Paleoclimate changes: a stochastic resonance model based on ice-core data analysis

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1. Introduction

Two different models based on the stochastic resonance mechanism are developed to reproduce the switch between two stable climate states which characterize the longer timescale dynamics. We introduced some kinds of novelties:

1. the double-well potential function is directly extracted from the observational paleoclimate datasets [1] by means of the Empirical Mode Decomposition (EMD).
2. the fast variables are described a single-well potential function obtained via the EMD from the datasets [1].
3. the cross-correlation analysis revealed similar results with observations datasets

2. Data

Throuhg the EMD we decomposed the oxigen isotope $\delta^{18}O$ records from EMDL Dronning Maud Land (EPICA) Ice Core and from NGRIP project into two dynamical components indicating differences between high-frequency processes with respect to low-frequency ones.

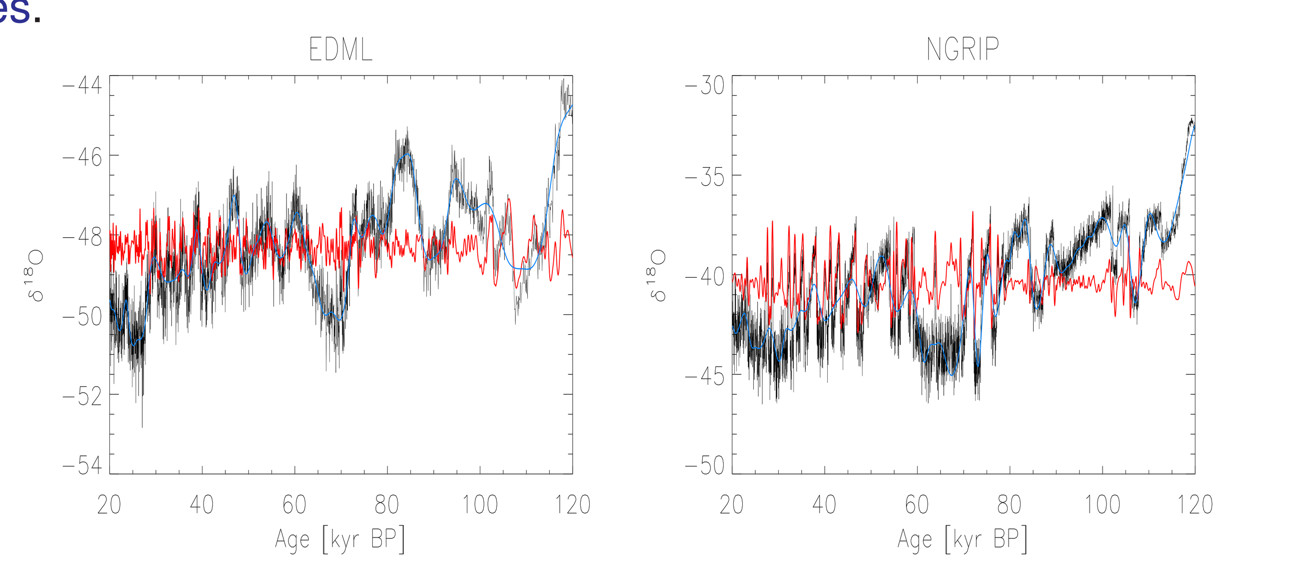

FIG. 1: The overlap between the original records (black lines), short timescale (red lines) and long timescale (blue lines) reconstructions: EDML [left] and NGRIP [right].

3. Langevin Model

- A simple 1-D Langevin model allows us to extract the potential functions (Fig. 2)

