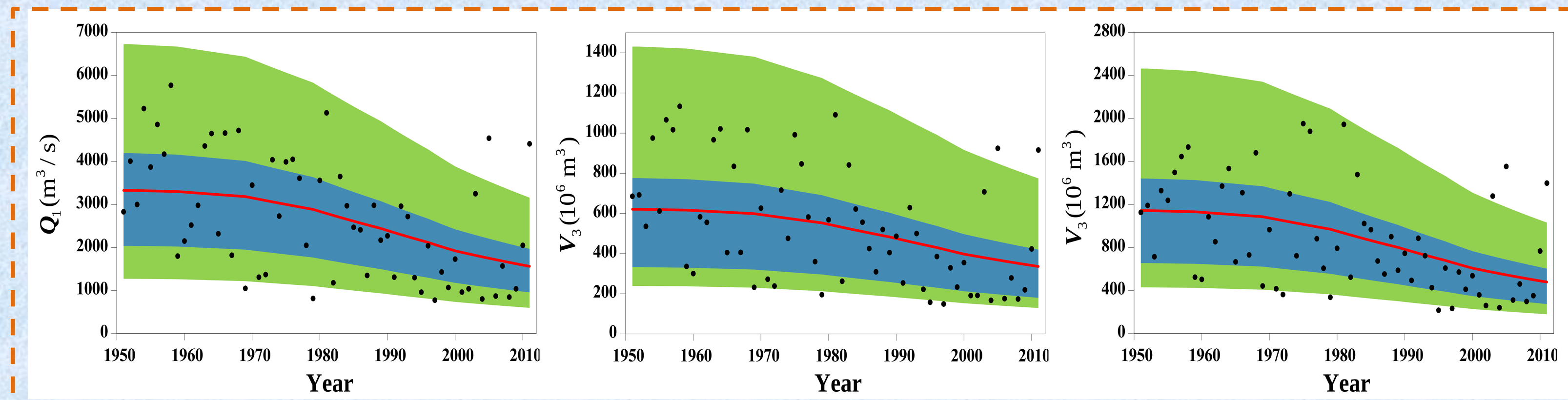
All parameters in the model are estimated by maxima likelihood estimation method (MLE), and to avoid model over-fitting the Akaike Information Criterion (**AIC**) is used to perform model selection

IV. Results

Nonstationarities in marginal distributions



Comparison between stationary and nonstationary models in fitting marginal distributions.

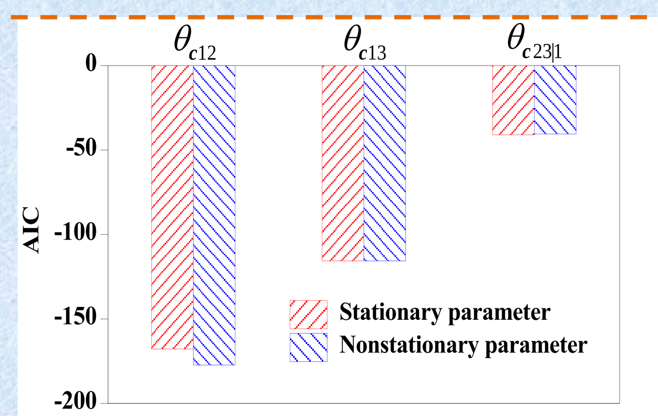


Evolution of marginal distributions with WSC as explanatory variable. The red line represents the mean of the flood series, the indigo blue region the area between 25% and 75% percentiles, while the green region the area between 5% and 95% percentiles.

