Two-way spatial coupling of magnetohydrodynamic and particle-in-cell methods for space plasma simulation

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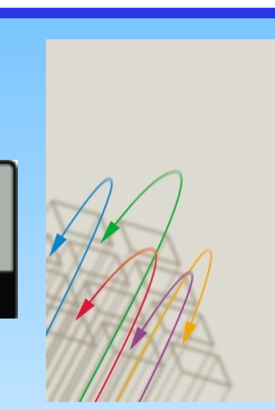
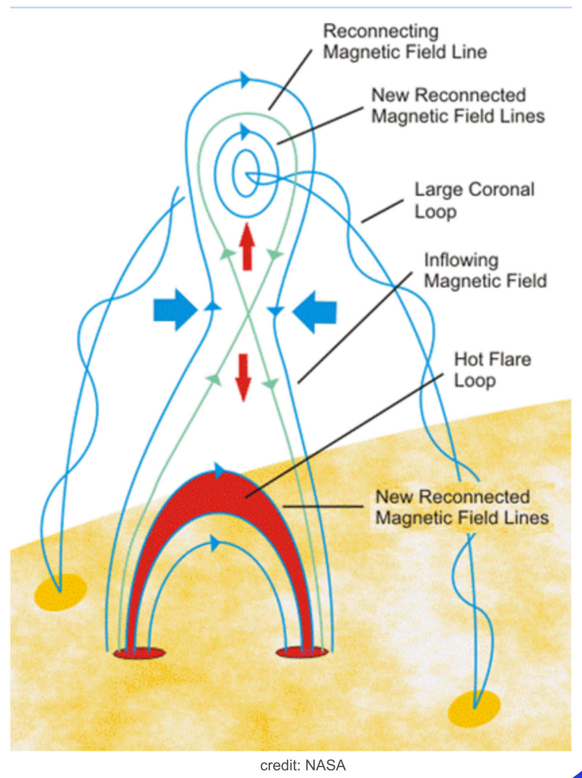


Motivation

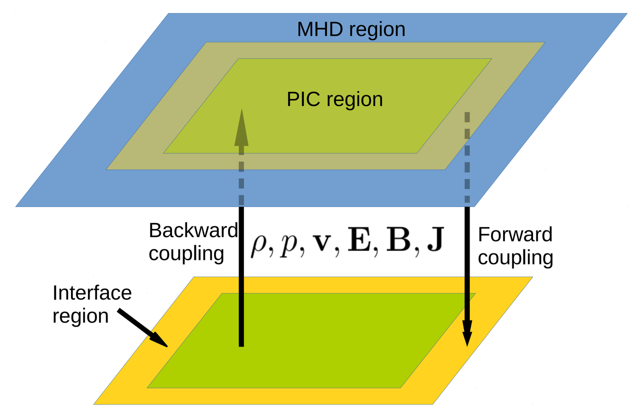
Several problems in solar physics involve multiple scales. For ex., in solar coronal flares the loop length is ~10^9m whereas kinetic scales are of ~10^{-1}m - a huge difference of scales that is beyond foreseeable simulation capabilities. Moreover, while fluid models work well at scales much larger than gyro-radius, kinetic models are required at the skin-depth scales. Therefore, these are **multi-scale and multi-physics problems. Global kinetic simulations are also not possible** for such large systems.



