Acoustic Remote Sensing of Rogue Waves

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Abstract/Introduction

We propose an early warning system for approaching rogue waves using the remote sensing of acoustic-gravity waves (AGWs) – progressive sound waves that propagate at the speed of sound in the ocean. It is believed that AGWs are generated during the formation of rogue waves, carrying information on the rogue waves at near the speed of sound, i.e. much faster than the rogue wave. The capability of identifying those special sound waves would enable detecting rogue waves most efficiently. A lot of promising work has been reported on AGWs in the last few years, part of which in the context of remote sensing as an early detection of tsunami, [1]. In this research, we used the model of the Draupner Wave of January 1st, 1995 as a source and calculated the induced AGW signature. In particular we studied the AGW signature associated with a special feature of this wave, and characteristic of rogue waves, of the absence of any local set-down beneath the main crest and the presence of a large local set-up, [2].

Fundamental Equations

We define the velocity potential $\Phi(x,y,z,t)$ and free surface elevation $\eta(x,y,t)$ for the problem and let $\zeta(x,y,t)$ be the impulsive free surface source term with support $\mathbb{S}_\zeta$. The speed of sound in water is c, g the acceleration due to gravity, ρ the density of water, r the radial distance from the origin to the sensor, $\varpi = 2\pi f$ is the *representative wavenumber/frequency* and $P(x,y,z,t)$ is the pressure.

$$\begin{cases} \nabla^2\Phi = \dfrac{1}{c^2}\dfrac{\partial^2\Phi}{\partial t^2}, \quad -h \le z \le 0 \ \text{ and } (x,y,t) \notin \mathbb{S}_\zeta \\ g\dfrac{\partial\Phi}{\partial z} + \dfrac{\partial^2\Phi}{\partial t^2} = 0, \quad \text{and}, \eta = -\dfrac{1}{g}\dfrac{\partial\Phi}{\partial t}, \quad \text{for } z = 0 \text{ and } (x,y,t) \notin \mathbb{S}_\zeta \\ \dfrac{\partial\Phi}{\partial z} = 0, \quad z = -h \quad -\infty < x,y,t < \infty \\ \dfrac{\partial\Phi}{\partial z} = \dfrac{\partial\zeta}{\partial t}, \quad z = 0, \text{ and } (x,y,t) \in \mathbb{S}_\zeta \end{cases}$$

Equation 1 - Problem

$$\mathrm{G}(\mathrm{r},\mathrm{z},\mathrm{t}) = -\left(\frac{c\rho[\varpi^2 - g]}{\pi^3 g}\right)\left\{\left\{\frac{1}{r}\{F' + rF''\}\right\}\right\}$$

$$F = \frac{-\pi}{2}J_0\left[\left\{\frac{\pi}{4h}\right\}\left(ct - \sqrt{(ct)^2 - r^2}\right)\right]Y_0\left[\left\{\frac{\pi}{4h}\right\}\left(ct + \sqrt{(ct)^2 - r^2}\right)\right]$$

Equation 2 - Green's Function

$$\zeta(x,y,t) = \begin{cases} A(r)\sin(2\pi f t), 0 \le \sqrt{x^2+y^2} \le R, \quad 0 \le t \le T \\ 0, \ otherwise \end{cases}$$

