Meshfree simulation of avalanches with the Finite Pointset Method (FPM)

Isabel Michel[1], Jörg Kuhnert[1], and Dimitrios Kolymbas[2]

[1]Fraunhofer Institute for Industrial Mathematics ITWM, Kaiserslautern, Germany (isabel.michel@itwm.fraunhofer.de) [2]Division of Geotechnical and Tunnel Engineering, University of Innsbruck, Austria

Aim

The meshfree **Finite Pointset Method (FPM)** developed by Fraunhofer ITWM has in the past been successfully applied to problems in computational fluid dynamics (see Kuhnert 2014) such as water crossing of cars, water turbines, and hydraulic valves.

Most recently the simulation of granular flows, e.g. soil interaction of cars (rollover), has been tackled. This advancement is the basis for the simulation of avalanches.

In this contribution the aim is to investigate the following **keyfeatures**:

- **boundary conditions** between avalanche and geometry/other phases
- **material characteristics** of avalanches

We consider one- and two-phase 3D flow scenarios based on the Drucker-Prager yield stress criterion with respect to a friction boundary condition at the geometry. Volume-volume-coupling is used for the two-phase scenario. Additionally, for the nonlinear barodesy model (see Kolymbas 2012) selected results are presented.

Background of FPM

FPM is based on a generalized finite difference algorithm which solves PDEs (conservation equations and material models) employing a **cloud of numerical points**. The point cloud moves with the flow in a Lagrangian framework. Hence, physical information is transported in a natural way.

The approximation of spatial partial derivatives is performed by a moving weighted least squares approach based on the information in the interaction radius of each point. Second order implicit time integration is used.

Advantages:

- **meshfree character** – minimal preparation time (geometry can be directly taken from CAD tools)
- **Lagrangian formulation** – efficient handling of free surfaces, phase boundaries, and moving geometry
- **independence of the PDEs character** – large variety of material models (viscous, non-viscous, elastic, plastic, mixture of the previous, barodetic, etc)
- **parallelization** – shared, distributed, and hybrid memory parallelization

Conservation equations

FPM solves the equations for conservation of mass, momentum, and energy in Lagrangian form:

$$\dot{\rho} = -\rho(\nabla^T\mathbf{v}), \quad \dot{\mathbf{v}} + \frac{1}{\rho}\nabla p = \frac{1}{\rho}(\nabla^T\mathbf{S})^T + \mathbf{g} - \beta\cdot\mathbf{v}, \quad \rho c_v \dot{T} = \nabla^T(\lambda\cdot\nabla T) + q$$

Stress tensor model

In FPM the options to integrate the material characteristics of the avalanche via the stress tensor are as follows.

- **Drucker-Prager model:** stress tensor $\mathbf{S}$ is split into a solid and a viscous part

$$\dot{\mathbf{S}}_{solid} = \mu_{eff}\left(\nabla\mathbf{v}^T + (\nabla\mathbf{v}^T)^T - \frac{2}{3}\nabla^T\mathbf{v}\cdot\mathbf{I}\right), \quad \left\|\mathbf{S}_{solid}\right\| \le \mathbf{S}_{yield}(p) = \left(C_{DP}(p+p_0) + \mathbf{S}_{yield}^{ficticious}\right)$$

$$\mathbf{S}_{visc}(\mathbf{v}) = \eta_{eff}\left(\nabla\mathbf{v}^T + (\nabla\mathbf{v}^T)^T - \frac{2}{3}\nabla^T\mathbf{v}\cdot\mathbf{I}\right)$$