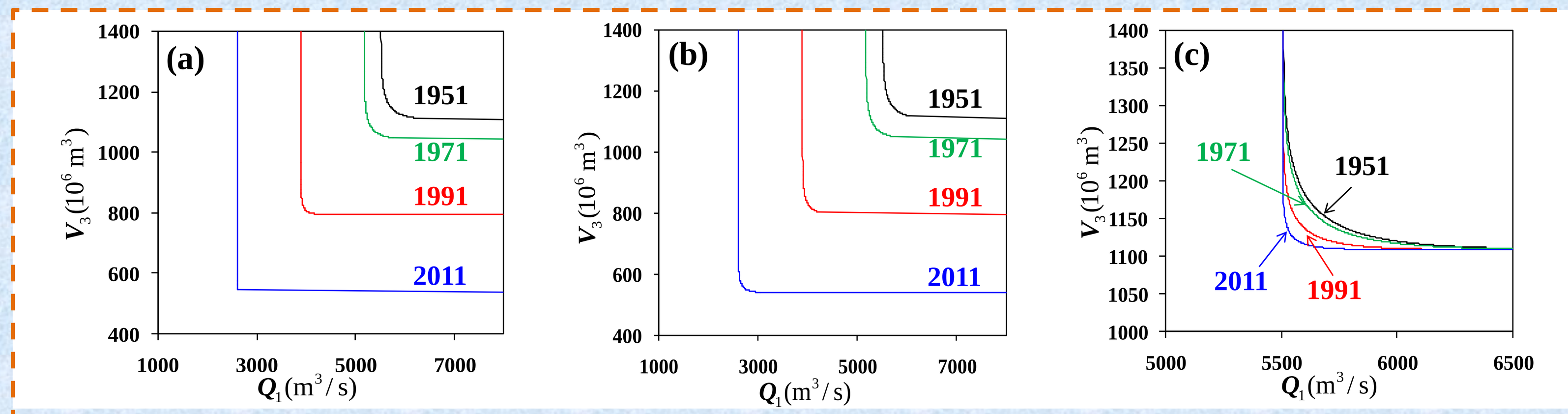
➢All marginal distributions are nonstationary

➢The nonstationary model describes well the variation of the univariate flood series

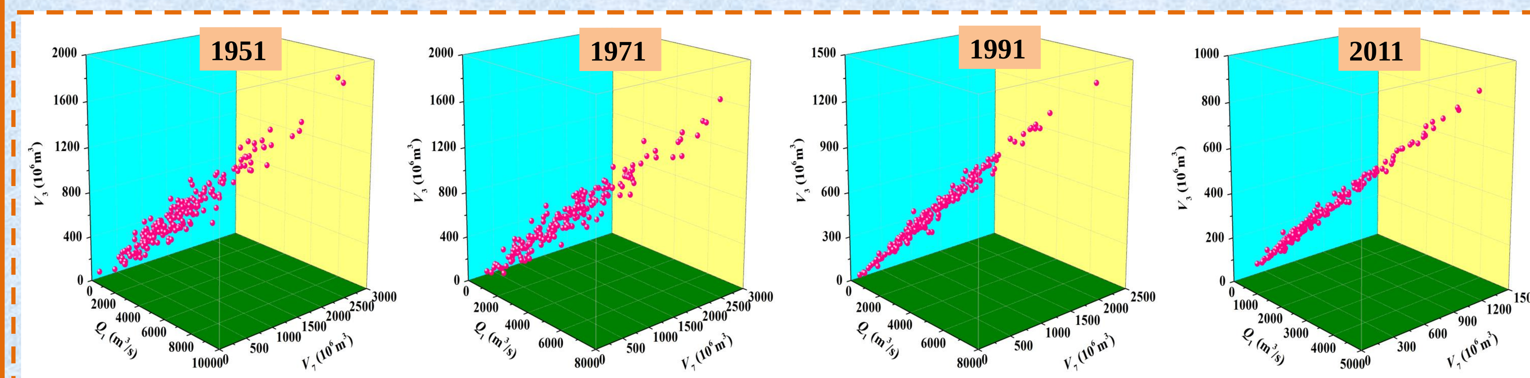
Nonstationarities in joint distribution



Comparison between the pair-copulas with stationary and nonstationary parameters in modeling dependence structure of the trivariate flood series



Level curves with the joint probability of 0.9 for pair (Q_1, V_3) in different periods under three scenarios. (a) both marginal distribution and copula parameters are nonstationary; (b) marginal distribution parameters are nonstationary, while copula parameter is taken the value in 1951; (c) copula parameter is nonstationary while the marginal distribution parameters are taken the values in 1951



Comparison between scatterplots of (Q_1, V_3, V_7) in the periods of 1951, 1971, 1991 and 2011. The scatters in the figure are generated by using the Monte Carlo method

➢ Only copula parameter θ_{c12} is nonstationary, while the other two parameters are stationary

➢Nonstationarity in margin alters location of level curve, while nonstationarity in dependence structure alters shape of level curve

➢ The distribution of scatterplots of (Q_1, V_3, V_7) tends to be more concentrated

V. Conclusions

➢ Time-varying copula model is a effect tool to distinguish and describe the nonstationarities in both marginal distribution and dependence structure of the multivariate flood series

➢Both the marginal distribution and dependence structure of the trivariate flood series of the Weihe River are nonstationary

➢The water and soil conservation project in the Weihe not only reduces the flood magnitude but also strengthens the dependence of trivariate flood series

Acknowledgement

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References

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