Kinetic study of electrostatic twisted waves instability

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ABSTRACT

The kinetic theory of twisted wave instability in a dusty plasma is developed in the presence of orbital angular momentum of the helical electric field in plasma with kappa distributed electrons, ions and dust particles. The kappa distributed electrons are considered to have a drift velocity.

The perturbed distribution function and helical electric field are decomposed by Laguerre Gaussian mode functions defined in cylindrical coordinated. The Vlasov-Poisson equation is obtained and solved analytically to investigate the growth rate of the electrostatic twisted waves in a nonthermal dusty plasma.

The growth rates of the dust ion acoustic twisted mode (DIATM) and dust acoustic twisted mode (DATM) are obtained analytically and also pictorial presented numerically. The instability condition is also discussed with different plasma parameters.

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INTRODUCTION

The helically phased light inherit s orbital angular momentum (OAM). The work of Allen on lasers with orbital angular momentum (OAM) has originated an attractive scientific and technological development in a wide range of growing fields such as microscopy and imaging, atomic and molecular nanotechnology manipulation, ultra-fast optical communication, quantum computing, ionospheric radar facility to observe 3D plasma dynamic, photonic crystal fibre, OAM entanglement of two photons, twisted gravitational waves , ultra intense twisted laser pulses and astrophysics.

In recent years, waves carrying orbital angular momentum has appealed to plasma community. Verbeeck et al., 2010 studied electron vortex beams. Magnetic tornadoes (rotating magnetic field structures in solar atmosphere) and Alfvenic Tornadoes (three dimensional modified kinetic Alfven waves) are reported to exhibit three types of morphology, i.e., spiral, ring and helical.

Recently, Arshad et al., 2015; 2016; developed non-Maxwellian kinetic theory of twisted Langmuir and ion acoustic modes in the presence of helical electric field. The validity of analytical and exact numerical results with respect to weak and strong damping is also illustrated.

TWISTED PLASMA DIELETRIC FUNCTION

The dielectric function for the twisted electrostatic waves is written as;

$$\epsilon(\omega,k,lq_\theta) = 1 + \sum_{\alpha=e,i,d} \frac{\omega_{p\alpha}^2}{k^2} \int \frac{\mathbf{q}_{eff}\cdot\partial_{\mathbf{v}} f_{0\alpha}}{(\omega - \mathbf{q}_{eff}\cdot\mathbf{v})} d\mathbf{v}$$

The susceptibility of the electrostatic waves carrying orbital angular momentum is given as;

$$\chi(\omega,lq_\theta,k) = \sum_{\alpha=e,i,d} \frac{\omega_{p\alpha}^2}{k^2} \int \frac{\mathbf{q}_{eff}\cdot\partial_{\mathbf{v}} f_{0\alpha} d\mathbf{v}}{(\omega - kv_z - lq_\theta v_\theta)}$$

The isotropic drifted generalized Lorentzian or kappa distribution can be defined in cylindrical coordinates as;

$$f_0 = \frac{1}{\pi^{3/2}\theta^3\kappa^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \times \left[1 + \frac{v_r^2+v_\theta^2}{\kappa\theta^2} + \frac{(v_z-v_d)^2}{\kappa\theta^2}\right]^{-\kappa-1}.$$

TWISTED DIATM AND DATM

The dielectric function for the dust ion acoustic twisted mode (DIATM) and dust acoustic twisted mode (DATM) can be expressed as;

$$\epsilon(\omega,k,lq_\theta) = 1 + \frac{2\omega_{pe}^2}{k^2\theta_e^2}\left[\frac{2\kappa_e-1}{\kappa_e} + \xi_{z_e} Z(\xi_{z_e}) + \xi_{\theta_e} Z(\xi_{\theta_e})\right] + \frac{2\omega_{pi}^2}{k^2\theta_i^2} \times \left[\frac{2\kappa_i-1}{\kappa_i} + \xi_{z_i} Z(\xi_{z_i}) + \xi_{\theta_i} Z(\xi_{\theta_i})\right],$$

