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The novel high-performance 3-D MT inverse solver

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1) Motivation

- Different inverse solvers give us different results
- Only a few solvers are freely available for academia
- New solvers should harness modern HPC to work with complex models and large datasets

2) Inverse problem

Mathematical formulation

Let $\mathbf{m} = (m_1, m_2, \dots m_n)$ be a vector of model parameters i.e $\sigma(x, y, z) = \nu(\mathbf{m})$

- Weighted residual: $R(\mathbf{m}) = \|f(\mathbf{m}) - \mathbf{d}^{obs}\|_W$
- Objective function: $F(\mathbf{m}) = R^2(\mathbf{m}) + \lambda\Omega(\mathbf{m})$
- Main stabilizer Ω property: the set $\Omega(\mathbf{m}) < C$ is compact [1]
- $\lambda > 0$ is a regularization parameter

Target: find argmin $F(\mathbf{m})$

Main challenges

- Choosing stabilizer Ω
- Choosing parametrization ν
- Fast solving of the nonlinear optimization problem:
  - Fast calculation of R i.e. fast forward problem solving
  - Fast calculation of the gradient of F

3) Inverse solver ExtrEMeJoyMT

Main features

1. Large set of stabilizers
2. Flexible "mask" parametrization (see Section 6)
3. Parallel IE solver for forward modelling
4. Adjoint approach for gradient calculation
5. BFGS method for optimization
6. Multi-language paradigm

Forward solver

$$\begin{aligned}
\mathbf{E}(M) &= \int_T \widehat{G}_E(M, M_0)\Delta_\sigma(M_0)\mathbf{E}(M_0)dT_{M_0} + \mathbf{E}^0(M) \\
\mathbf{H}(M) &= \int_T \widehat{G}_H(M, M_0)\Delta_\sigma(M_0)\mathbf{E}(M_0)dT_{M_0} + \mathbf{H}^0(M) \qquad (1) \\
\Delta_\sigma(M_0) &= \sigma(M_0) - \sigma_b(M_0)
\end{aligned}$$

Two different IE based solvers are implemented:

ExtrEMeMT (cf. [3])	GIEM2G (cf. [2])
Based on collocation approach	Based on Galerkin approach
Parallel and sequential modes	Only parallel mode
Faster at small number of nodes	Faster at hundreds and thousands of nodes

Stabilizer

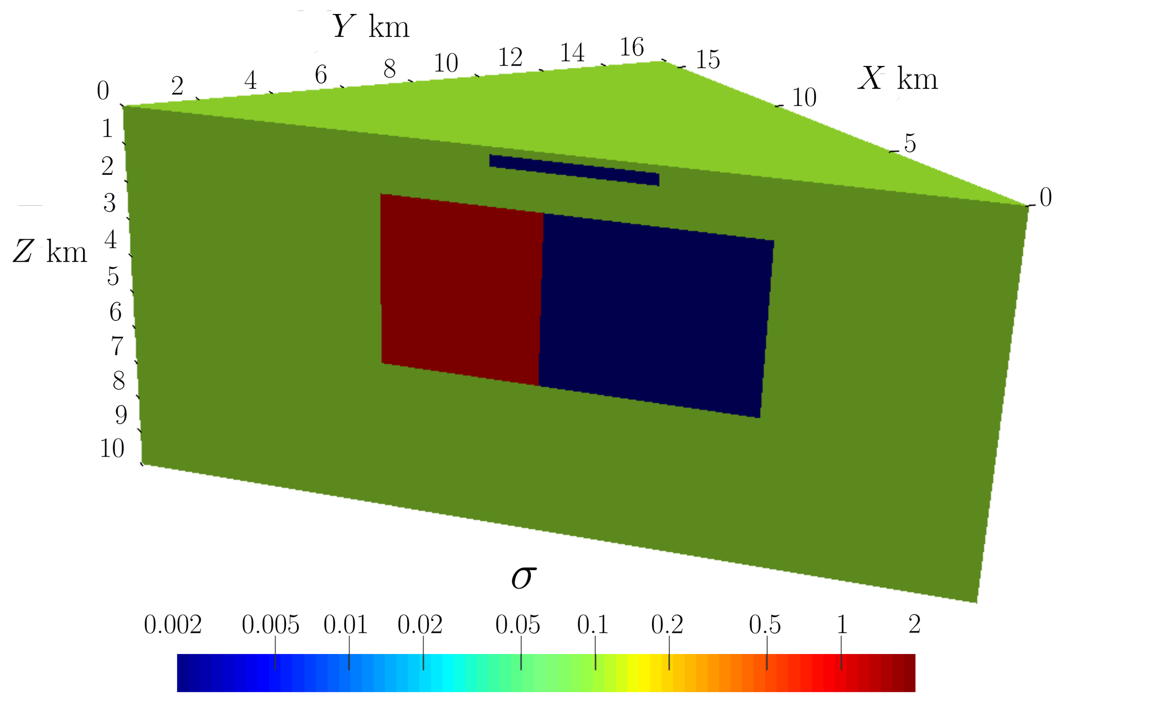
$$\begin{aligned}
\Omega = &\int_{V_{inv}} \alpha(x, y, z)\sigma_a^2(x, y, z)dxdydz + \\
&\int_{V_{inv}} \frac{\beta(x, y, z)}{\sigma_a^2(x, y, z)}dxdydz + \qquad (2) \\
&\int_{V_{inv}} \gamma(x, y, z)|\,\mathbf{grad}\log\sigma(x, y, z)|^2 dxdydz
\end{aligned}$$