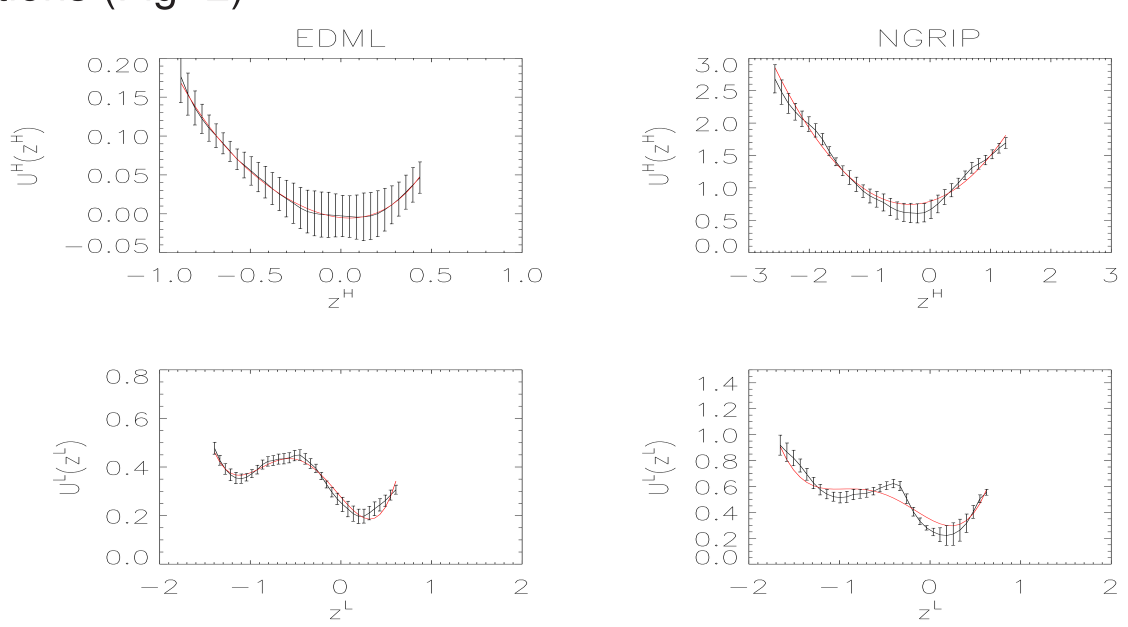

FIG. 2: Potential $U(z)$ calculated from the data (black curves and error bars) and polynomial best fits (red dashed lines)

- An analytical expression is found by performing a second- and forth-order polynomial fit for the high-frequency ($U^H(z)$) and long timescales ($U^L(z)$) components:

$$U^H(z) = \sum_{i=0}^{2} a_i^H z^i \quad (1)$$

$$U^L(z) = \sum_{i=0}^{4} a_i^L z^i \quad (2)$$

4. The "classical" stochastic resonance model

- A "classical" stochastic resonance model [2] for the Northern and Southern hemisphere changes by using $U^L(z)$ is built

$$\begin{aligned} \frac{dz_N}{dt} &= -\frac{\partial U^L(z_N)}{\partial z_N} + A_N^L \cos(\omega_N^L t) + \sigma_N^L W(t) \\ \frac{dz_S}{dt} &= -\frac{\partial U^L(z_S)}{\partial z_S} + A_S^L \cos(\omega_S^L t) + \sigma_S^L W(t) \end{aligned} \quad (3)$$

- the amplitude of the periodic term is obtained as the semi-difference between maximum and minimum values of the long timescale reconstructions
- the frequency is related to their characteristic timescales ($\omega = 2\pi/\tau$).
- z_N and z_S identify the system variables for the NGRIP and EDML datasets (e.g. the oxygen isotope δ^{18}O concentration)
- the double-well potential functions are characterized by two stable equilibrium points, corresponding to the local minima of $U(z)$, separated by an unstable fixed point, corresponding to the local maximum of $U(z)$.
- as shown in Fig. 2 (panels c,d), $U(z)$ has an asymmetric form both for NGRIP and EDML datasets, indicating the existence of two stable states which are not equiprobable and the corresponding to two different escape times between 10-12 kyr.