and

$$\epsilon(\omega,k,lq_\theta) = 1 + \frac{2\omega_{pe}^2}{k^2\theta_e^2}\left[\frac{2\kappa_e-1}{\kappa_e} + \xi_{z_e} Z(\xi_{z_e}) + \xi_{\theta_e} Z(\xi_{\theta_e})\right] + \frac{2\omega_{pi}^2}{k^2\theta_i^2}\left[\frac{2\kappa_i-1}{\kappa_i} + \xi_{z_i} Z(\xi_{z_i}) + \xi_{\theta_i} Z(\xi_{\theta_i})\right] + \frac{2\omega_{pd}^2}{k^2\theta_d^2}\left[\frac{2\kappa_d-1}{\kappa_d} + \xi_{z_d} Z(\xi_{z_d}) + \xi_{\theta_d} Z(\xi_{\theta_d})\right]$$

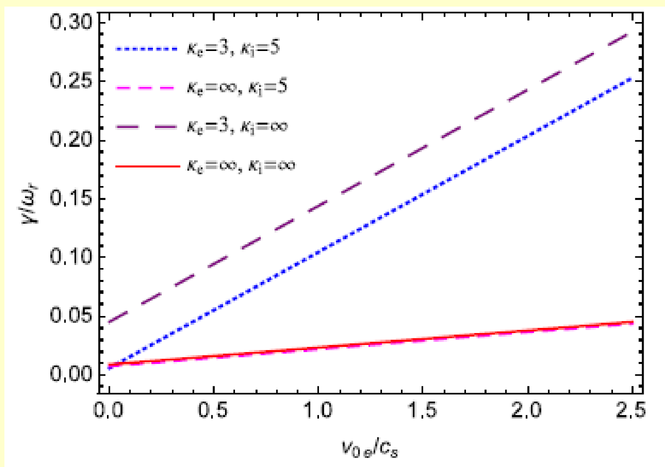

FIG. 1 The normalized growth rates of DIATM are plotted against the normalized drift velocity.

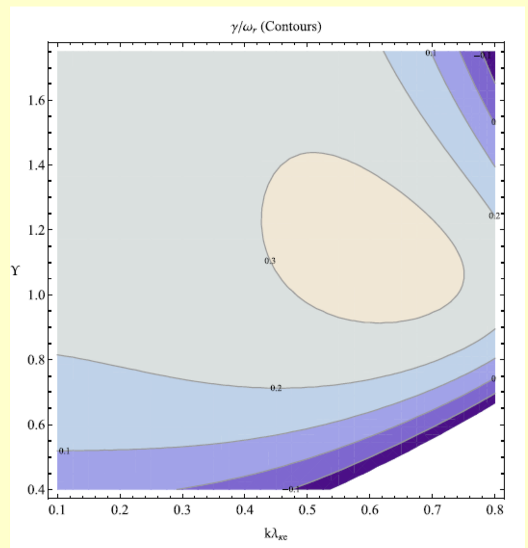

FIG. 2 The contour of normalized growth rates of DIATM are plotted against the normalized wave number and azimuthal parameter.