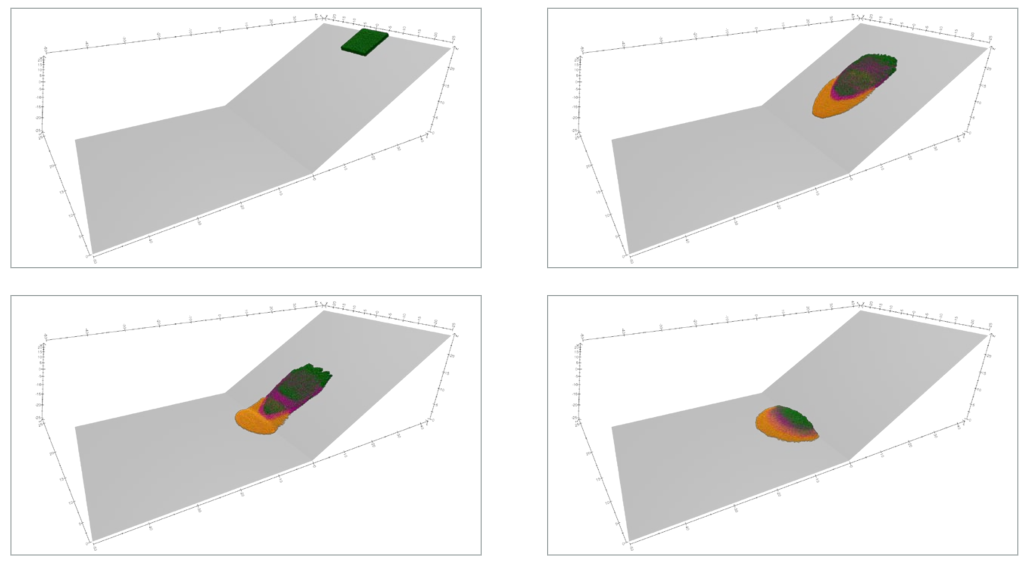
- **Barodesy model:** stress tensor and additional void ratio are given by ODEs (see Ostermann et al. 2013)

$$\dot{\mathbf{S}} = \mathbf{WS} - \mathbf{SW} + \mathbf{H}(\mathbf{S},\mathbf{D},e), \quad \dot{e} = (1+e)\cdot\mathrm{tr}(\mathbf{D})$$

$$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{v}^T + (\nabla\mathbf{v}^T)^T), \quad \mathbf{W} = \frac{1}{2}(\nabla\mathbf{v}^T - (\nabla\mathbf{v}^T)^T)$$

Friction boundary condition

The interaction between avalanche and geometry is characterized by friction. For a simplified one-phase flow scenario we compare three different friction boundary conditions from „slip“ to „no-slip“ behavior in Figures 1-2.



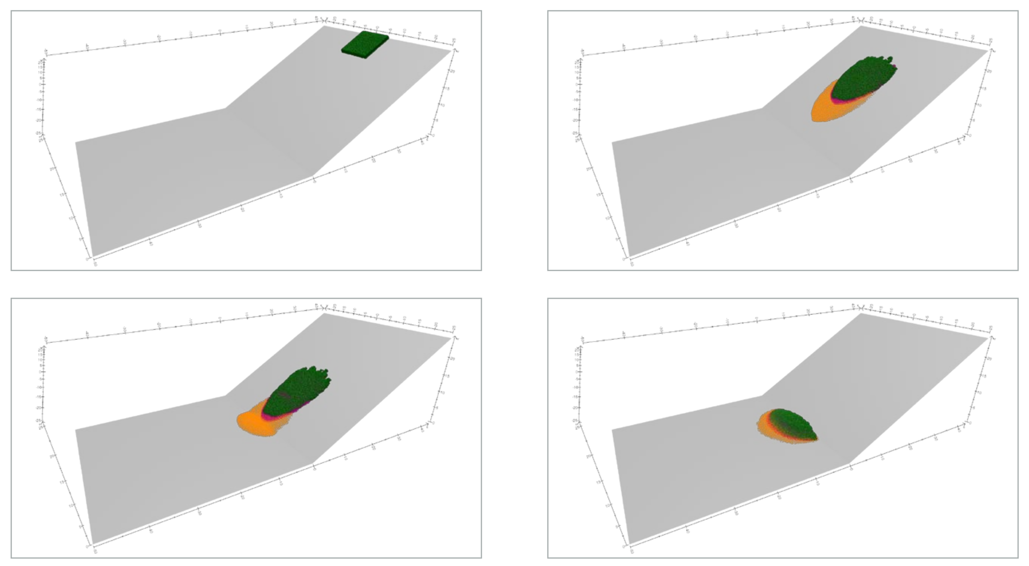
30° inclination
1m initial release height

v_{max} = 9.35m/s
0.52m deposition depth

v_{max} = 8.34m/s
0.84m deposition depth

v_{max} = 8.27m/s
1.11m deposition depth

Figure 1: ***One-phase scenario*** *– initial to final state of the 3D FPM simulations for the Drucker-Prager model (orange: friction coefficient 10^3, magenta: friction coefficient 10^4, green: friction coefficient 10^5)*