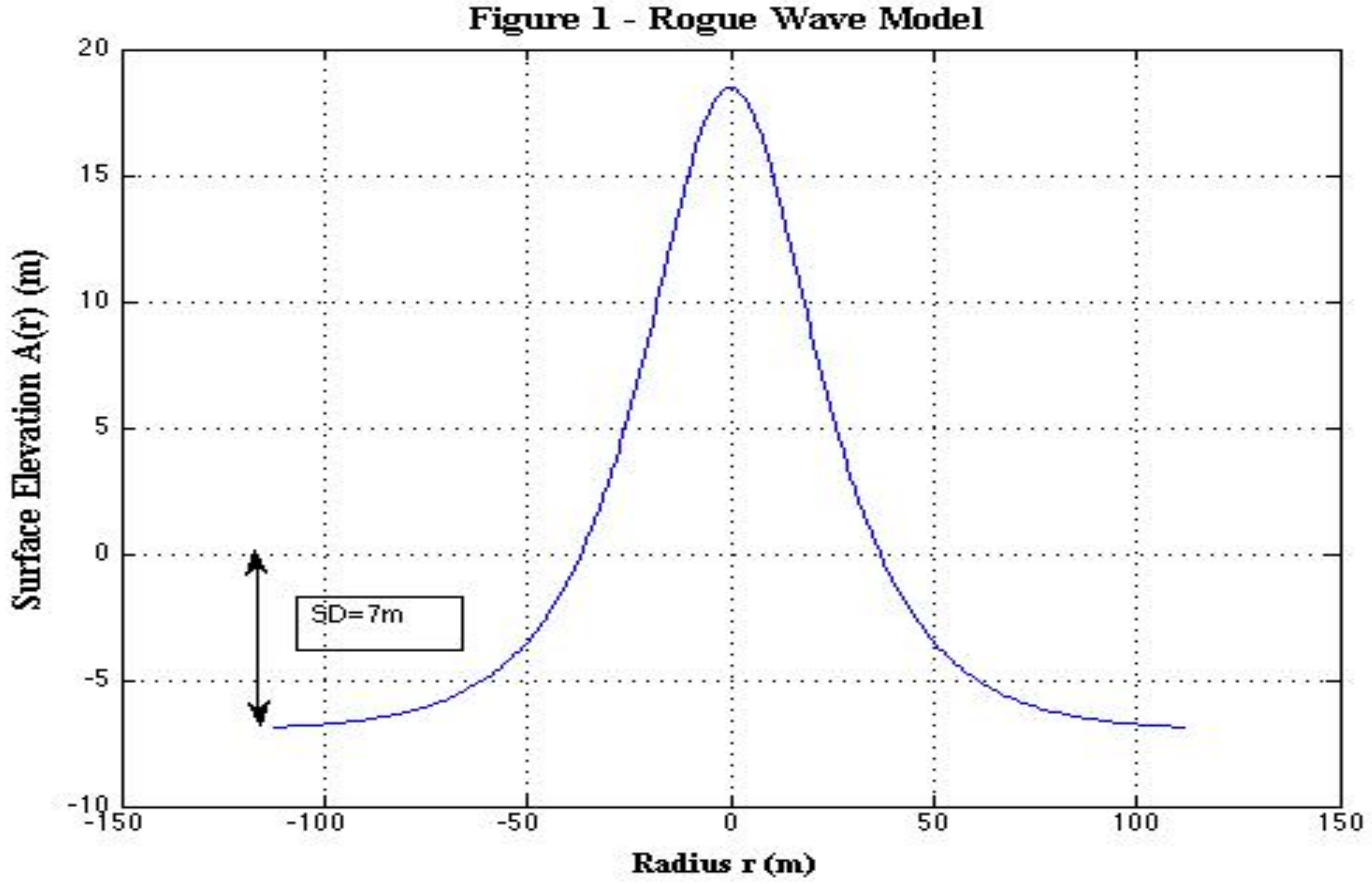
$$\mathrm{P}(x,y,z,t) = \iiint_{-\infty}^{\infty} \mathrm{G}(x',y',z,t')\,\zeta(x-x',y-y',t-t')\,\mathrm{d}x'\mathrm{d}y'\mathrm{d}t'$$

Equation 3 - Source and Solution

Rogue Wave Model

Based on a model of the Draupner wave [2], we take a radial symmetric source with the hyperbolic secant profile function A(r) shown in Figure 1 and a period of 12 seconds. The depth is 4000m and the radial distance to the observer is 1000 km.

Figure 1 - Rogue Wave Model



Results

The AGW pressure signal for the rogue wave model with wave height 25.5m and a set-down of 7m is given in Figure 2 and its spectrum is given in Figure 3.

