

# **NEXD**: A Software Package for High Order Simulation of Seismic Waves using the Nodal Discontinuous Galerkin Method

**M. S. Boxberg, A. Lamert, T. Möller,  
L. Lambrecht, W. Friederich**

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# Features of NEXD



	Feature	1D	2D	3D
<b>Material</b>				
elastic	✓	✓	✓	
viscoelastic	✗	✓	✓	
poroelastic	✓	✓	(✓)	
fractures	✓	✓		✗
<b>Boundary conditions</b>				
reflecting	✓	✓	✓	
periodic	✓	✗	✗	
absorbing	✓	✓	✓	
absorbing (PML)	✗	✓	✓	
<b>Other</b>				
adjoint methods	✗	✓	(✓)	
Cubit meshes	✗	✓	✓	
MPI (parallel computing)	✗	✓	✓	

**Top:** Overview of current features and development

✓: implemented and published, ✓: implemented, (✓): work in progress, ✗: not implemented (yet)

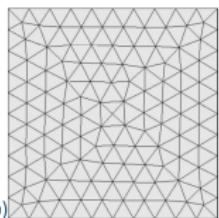
# Nodal Discontinuous Galerkin Method



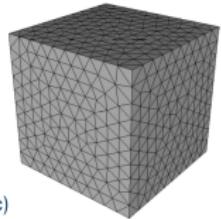
## Space discretization

- a) Linear meshes (1D)
- b) Triangular meshes (2D)
- c) Tetrahedral meshes (3D)

a)

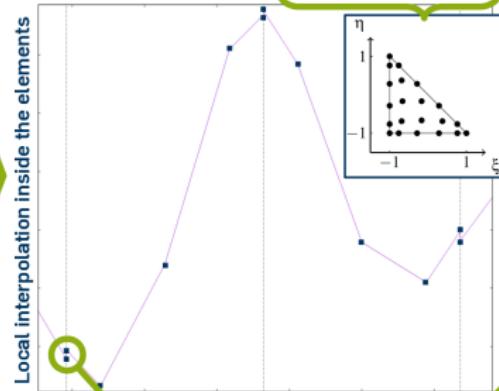


b)



c)

Interpolation points: GLL (1D), warp & blending (2D, 3D)



Riemann-Problem  
(initial value problem for  
piecewise constant data  
with a single discontinuity)

Numerical Flux

wave equation in velocity-stress formulation (here: 1D)

$$\partial_t \vec{Q} + \mathbf{A} \partial_x \vec{Q} = \mathbf{E} \vec{Q} + \mathbf{F} \vec{s}$$

weak form

(multiplied with test functions & integrated)

$$\int_{D^k} (\partial_t \vec{Q}_h^k + \mathbf{A} \partial_x \vec{Q}_h^k - \mathbf{E} \vec{Q}_h^k - \mathbf{F} \vec{s}_h^k) \psi_n dx \\ = \int_{\partial D^k} n_x [\mathbf{A} \vec{Q}_h^k - (\mathbf{A} \vec{Q}_h^k)^*] \psi_n dx$$

semi-discrete numerical scheme for the weak form

$$\partial_t \vec{Q}^k = -J^{k-1} \mathbf{A} D_{r,(i,j)} \vec{Q}^k + \mathbf{E} \vec{Q}^k + \mathbf{F} \vec{s} - \text{Flux}$$

(Galerkin Principle used)