30° inclination
1m initial release height

v_{max} = 9.07m/s
0.58m deposition depth

v_{max} = 8.09m/s
0.82m deposition depth

v_{max} = 7.72m/s
1.07m deposition depth

Figure 2: ***One-phase scenario*** *– initial to final state of the 3D FPM simulations for the barodesy model for loose granular material based on a Drucker-Prager-type linearization (orange: friction coefficient 10^3, magenta: friction coefficient 10^4, green: friction coefficient 10^5)*

Volume-volume-coupling

Incorporating another phase, e.g. trees or rocks, can be achieved by a volume-volume-coupling in FPM: For the avalanche the second phase is a porous medium (Darcy's law). Furthermore, the pressure of the avalanche is mapped to the free surface of the second phase. Figure 3 illustrates the simulation result for a simplified two-phase scenario.

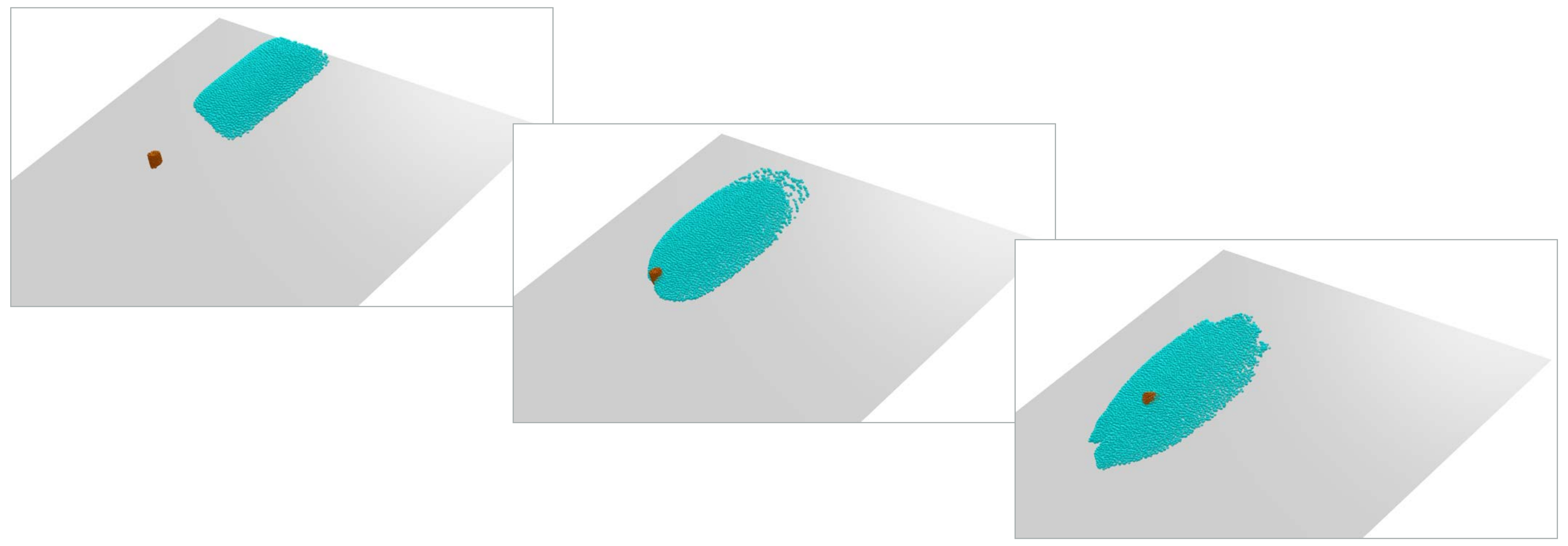


Figure 3: ***Two-phase scenario*** *– 3D FPM simulation based on the Drucker-Prager model with friction coefficient 10^4 (cyan: avalanche, brown: rock) before and during interaction of the two phases*

Wolfsgruben avalanche

A first application to the Wolfsgruben avalanche site using the one-phase approach is presented in Figure 4.