$\alpha > 0 \quad \beta \geq 0 \quad \gamma \geq 0$

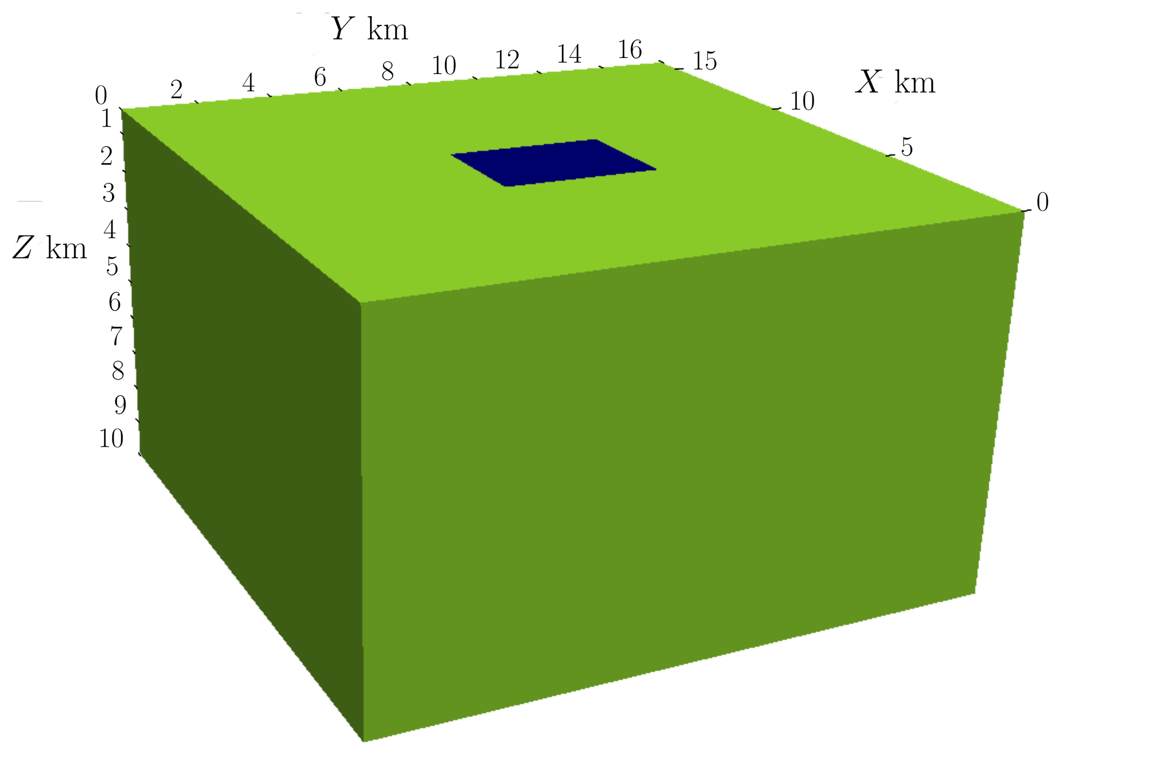
Functions α, β, γ are under researcher control