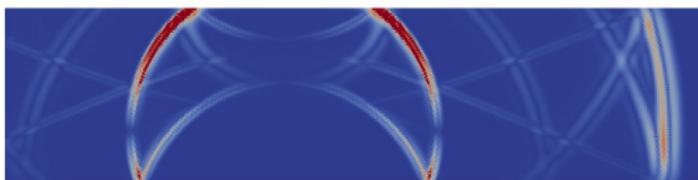
Time stepping scheme

$$\partial_t \vec{Q}^k \rightarrow ?$$

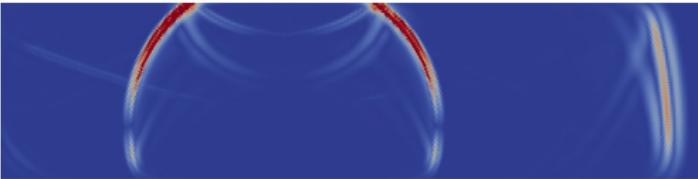
- a) Euler method
- b) different Runge-Kutta implementations

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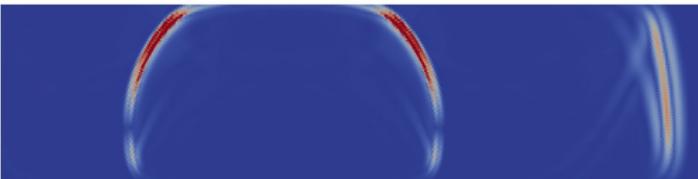
# Absorbing Boundaries: PML