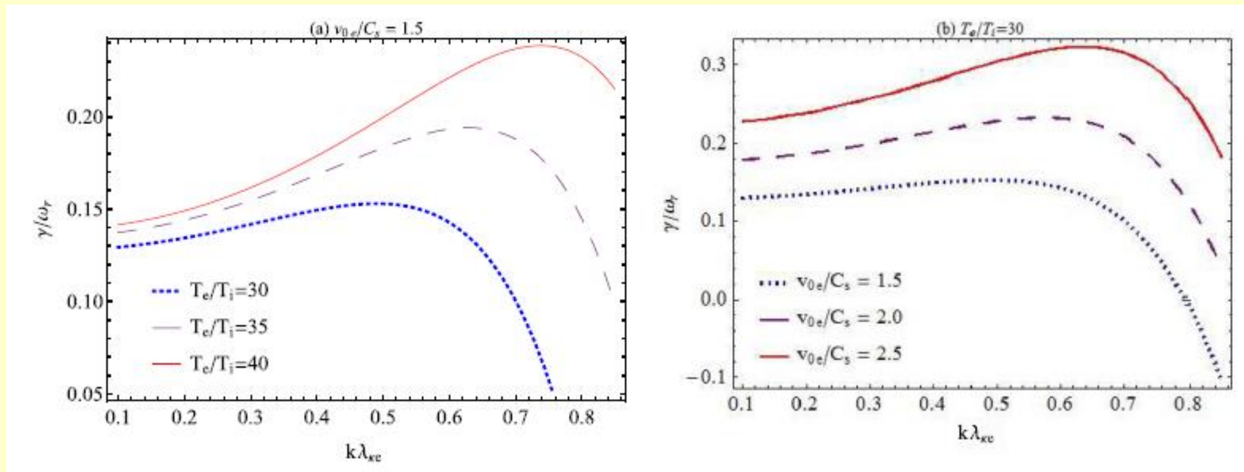

FIG. 3 The normalized growth rates of DIATM are plotted against the normalized wave number.

LAGUEERE GAUSSIAN MODE FUNCTION

The Laguerre Gaussian (LG) mode function is defined as;

$$F_{pl}(r,z) = C_{pl} X^{|l|} L_p^{|l|}(X) \exp(-X/2)$$

Where the constant and associated Laguerre polynomial is defined as;

$$C_{pl} = \sqrt{(l+p)!/4\pi p!},$$

And

$$L_p^{|l|}(X) = \exp(X) d^p/dX^p \left[X^{l+p}\exp(-X)\right]/p!X.$$

By definition of Laguerre Gaussian (LG) mode function $F_{pl}((r,z)$, the modified potential can be written as follows

$$\phi(\mathbf{r},t) = \sum_{pl} \widetilde{\phi}_{pl} F_{pl}(r,z) e^{il\theta} e^{ikz-i\omega t}$$

To decompose the perturbed distribution function in Laguerre Gaussian (LG) modes, we employ;

$$\widetilde{f}_\alpha(\mathbf{v}) = \sum_{pl} \widetilde{f}_{pl}(\mathbf{v}) F_{pl}(r,z) e^{il\theta} e^{ikz-i\omega t}.$$

CONCLUSIONS

The growth rate of DIATM and DATM are calculated analytically using the weak damping or growth theory of the waves in dusty plasmas in which kappa distributed electrons are considered to be having a drift velocity.

The growth of the instability of DIATM and DATM is increased with the decrease in the spectral index of drifted electrons, increase in the temperature ratio of (electron to ion ratio and ion to dust for the DIATM and DATM respectively) and dimensionless drifted velocity.

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KINETIC THEORY OF TWISTED MODES IN NON-MAXWELLIAN PLASMA CARRYING ORBITAL ANGULAR MOMENTUM

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Plan of Talk

- Motivation/ Introduction
- Theoretical modeling
- Twisted Plasma dispersion function
- Three dimensional Generalized Lorentzian (kappa) and Maxwellian distribution functions.
- Ion acoustic twisted waves
- Dust ion acoustic twisted mode and dust acoustic twisted mode

Motivation

- In space plasmas most the astronomical objects are surrounded by the atmosphere which follow the ring, spiral or elliptical like morphology due to the existence of the helical fields around them.

- These morphologies provides a clear evidence for the existence of twisted plasma, gravitational and radio waves in such environment.