4) True model

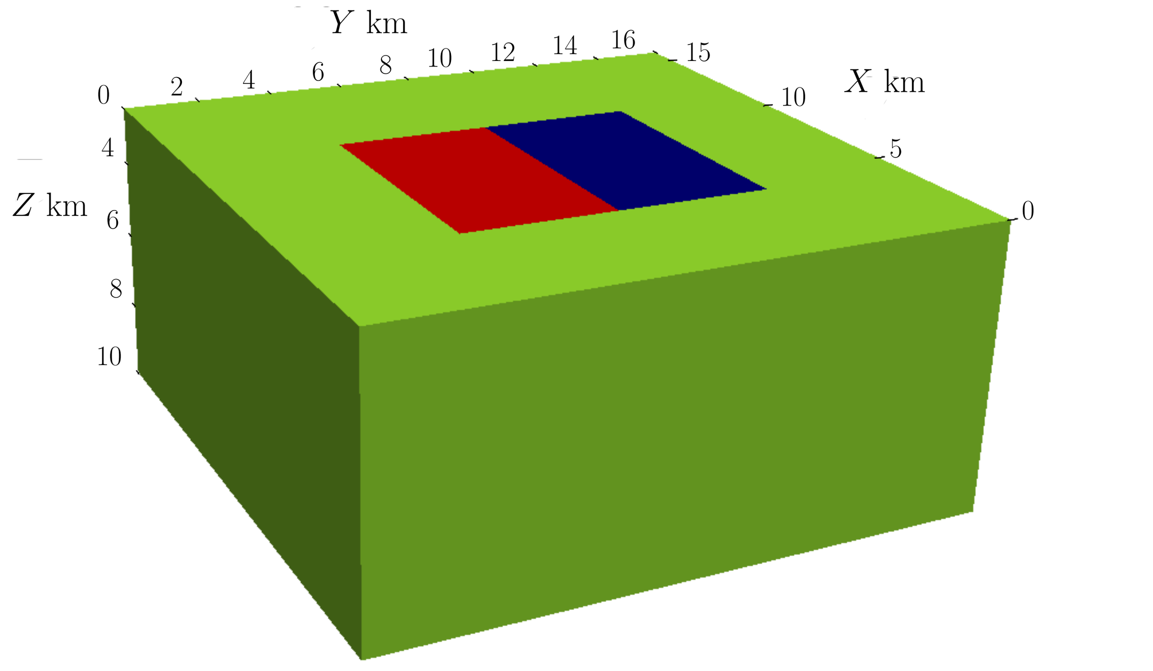
- Due to IE approach only anomaly domain is discretized
- N_x=280, N_y=320, N_z=232, cubic cells with 25 m edges
- 225 receivers at uniform grid with 1km step
- 16 frequencies from 10^{-4} to 10^2