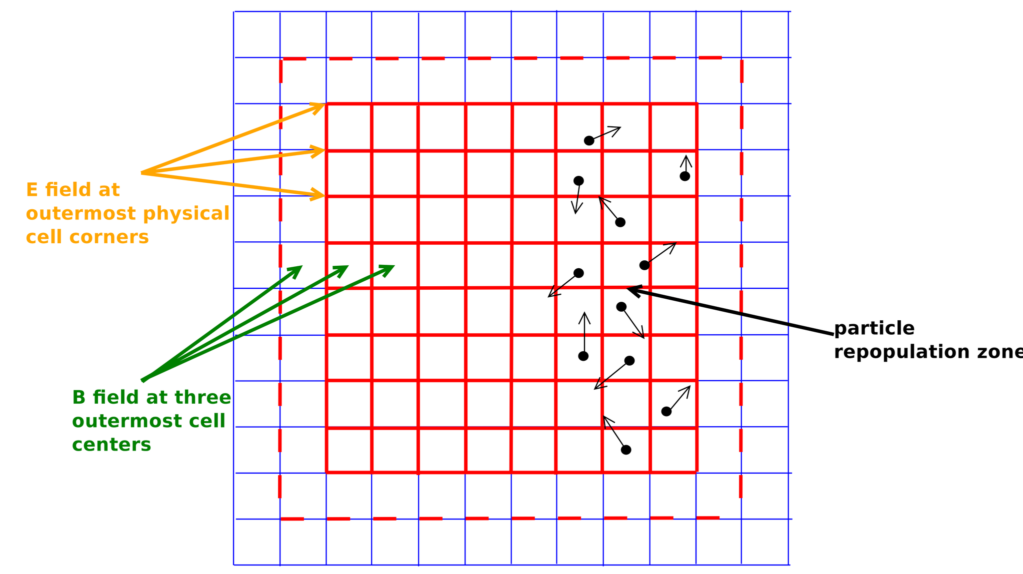
credit: NASA

Idea - coupling MHD with PIC

The idea is to **simulate the global system by MHD and simulate only a small region where kinetic physics is important by PIC**. MPI-AMRVAC is used for MHD as it has adaptive mesh refinement and Hall-MHD.



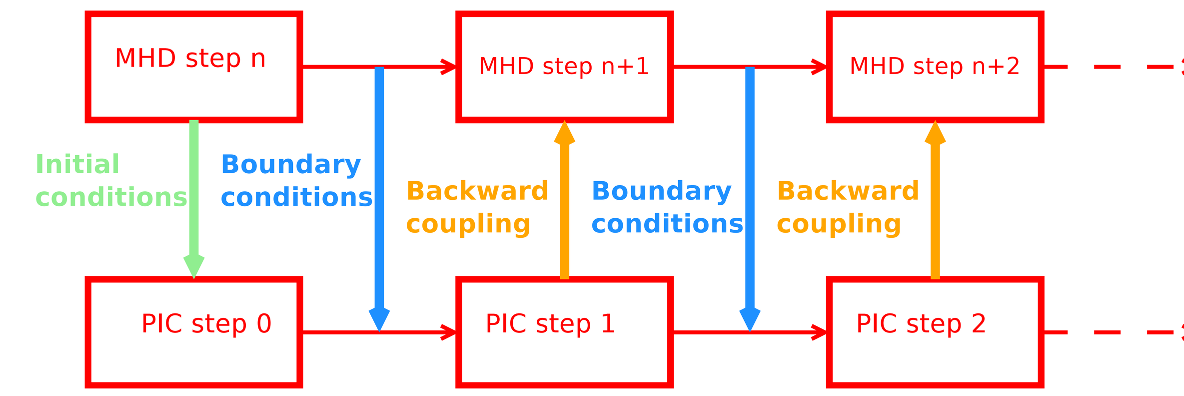
iPIC3D, an implicit moment method code is used for the PIC domain, as it can have 5X-10X larger time steps and 10X-50X larger grid spacing compared to explicit PIC codes



iPIC3D has been similarly coupled with the BATS-R-US MHD code in Daldorff et al., JCP, vol. 268 (2014) 236-254. In this work the PIC grid overlaps the MHD grid.

Time stepping method

The PIC simulation is started at some MHD time step n. The MHD state is advanced to step n+1 and the time averaged boundary conditions are passed to PIC, to advance its state to step 1. Then PIC moments are passed back to MHD to update its solution at step n+1, and the process is repeated.



Forward coupling

In forward coupling, **MHD provides initial and boundary conditions to PIC**. The electric and magnetic fields are directly copied from the MHD solution. At every time step, electric field solver utilizes the MHD values at the outermost active cell corners as the boundary conditions. After every time step, the magnetic field at the three outermost cell centers are fixed by MHD values

The **particles are initialized with a Maxwellian distribution** of density set by

$$n \equiv n_i = n_e = \frac{\rho_{MHD}}{m_i + m_e}$$

Their velocity is set by $\mathbf{v} = \mathbf{u}_0 + \mathbf{u}_{th}$

Their drift velocity u_0 is derived by splitting the MHD flow and current density into two-fluid velocities, giving

$$\mathbf{u}_{0,s} = \frac{[1+(m_{s'}/m_s)][(q_{s'}/m_{s'})\mathbf{u}_{MHD} - (\mathbf{J}/\rho_{MHD})]}{[(q_{s'}/m_{s'}) - (q_s/m_s)]}$$

The thermal velocity u_{th} has a Maxwellian distribution

$$f(v) = \left(\frac{2v}{v_{th}^2}\right)\exp\left(\frac{-v^2}{v_{th}^2}\right).$$

