Bayesian Model Evidence as a Model Evaluation Metric

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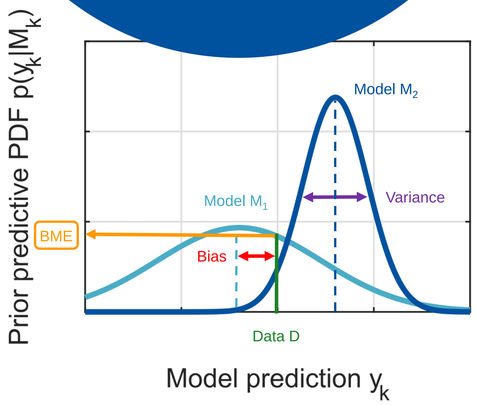
We thank the DFG for financial support within the IRTG "Integrated Hydrosystem Modelling" (IRTG 1829, Tübingen), and within the Cluster of Excellence in Simulation Technology (EXC 310/1, Stuttgart).

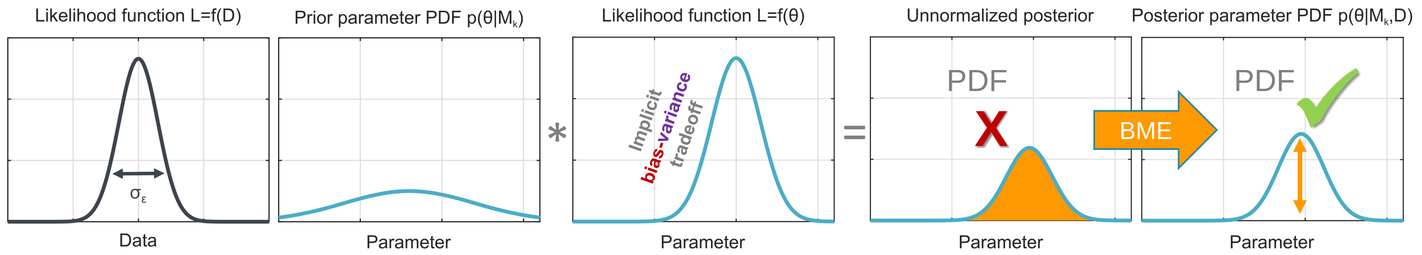
How to assess model quality in the face of uncertainty?



Uncertainty (input, parameters) and errors (model structure, measurements) trigger a **predictive distribution** instead of a deterministic forecast. How to rate model quality aspects **bias** and **variance**?

Bayesian model evidence

[Raftery, 1995]: $p(\boldsymbol{D}|M_k) = \int_{\Theta_k} p(\boldsymbol{D}|M_k,\boldsymbol{\theta}_k)\, p(\boldsymbol{\theta}_k|M_k) d\theta_k = \frac{p(\boldsymbol{D}|M_k,\boldsymbol{\theta}_k)\, p(\boldsymbol{\theta}_k|M_k)}{p(\boldsymbol{\theta}_k|M_k,\boldsymbol{D})}$



Bayesian model evidence…

Predictive log-score [Good, 1952] for independent predictions:

$$PLS = \log p(\boldsymbol{D}|M_k) = \sum_{j=1}^{N_d} \log p(D_j|M_k)$$

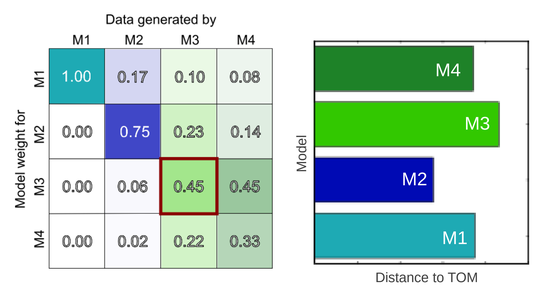
Information criteria (e.g., AIC [Akaike, 1973], BIC [Schwarz, 1978], KIC [Neuman, 2003]) for linear models and multi-Gaussian distributions [Schöniger et al., 2014]:

$$IC \approx -2 \log p(\boldsymbol{D}|M_k)$$

… related metrics…

- Investigating model quality for competing models with **Bayes factors** [Kass & Raftery, 1995]
- Weighting and combining models: **Bayesian model selection and averaging** [Hoeting et al., 1999]
- Ranking models under limited data: **Model justifiability analysis** [Schöniger et al., 2015a]
- Ranking models under noisy data: Comparison with **theoretically optimal model** TOM [Schöniger et al., 2015b]

$$BF = \frac{p(\boldsymbol{D}|M_1)\, P(M_1)}{p(\boldsymbol{D}|M_2)\, P(M_2)}$$



… and further analysis options to dig deeper!

References:

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