Black Hole

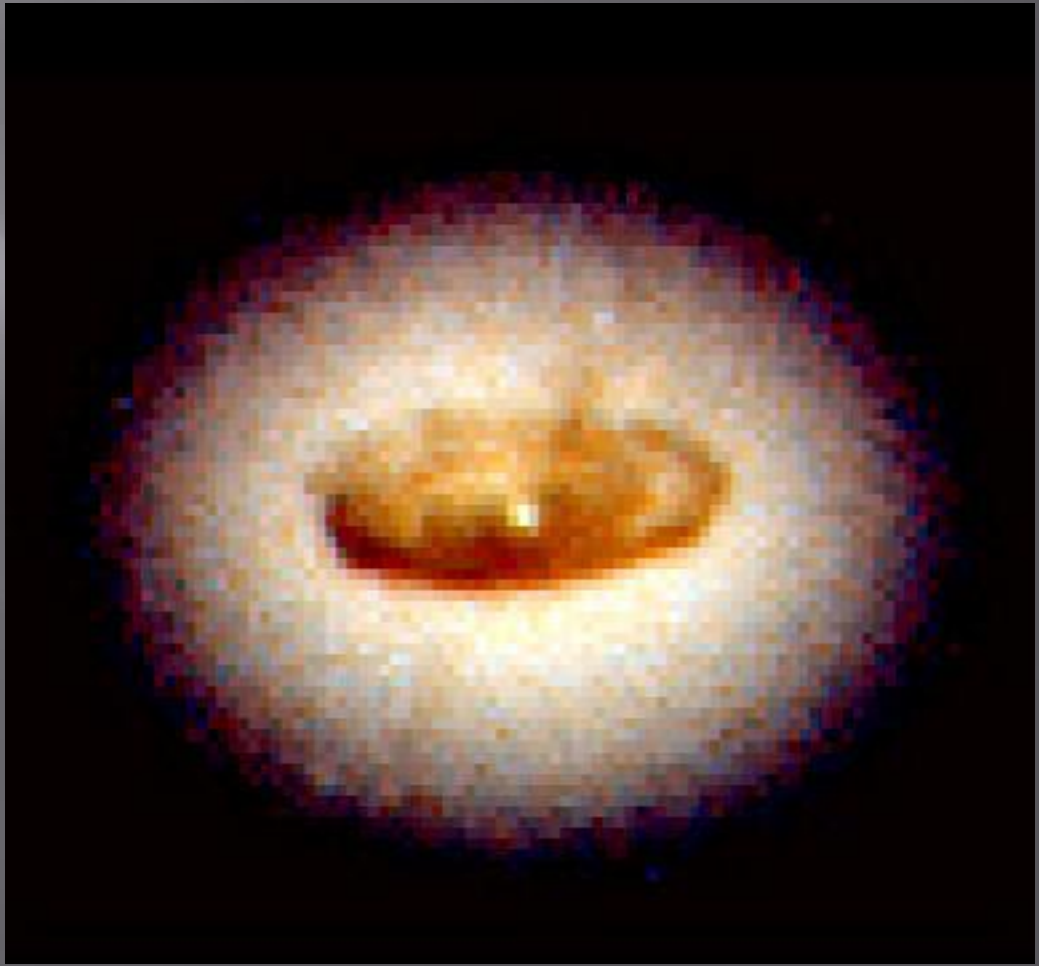
A black hole is a region of space-time, exhibiting such strong gravitational effects that neither particles nor radiation can escape from inside of it.

Solar Corona

Solar corona is an aura of plasma that surrounds the sun and other stars.

Accretion disks

An accretion disk is a structure (often a circumstellar disk) formed by the diffused material in orbital motion around massive central body.
The central body is typically a star.



Galaxies

A **galaxy** is a gravitationally bound system of stars, stellar remnants, interstellar gas, dust, and dark matter.

Background of the studies

- Electromagnetic (EM) fields are used in increasingly many different contexts, ranging from fundamental research and development to communications and household appliances.

- However, there are still properties of the classical EM field, well known already to the pioneers of electromagnetism a century ago, that are not yet fully utilized.

- This is because of either actuator/sensor/detector limitations or because of lack of familiarity with the more subtle aspects of the electromagnetic field.

Contd.

- One of the underutilized properties is the complete, instantaneous magnetic or electric field vector itself.

- It is because all three spatial dimensions of the field vector are taken fully into account so that one can make use of the information embedded in both the magnitude and the direction of the field vector.

Contd.

- Because of the technical complexities involved, sensing of three dimensional electromagnetic fields, have typically been made with sensors capable of capturing only one or two of the three spatial components of the field vectors.

- Radio antennas erected in one or two spatial dimensions, effectively resulting in a waste of available EM information.

- One obvious example being the inadequacy to sense, simultaneously, the transverse part as well as the longitudinal component of the EM field in a beam which has a non-planar phase front.

Contd.

- Simple one-dimensional sensing antennas are what is typically used for picking up radio and TV broadcasts on our domestic radio and TV sets.