■ without PML

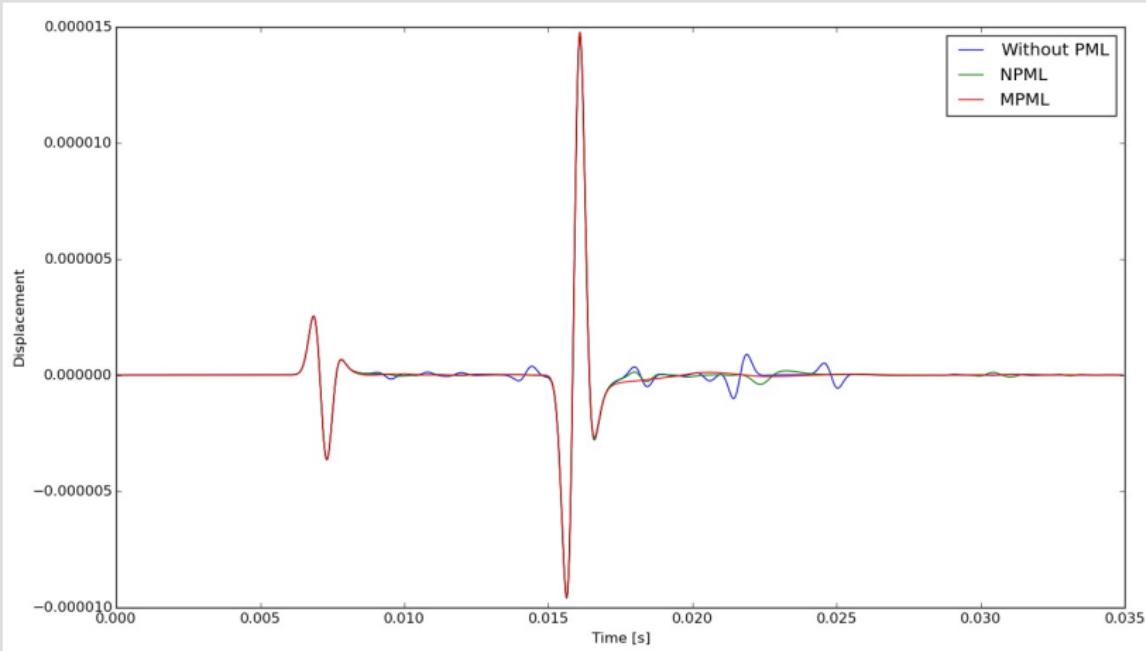


■ NPML

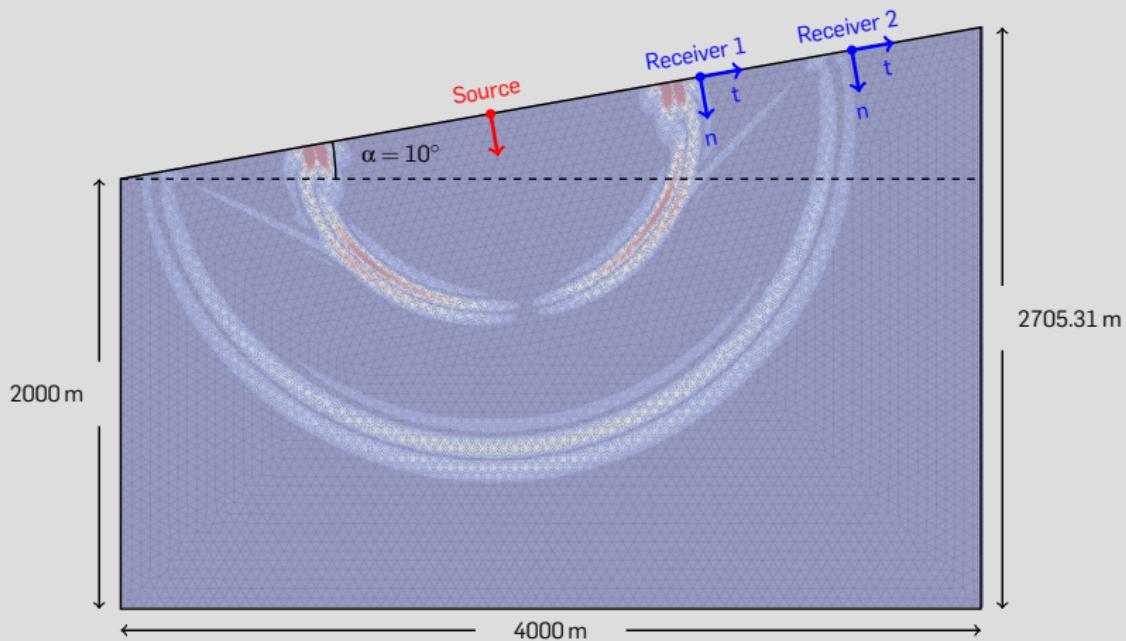


■ MPML