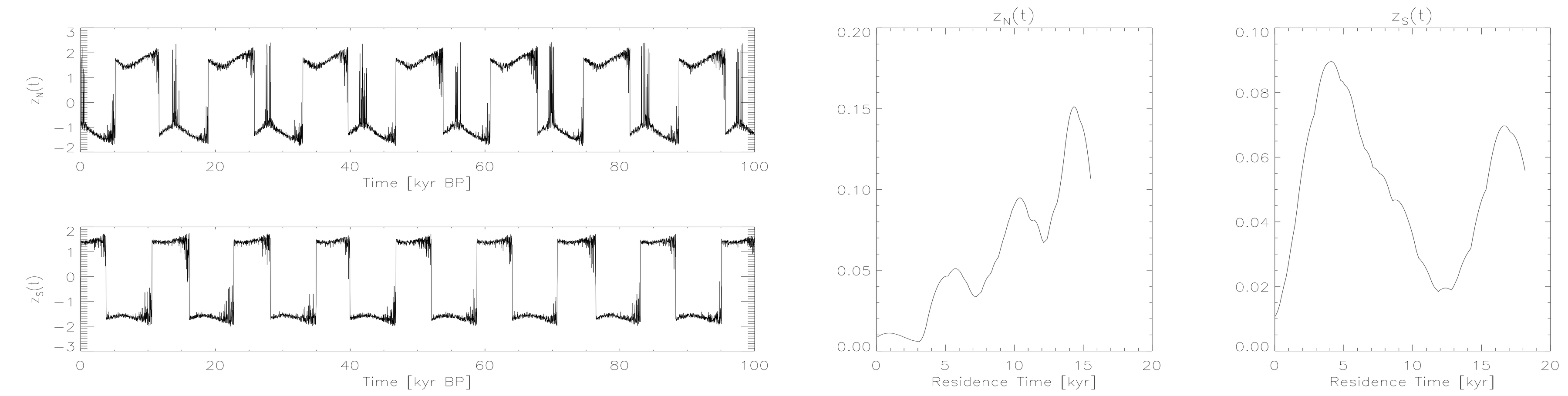

FIG. 3: Numerical solutions of Eqs. (3) [lef panels] and Residence Time Distributions [right panels]

5. The "modified" stochastic resonance model

- A crucial point into climate modeling is that the fast variables not always can be considered as noise-induced but need to be considered as an effective variable of the system [3].
- A simple way to consider them into a climate model is to derive a potential function corresponding to these variables.
- So, we introduce a "modified" stochastic resonance model in which the noise term is substituted by the single-well potential function U^H as derived through the potential analysis and related to the occurence of the DO events which represent the fast component of the dynamics of the system

$$\begin{aligned} \frac{dz_N}{dt} &= [-U' + A_N \cos(\omega_N t)]_L + [-U' + A_N \cos(\omega_N t)]_H \\ \frac{dz_S}{dt} &= [-U' + A_S \cos(\omega_S t)]_L + [-U' + A_S \cos(\omega_S t)]_H \end{aligned} \quad (4)$$

- the subscripts L and H denote the long timescale and the high-frequency components respectively

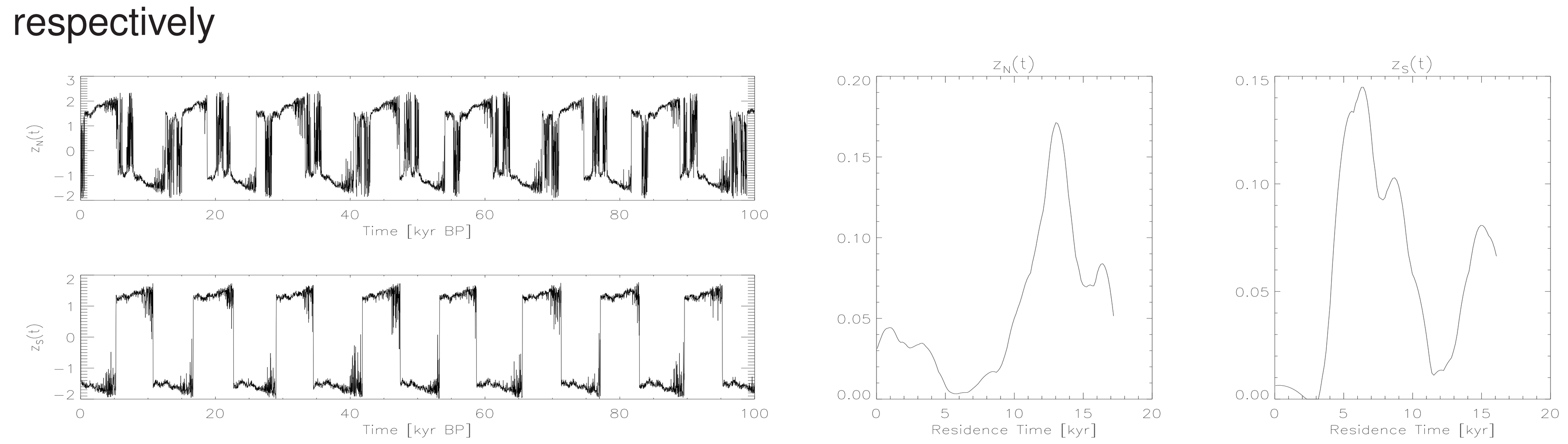

FIG. 4: Numerical solutions of Eqs. (4) [lef panels] and Residence Time Distributions [right panels]