Section at $z = 350$m

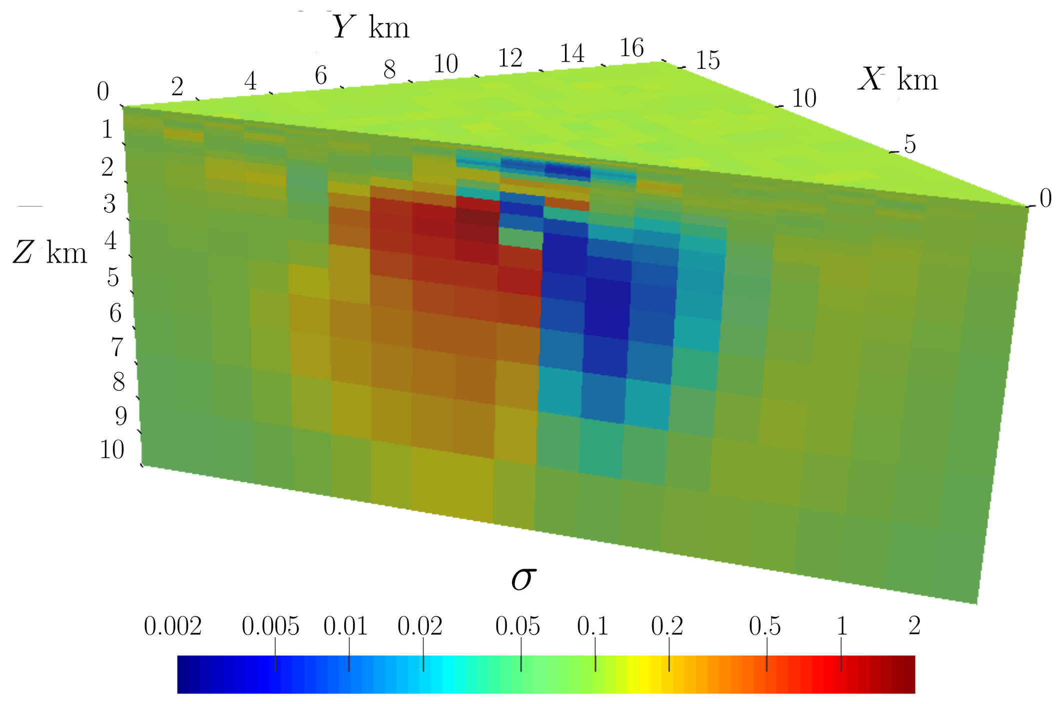


Section at $z = 2500$m



5) Inversion results

- Both impedances and tippers were inverted together
- 5% random noise was added to the responses
- Modelled domain is 16 × 16 × 10km
- Discretization for forward modelling: $N_x = N_y = 160$, $dx = dy = 100$ m, N_z=120, dz from 0.2m to 600m as geometric sequence



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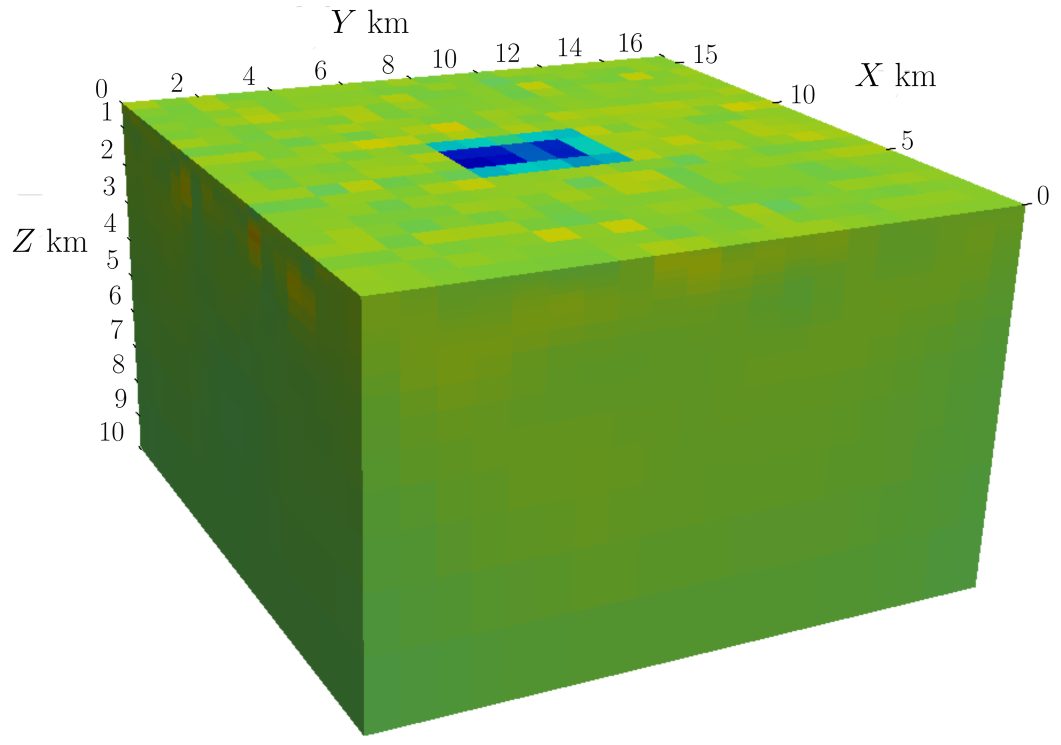