- Two-dimensional sensing of the 3D vector field (e.g., crossed dipole antennas) is used in many modern radio telescopes, including the LOFAR (Low Frequency Array) distributed radio telescope currently under construction in the Netherlands, Germany and France

Contd.

One particularly interesting property, used in modern physics, e.g., in experiments on trapping and manipulating of atoms, molecules, and microscopic particles by the help of laser fields, is the electromagnetic orbital angular momentum, (OAM).

However, while OAM has been used to efficiently encode information for free-space communications in the optics frequency range, OAM has so far not been used to its full extent in the radio domain, except for some proof-of-concept experiments in the microwave range.

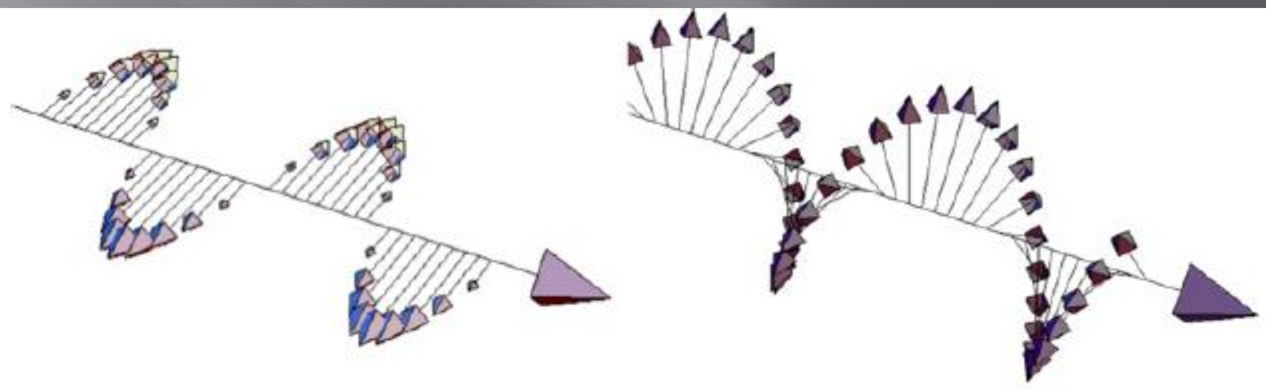
Momentum of EM Field

- The momentum of an electromagnetic field or wave has both linear and angular components.

- The angular momentum of electromagnetic radiation can be further categorize into two distinct intrinsic and extrinsic parts;
- Spin angular momentum (SAM) ,
- Orbital angular momentum (OAM).

Spin Angular Momentum (SAM)