Figure 2 Pressure vs time for AGW of rogue wave

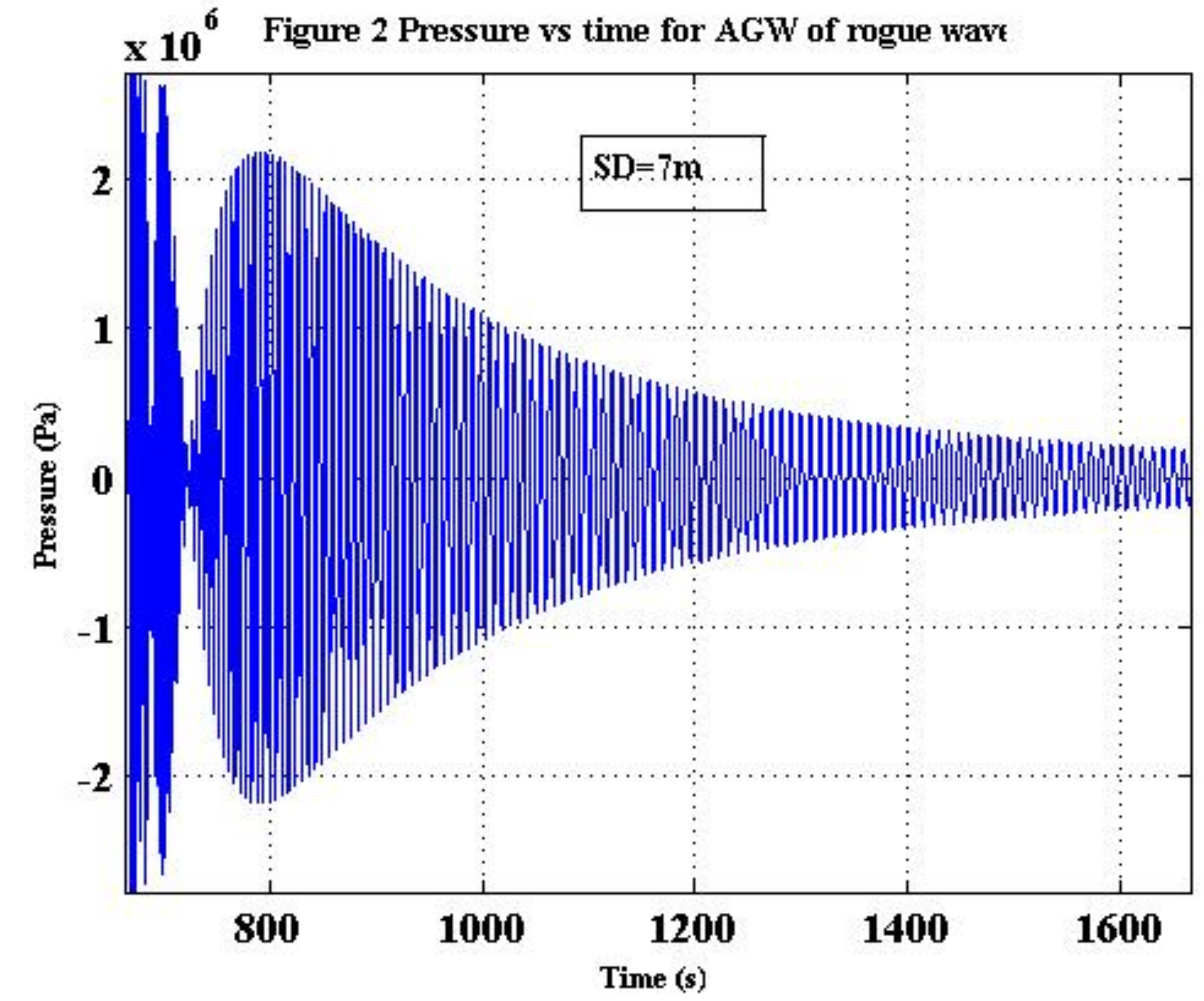
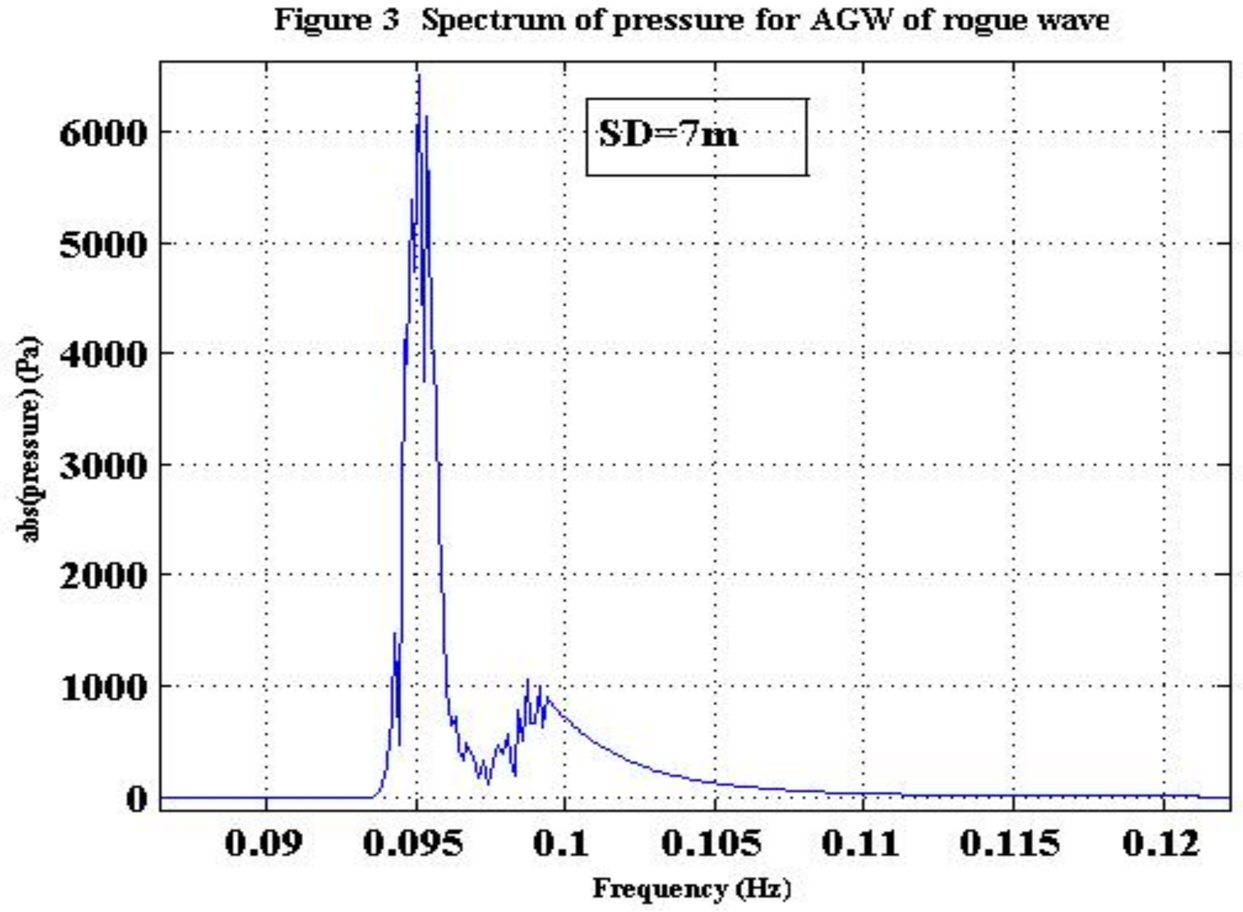


Figure 3 Spectrum of pressure for AGW of rogue wave



Conclusions

The AGW pressure generated by a rogue wave based on the Draupner wave model is measurable, easily distinguished from background noise and distinctive. In addition, the signal is stronger for waves with small set-down which is a distinguishing feature of rogue waves and makes this method particularly attractive for remote sensing of rogue waves.

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References

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