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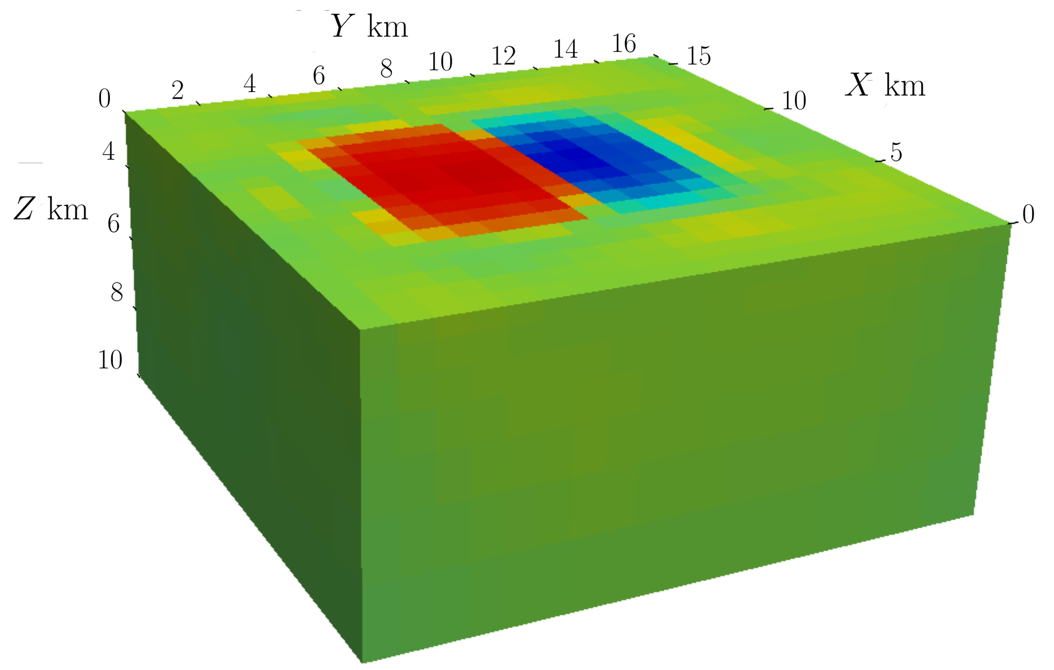
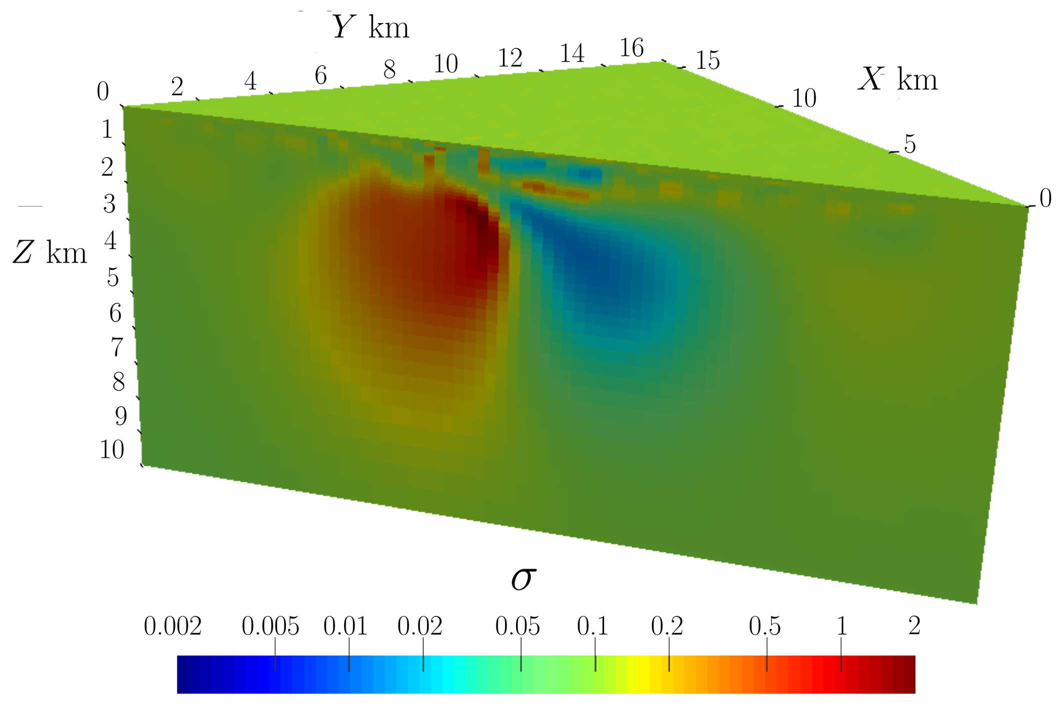