The thermal speed v_{th} is determined from MHD pressure

$$v_{th,s} = \sqrt{\frac{p_s}{\rho_{MHD}}\left(1+\frac{m_{s'}}{m_s}\right)}.$$

The pressure is split into ions and electrons

$$p_e = \zeta p \qquad p_i = (1-\zeta)p$$

Backward coupling

After every PIC time step, moments of the distribution function are calculated to gather the fluid variables, which are then passed back to MHD. The **MHD solution in the PIC domain is updated by taking a weighted average of the MHD and PIC solutions** as follows,

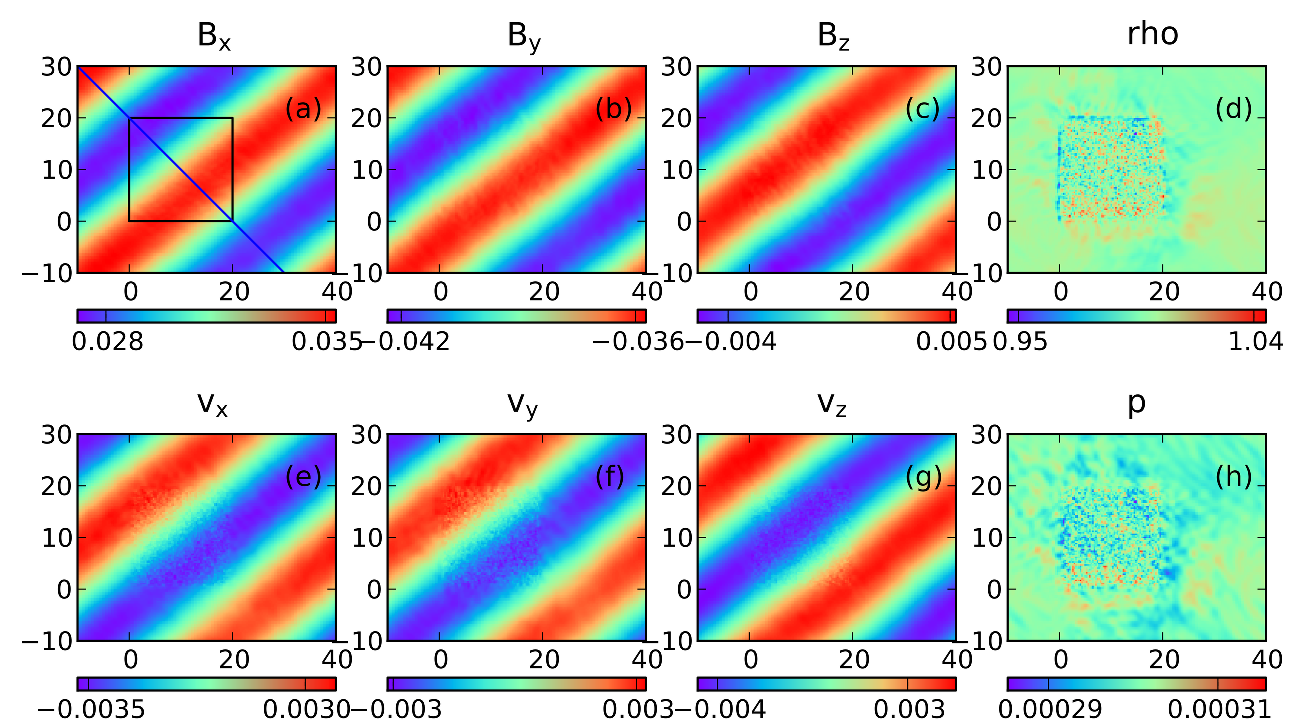
$$\hat{\psi} = (1-w)\psi_{MHD} + w\psi_{PIC}$$

The weight function w determines the weight given to the PIC solution. It is shaped such that zero weight is given to the PIC solution at the MHD-PIC interface, while it rapidly rises within a layer of width delta so that the PIC solution receives almost unity weight in the interior. The weight function for ex.,