6. Discussion

- We note an increase of the level of fluctuations due to the potential term related to the DO occurrence, with respect to the previous case.
- This increase is particularly evident into the $z_N(t)$ variable because, considering the EMD results [1], the DO contribution is most pronounced into the NGRIP dataset with respect to the EDML reconstruction.
- This can be also seen into the potential function related to the high-frequency components (Fig. 2 panels a,b) which shows a deeper well for the NGRIP with respect to EDML.
- The Residence Time Distributions (RTDs) is changed with respect to Fig. 3 according to the different physical informations introduced in this case.
- Particularly, we note that a residence time of about 1-2 kyr occurs as a result of the inclusion of the potential term related to the DO events which involves characteristic timescales between 0.7-3.3 kyr.
- This is present in both system variables $z_N(t), z_S(t)$ but it is more evident (with a greater probability density) into the northern hemisphere variable.
- In both cases, two clear peaks corresponding to the residence times of about 5 and 15 kyr are shown as a consequence of the potential functions related to the longer timescale dynamics and to the characteristic timescale of the periodic term.

7. Cross-correlation analysis

- To understand the efficiency of the two different models to reproduce climate changes and to obtain physical informations regarding these changes we perform the cross-correlation analysis for both models.

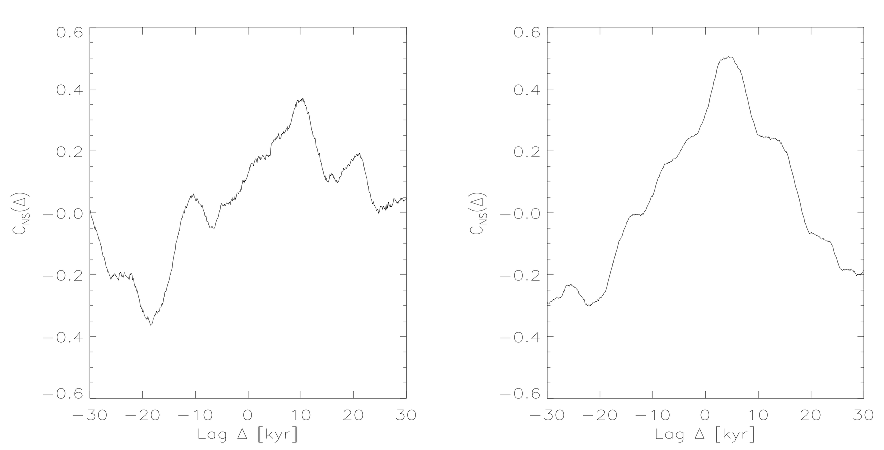

FIG. 5: Cross-correlation analysis between z_N and z_S for the "classical" model [lef panel] and for the "modified" modes [right panel]

- The cross-correlation coefficient shows that there are two different lags for the two models but with the same relation between the two signals.
- Particularly, we note that for the "classical" model the southern changes lead those of the northern with a characteristic time-delay of about 10 kyr which is not in agreement with observational cross-correlation analysis [1].
- Conversely, when the cross-correlation is applied to the "modified" stochastic resonance model a lag of about 4 kyr is found by which $z_N(t)$ is lagged with respect to $z_S(t)$, in agreement with previous work [1].

8. Conclusions

Using two types of stochastic resonance models we found that:

1. the "classical" stochastic resonance model is able to reproduce the switchings between the two stable states but these transitions are driven by a noise term which is not directly related to a physical mechanism
2. the "modified" model allows to consider the physical mechanism and the effects related to the fast variables of the system such as the occurrence of DO events, particularly evident during the last glacial period in Northern hemisphere both in observational dataset and in simulation results

References

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