- Inverse domain contains 20 × 20 × 40 cells, i.e. one inverse cell is merge of 8 × 8 × 4 cells used in forward modelling
- Final nRMS=0.84, 169 iterations



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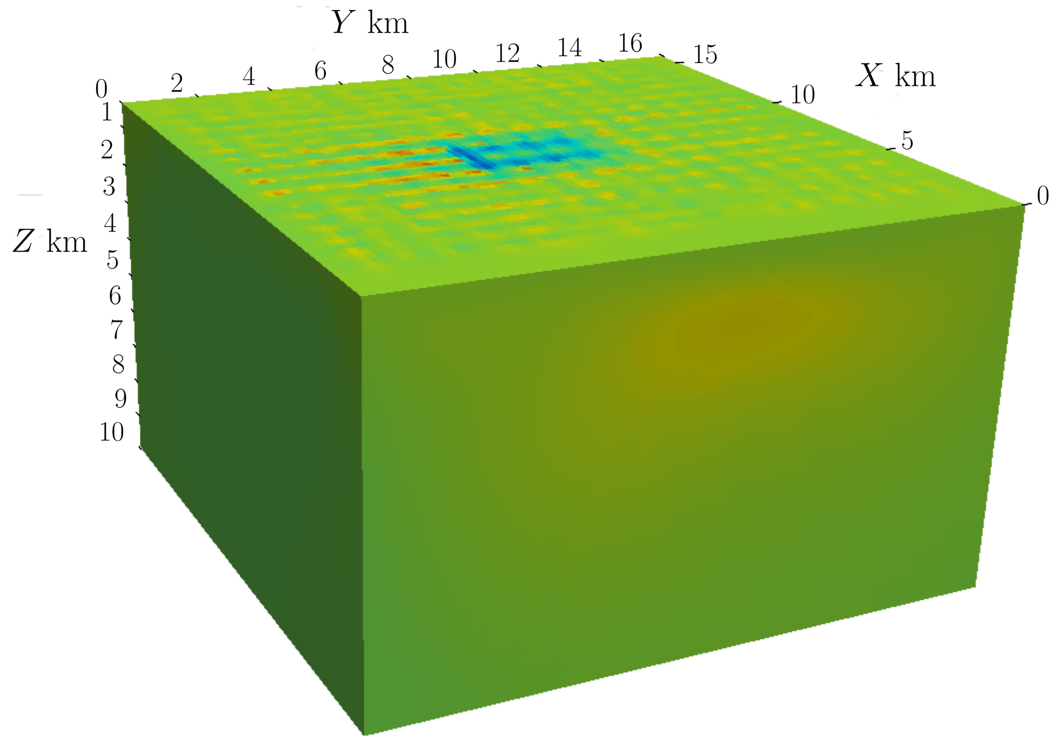
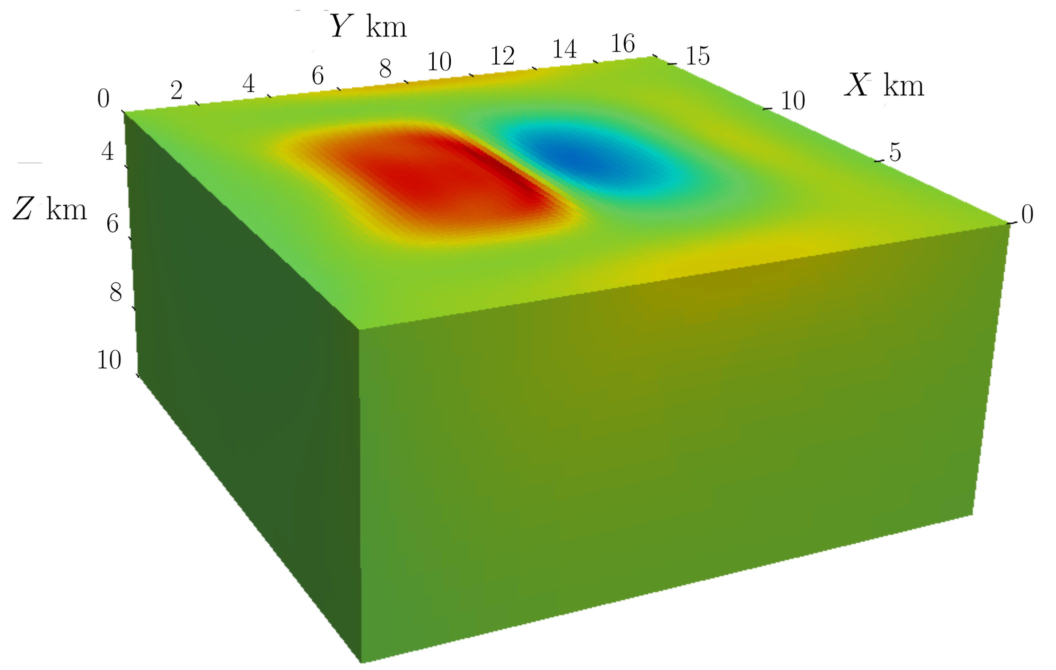