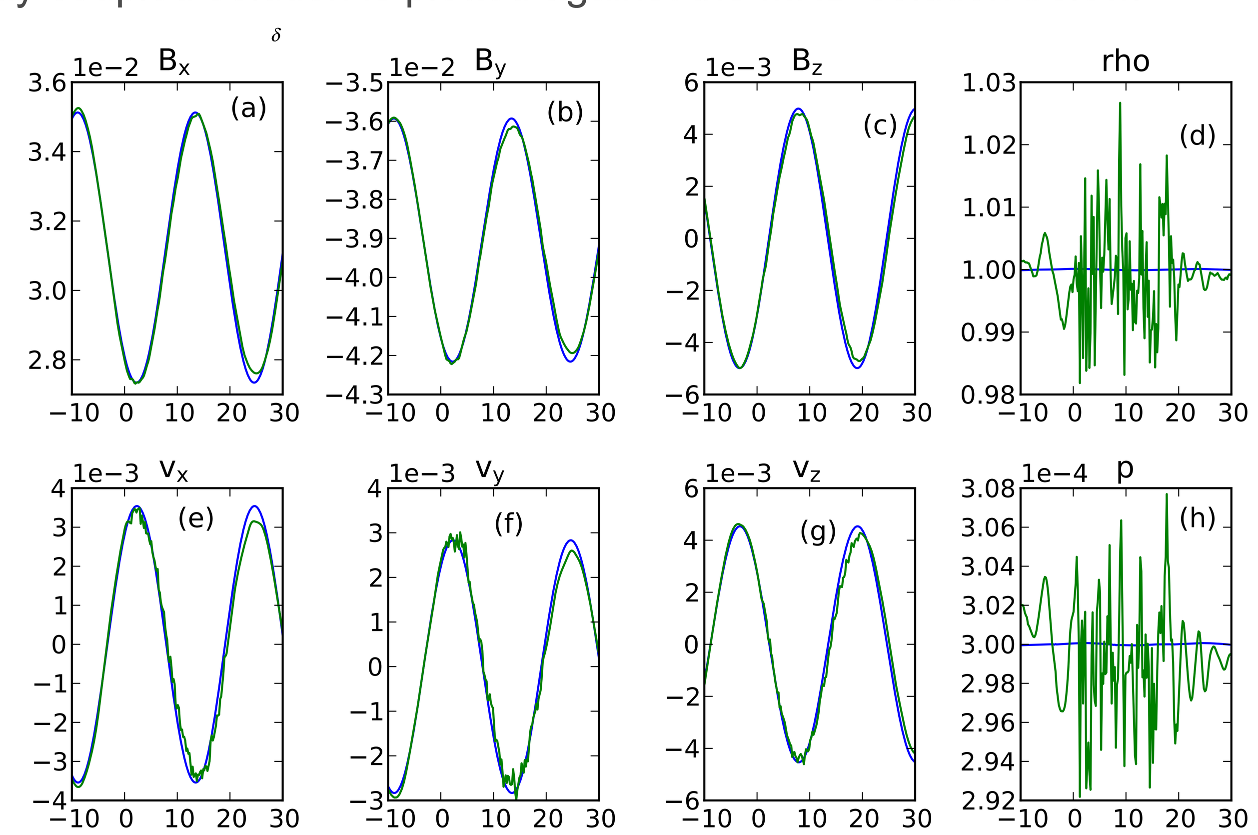
$$w = (1-\exp(-(x-x_1)^2/\delta^2))(1-\exp(-(x-x_2)^2/\delta^2))$$
$$\times(1-\exp(-(y-y_1)^2/\delta^2))(1-\exp(-(y-y_2)^2/\delta^2))$$

Tests with plasma waves

The two-way coupling has been tested successfully for energy and momentum conservation in steady-state systems. Below we show **results of whistler wave propagation through the PIC domain** shown in black box. The 8 MHD quantities are showing good coupling.