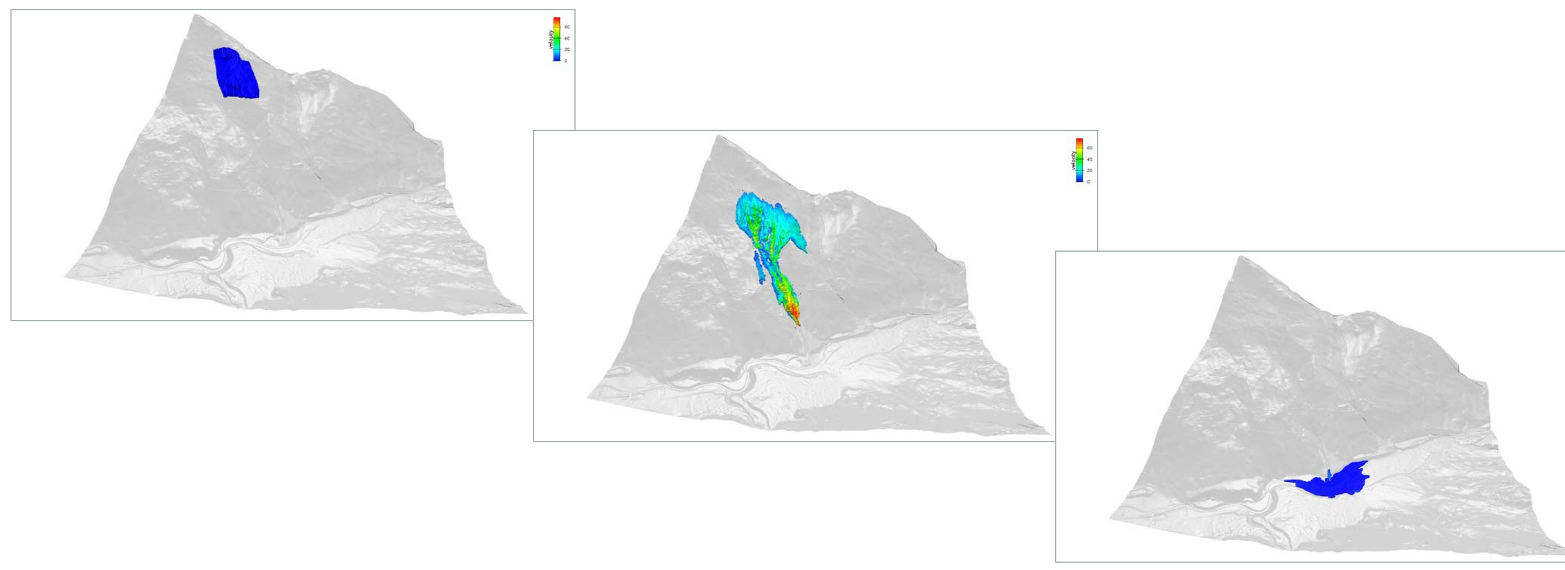


Figure 4: ***One-phase scenario*** *– initial to final state of the 3D FPM simulation for the Wolfsgruben avalanche site based on the Drucker-Prager model with friction coefficient 10^3 (v_{max} = 80m/s)*

The digital elevation model for the Wolfsgruben avalanche including the release zone (see Fischer et al. 2015) has been kindly provided by BFW and WLV.

BFW. die-wildbach.at

Fischer JT, Kofler A, Fellin W, Granig M, Kleemayer K (2015) Multivariate parameter optimization for computational snow avalanche simulation. *Journal of Glaciology*, 61(229), doi: 10.3189/2015JoG14J168

Kuhnert J (2014) Meshfree Numerical Scheme for Time Dependent Problems in Fluid and Continuum Mechanics. In: S. Sundar (ed.) Advances in PDE Modeling and Computation, Ane Books, pp. 119-136

Kolymbas D (2012) Barodesy: a New Constitutive Frame for Soils. *Géotechnique Letters*, 2:17-23

Ostermann I, Kuhnert J, Kolymbas D, Chen C-H, Polymerou I, Smilauer V, Vrettos C, Chen D (2013) Meshfree generalized finite difference methods in soil mechanics – part I: theory. *International Journal on Geomathematics*, 4:167-184

Fraunhofer ITWM