Section at $z = 2500$m

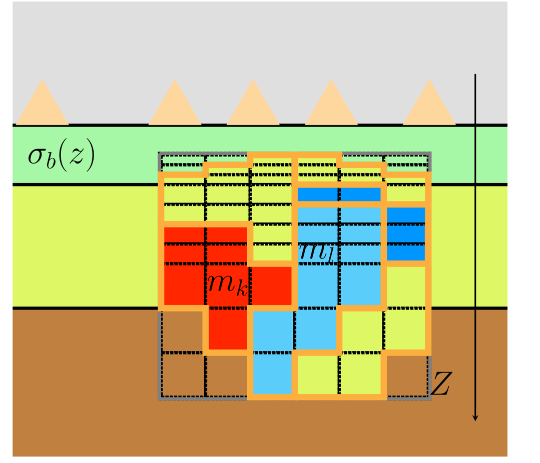


- Inverse domain contains 80 × 80 × 120 cells, i.e. one inverse cell is merge of 2 × 2 × 1 cells used in forward modelling
- Final nRMS=0.87, 104 iterations

6) Mask parametrization

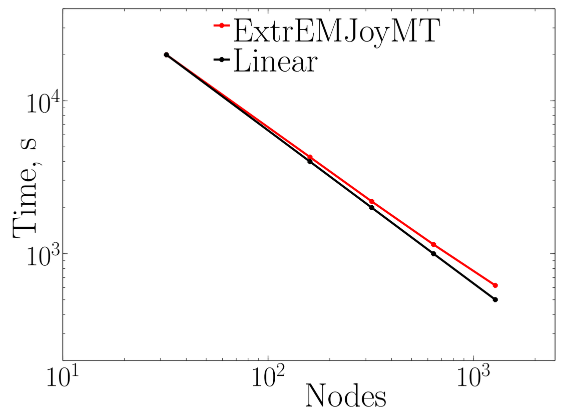
- One element of inverse domain i.e. m_k is the union of cells from forward modelling domain
- The rule how to merge cells is defined by researcher

- forward domain discretization
- inversion domain discretization
- modeling domain



7) Scalability

- Two kinds of parallelism are implemented:
  - Forward problems at different N_f frequencies are computed independently
  - Internal parallelism along X dimension in IE solver
- Hybrid OpenMP+MPI approach
- Maximum number of used nodes is $2N_xN_f$



8) Conclusion

New 3D inverse solver ExtrEMeJoyMT is presented:

- Flexible inversion domain parametrization
- Wide class of stabilizers
- Two modern IE solvers for forward problems
- Adjoint approach for gradient calculation
- Perfect scalability on HPC

9) Contacts

I will be here at

17.30 – 19.00

Feel free to take my card and email me

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10) References

[1] Tikhonov, A. N. & Arsenin, V. Y. Solutions of Ill-Posed Problems. Wiley, New York, 1977

[2] Kruglyakov, M. & Bloshanskaya, L. High-performance parallel solver of electromagnetics integral equations based on Galerkin method. Mathematical Geoscience, also at http://arxiv.org/abs/1512.06126, under review (2016).

[3] Geraskin, A., Kruglyakov, M. & Kuvshinov, A. Novel robust and scalable 3-D EM forward solver based on contracting integral equation method and modern programming techniques. Computers & Geoscience, under review (2016).