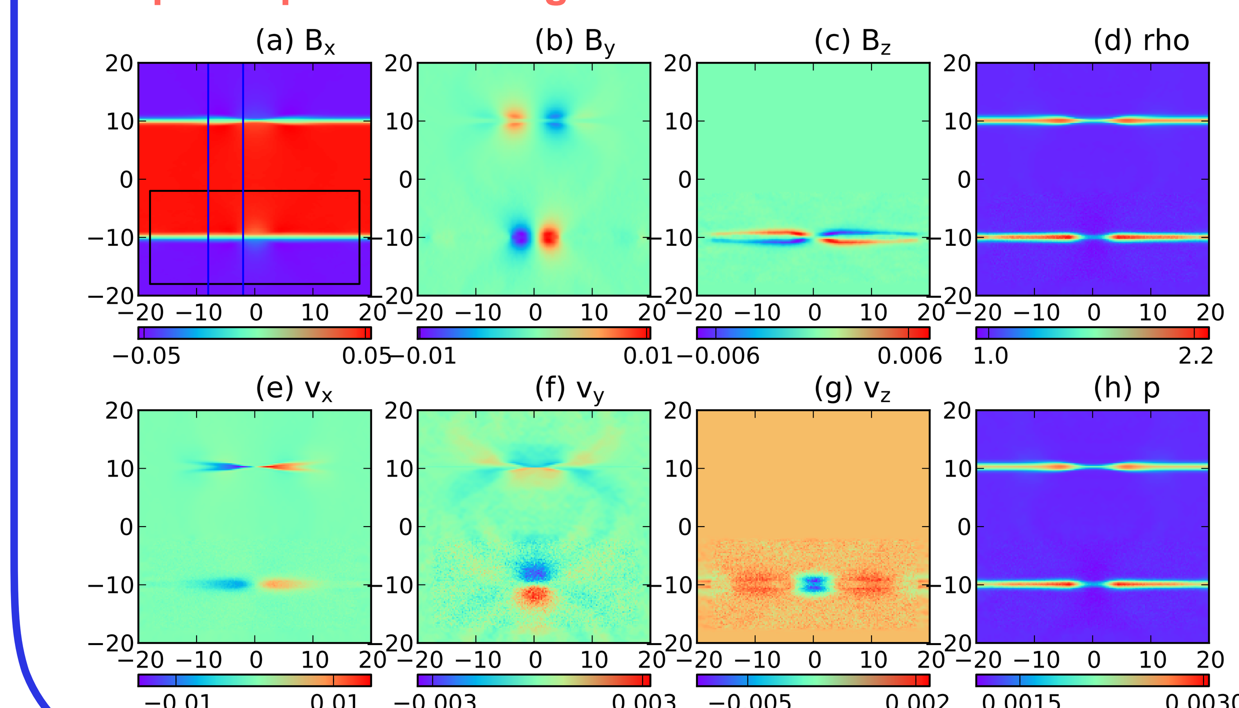
The above figure is made after a coupling time of 1.06 wave period. A trace is taken along the black diagonal line shown above and the two-way coupled solution is plotted against the MHD solution below.

