ALIF: a new promising technique for the decomposition and analysis of nonlinear and nonstationary signals

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Problem

Given a nonstationary signal $s(x)$, $x \in \mathbb{R}$

Problem: without any a priori knowledge on the signal decompose $s(x)$ in a few simple components called IMFs and perform their time-frequency analysis.

Definition: IMF is a function whose mean value between an upper and lower envelope is zero everywhere.

OBS: traditional methods, like Fourier and wavelet Transform, have problems dealing with nonstationary signals and require a priori assumptions on the signal. Newly developed methods, like Short Time Fourier Transform or Synchrosqueezing wavelet Transform, suffer due to the uncertainty principle in the time frequency representation.

Algorithm ALIF

Algorithm IMF = ALIF(s)

IMF = {}

while *number of extrema of* $s \geq 2$ **do**

- $s_1 = s$
- evaluation of the filter $w(x,t)$ support length $l(x)$ for $s(x)$
- **while** *stopping criterion not satisfied* **do**
  - $s_{n+1}(x) = s_n(x) - \int_{-l(x)}^{l(x)} s_n(x+t)w(x,t)\mathrm{d}t$
  - $n = n+1$
- IMF = IMF $\cup \{s_n\}$
- $s = s - s_n$

IMF = IMF $\cup \{s\}$

Convergence Theorem [1]

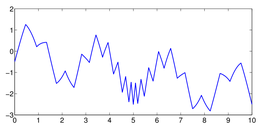
Given a filter $w \in L^2([-l,\, l])$ and $s(x) \in L^2(\mathbb{R})$ and set the filter w support length $l(x)$ fixed in x,
If $|1 - \widehat{w}(\xi)| < 1$ or $\widehat{w}(\xi) = 0$, then $\{s_n\}_{n\geq 1}$ converge and

$$\lim_{n\to\infty} s_n(x) = \int_{-\infty}^{\infty} \widehat{s}(\xi)\chi_{\{\widehat{w}(\xi)=0\}} e^{2\pi i \xi x}\mathrm{d}\xi$$

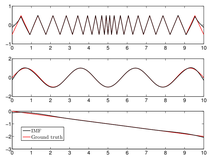
where $\widehat{w}$ and $\widehat{s}$ Fourier transform of w e s.

Artificial Examples

Example 1

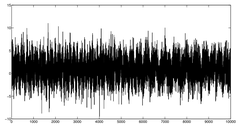


Artificial signal

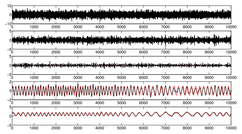


IMF

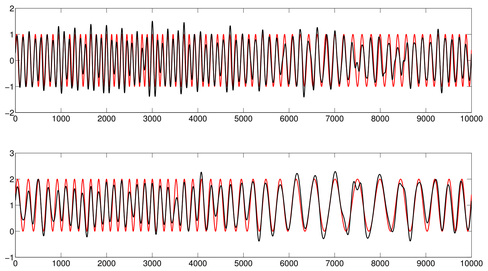
Example 2 [1]



Signal with SNR −10 dB



IMFs vs ground truth



Last two IMFs vs ground truth

Contacts

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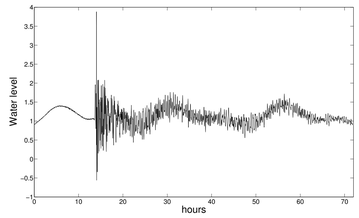
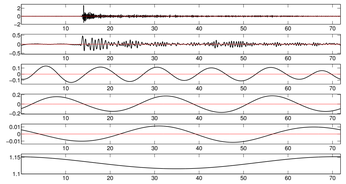
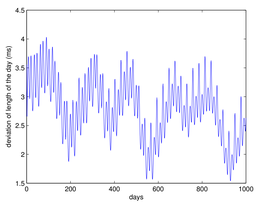
Real life examples 1D

Tsunami dataset [1]

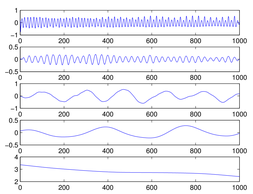




Length of the day [1]



Length of day signal

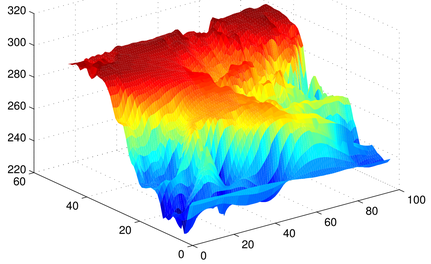


IMFs

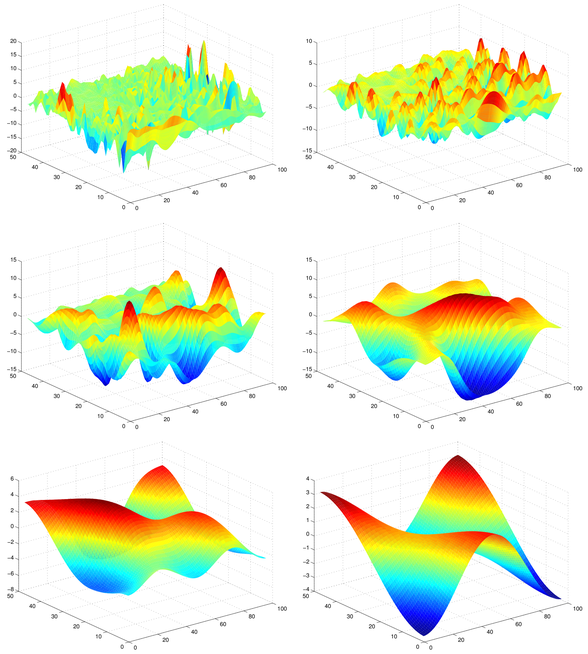
Radio power scintillation

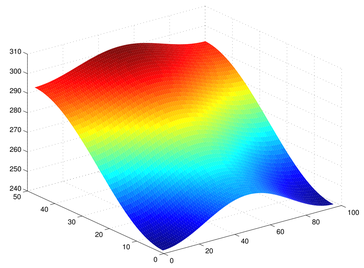
Ref. poster **Complex Dynamics of Equatorial Scintillation** by M. Piersanti et al. (ST 4.3)
Hall X4, board number X4.385
Thursday, 27 Apr 2017, 17:30–19:00

Real life examples 2D [2]



Global air temperature [K], January 1 2014





IMFs

Conclusions

- ALIF allows to decompose and analyze nonstationary signals in 1D, 2D, and higher dimensions.
- ALIF is robust to noise, nonlinear and completely data adaptive.
- The uncertainty principle is no more a problem since we first decompose and then compute the time–frequencies plots.

Bibliography

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[2] A. Cicone, and H. Zhou. Multidimensional Iterative Filtering method for the decomposition of high-dimensional nonstationary signals. To appear in *Numerical Mathematics: Theory, Methods and Applications* arXiv:1507.07173.

[3] A. Cicone, J. Liu and H. Zhou. Hyperspectral Chemical Plume Detection Algorithms Based On Multidimensional Iterative Filters Decomposition. *Phil. Tran. R. Soc. A*, 374(2065) 2016: 2015.0196. doi: 10.1098/rsta.2015.0196 arXiv:1512.01979