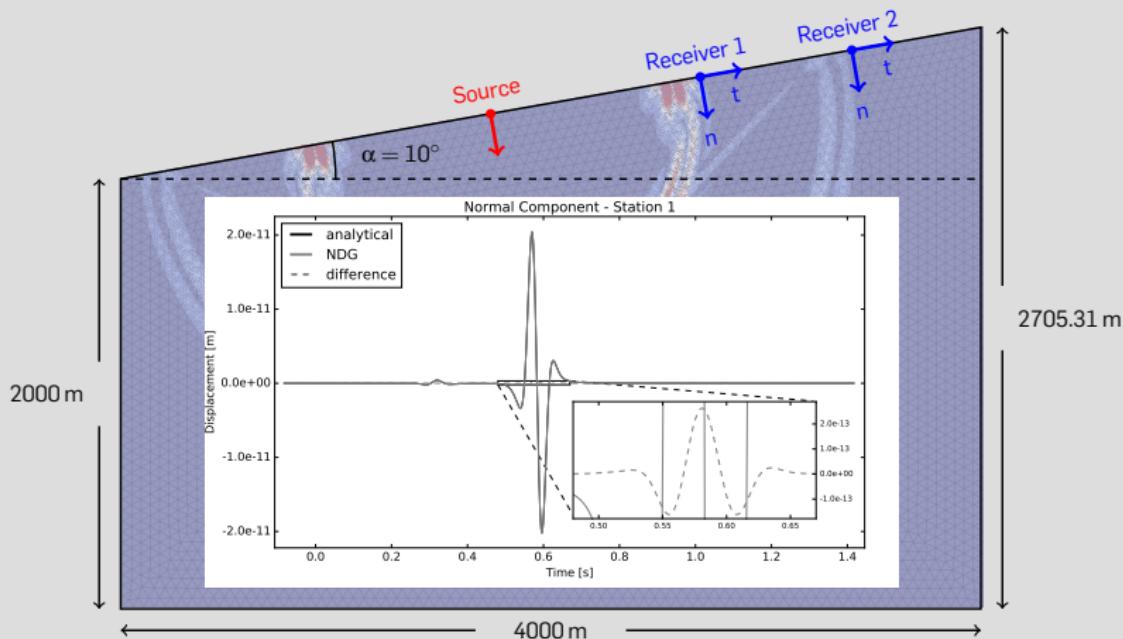
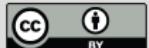
# Absorbing Boundaries: PML