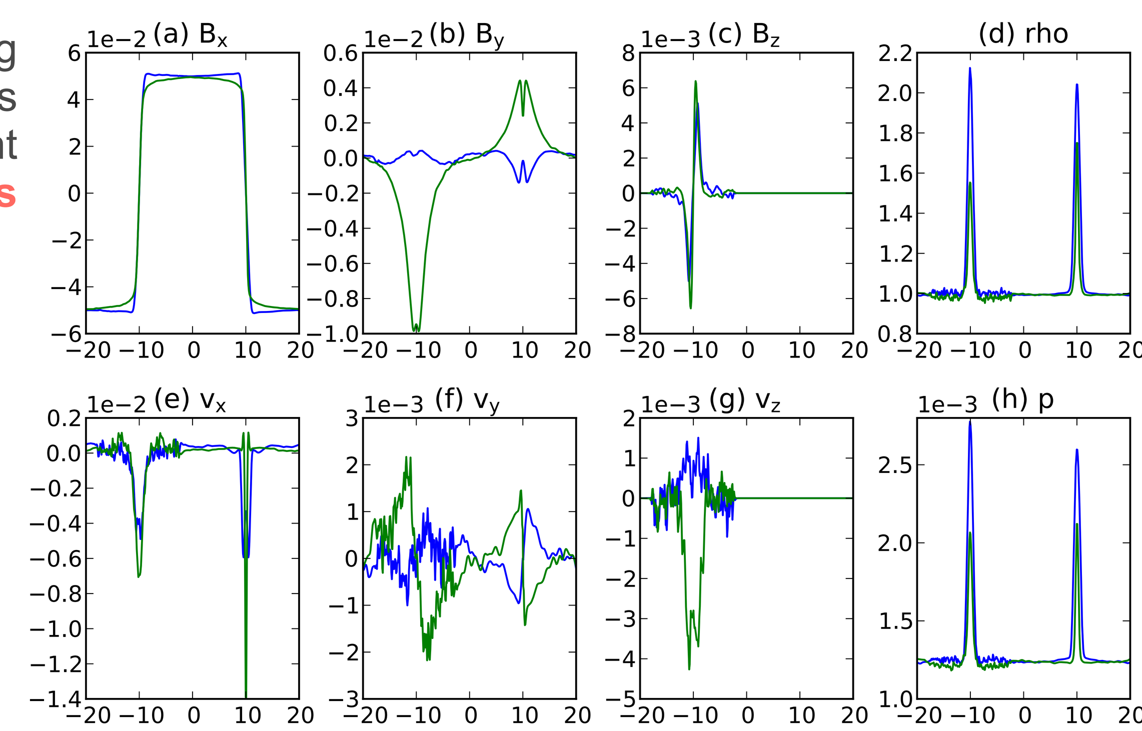


We see very good coupling at long wavelengths even for the fast magnetosonic wave in all the 4 MHD quantities. Thus, **the two-way coupling works well for wave propagation**

Geospace Environmental Modeling (GEM) simulation

GEM challenge is a benchmark problem for simulating reconnection in magnetospheres. A double GEM challenge is simulated with MHD, including PIC feedback in bottom current layer around the X- point. **PIC feedback correctly reproduces the quadrupolar Hall magnetic field.**

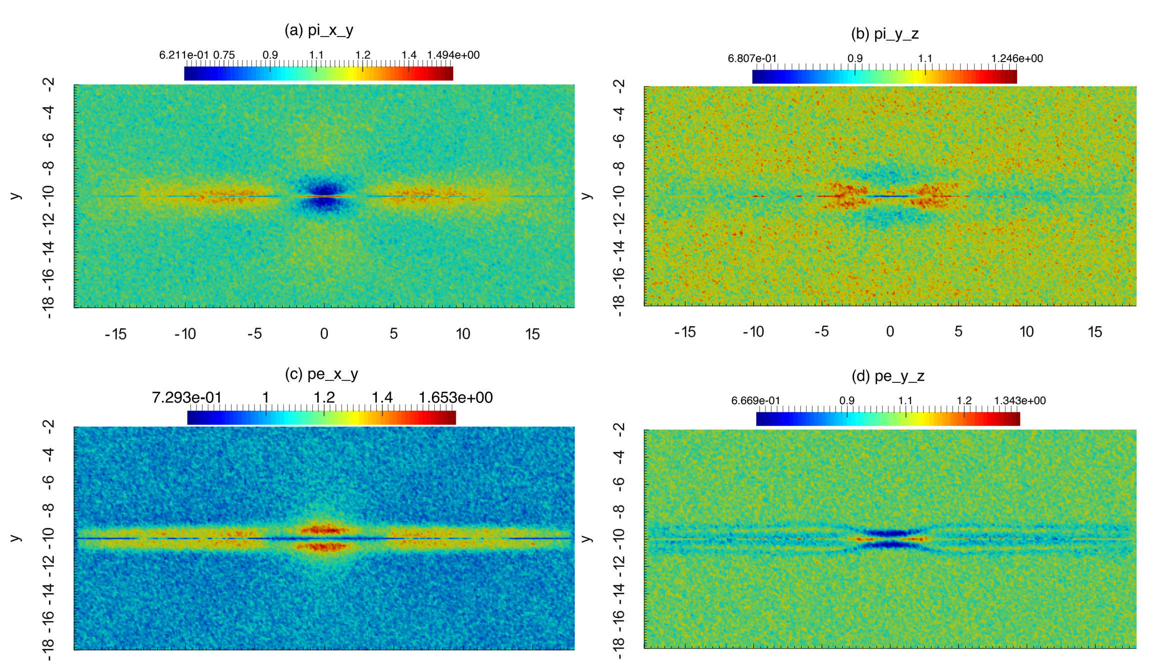




Trace of quantities through the two lines shown in left figure. The left portion includes PIC feedback. We see **no distortion at the coupling interface and smooth propagation of waves across the interface.**

Anisotropy & Agyrotropy

The **PIC is able to produce effects that cannot be captured by MHD**. For ex. the temperature anisotropy in the PIC domain is shown below.



Agyrotropy (difference in the two perpendicular pressures) is also observed. Electrons show stronger anisotropy and agyrotropy.

Conclusions

- Two-way coupling between MPI-AMRVAC and iPIC3D have been implemented, and the results have been submitted for peer-review.
- There are several differences compared to Daldorff et al., 2014 - like the handling of boundary conditions, grid setup, time stepping, backward coupling
- The coupling has been successfully tested for energy and momentum conservation, variety of plasma waves propagation, and magnetic reconnection
- Coupling is suitable for application to large scale systems of waves, reconnection, and shocks

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