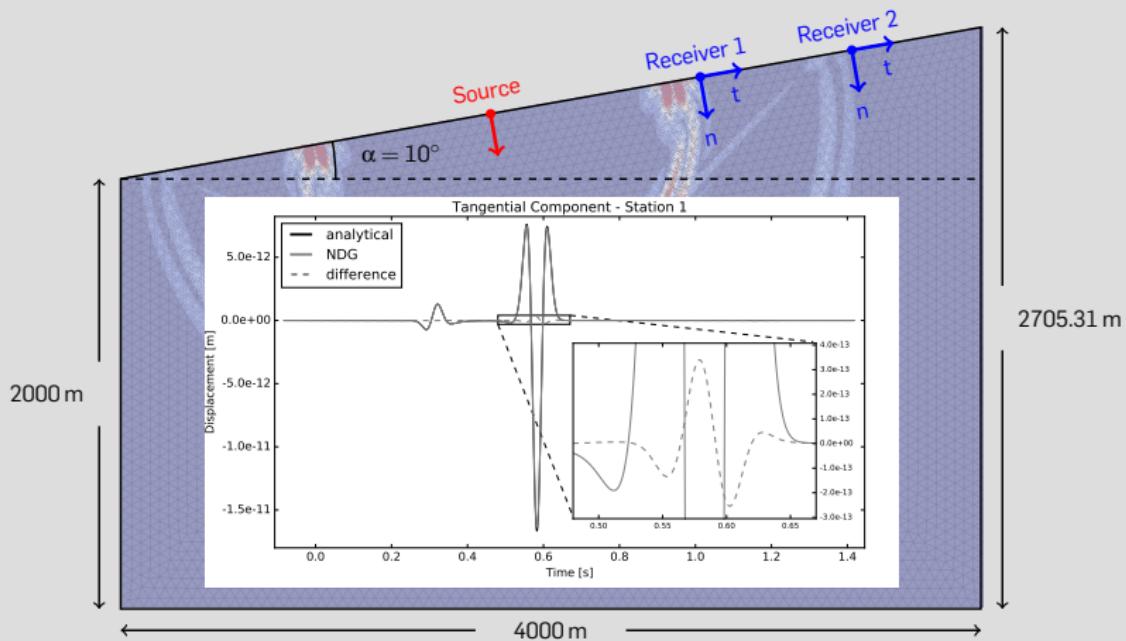
# Lamb's problem



# Lamb's problem



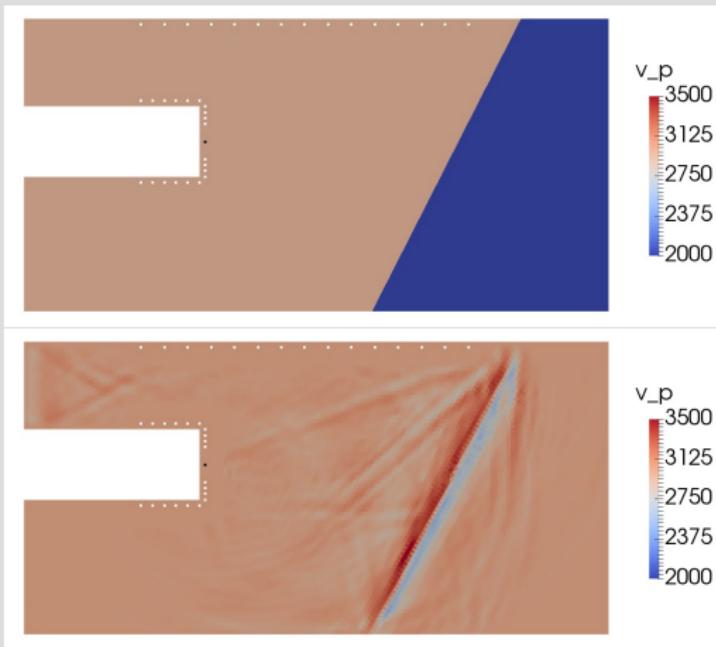
# Lamb's problem



# Full Waveform Inversion for Mechanized Tunneling



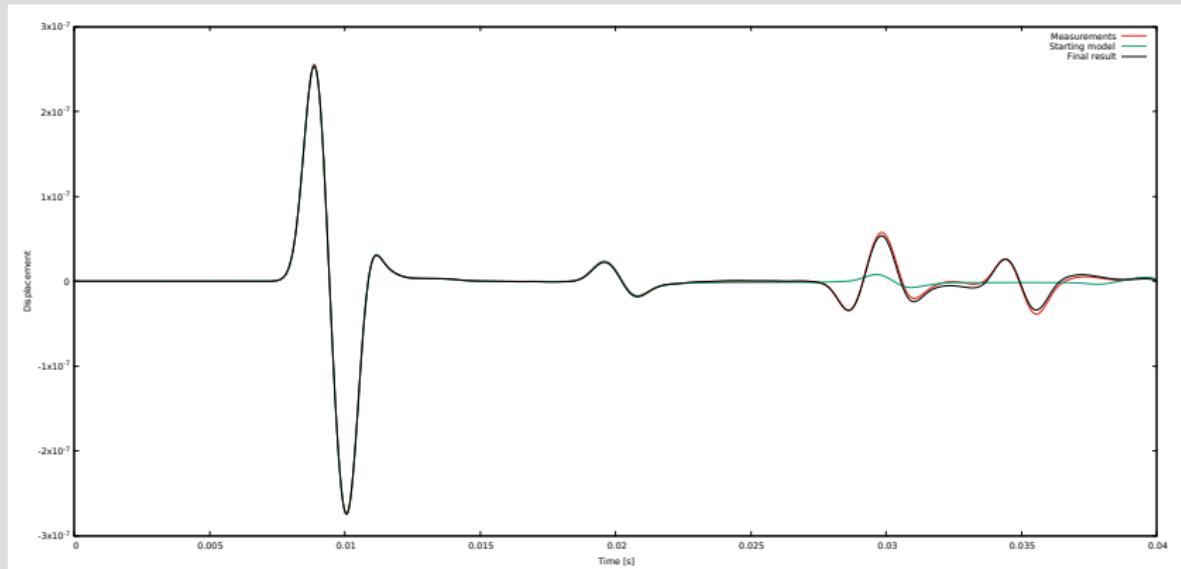
- Full Waveform Inversion (acoustic) using adjoint and conjugate gradient method
- 500 Hz Ricker source at tunnel face
- Receivers at tunnel face, tunnel walls and surface
- Reflecting interface well recovered



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Absorbing Boundaries: PML Lamb's Problem FWI for Mechanized Tunneling

# Full Waveform Inversion for Mechanized Tunneling



Station at surface

# GitHub



- NEXD is open source software (GNU GPL v3.0)
- Download at <http://github.com/seismology-RUB>
- More information at <http://www.rub.de/nexd>
  
- More on poroelastic solver: **EGU2017-12821**  
Poster, Mon, 17:30-19:00, X3.98
- More on fractures: **EGU2017-15012**  
Poster, Thu, 17:30-19:00, X1.57