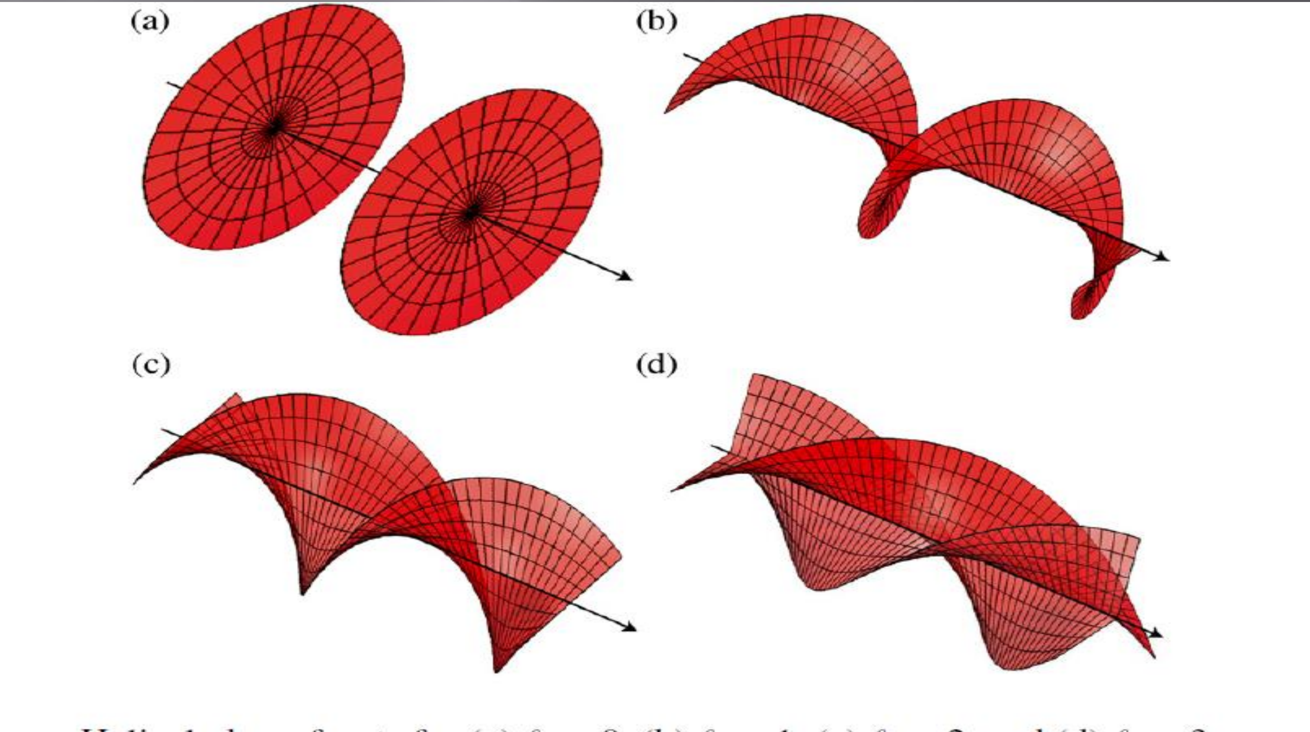
- The spin angular momentum (SAM) is the intrinsic component of the angular momentum which is mostly associated with the photon spin and wave polarization. The expression of the SAM is written as σħ.



The spin angular momentum (SAM) of light is connected to the polarization of the electric field. Light with linear polarization (left) carries no SAM, whereas right or left circularly polarized light (right) carries a SAM of $\pm\hbar$ per photon.

Orbital Angular Momentum (OAM)

- The orbital angular momentum (OAM) is the extrinsic component of the angular momentum which is mostly associated with the azimuthal phase of the wave and its expression is defined as $\ell\hbar$.



Helical phase fronts for (a) $\ell = 0$, (b) $\ell = 1$, (c) $\ell = 2$, and (d) $\ell = 3$.

Contd.

The work by Allen et al., 2003 on lasers with orbital angular momentum (OAM) has originated an attractive scientific and technological development in a wide range of growing fields, such as

- Microscopy and imaging,
- Atomic and nanoparticle manipulation
- Ultrafast Optical communications
- Quantum computing
- Ionospheric radar facility to observe 3D plasma dynamics in ionosphere.
- Photonic crystal fibre
- OAM entanglement of two photons
- Twisted gravitational waves.
- Ultra intense twisted laser pulses
- Astrophysics.

Contd.

- Few years ago, Mendonca switches the concept of the orbital angular momentum to plasma physics.

- But most of the study is either based on the conventional fluid theory or experimental techniques.

- In 2012, Mendonca first time introduced the kinetic description of the electron plasma modes carrying orbital angular momentum.

Contd.

- Khan et al., 2014 modified the earlier model with the kinetic description of the ion acoustic plasma vortices.
- But both the models are based on the well known Maxwellian distribution of the plasm particles.
- This Maxwellian distribution does not fit well for the some of the laboratory and most of the space plasmas and most suitable distribution for such systems is the power law distributions like kappa distribution [Arshad et al., 2014].

Why kappa distribution for the modeling of the space plasmas

- The Maxwellian distribution is applied for the system in thermodynamical equilibrium. But most of the realistic system are far from this state.
- In laboratory experiments and most of the astrophysical environments, the charged particles exhibit non-Maxwellian or non-thermal distribution.
- This may be due to variation in the parameters like (number density, temperature, magnetic field, pressure), interplay between different plasma species and background turbulence etc.

Generalized Lorentzian or kappa distribution function

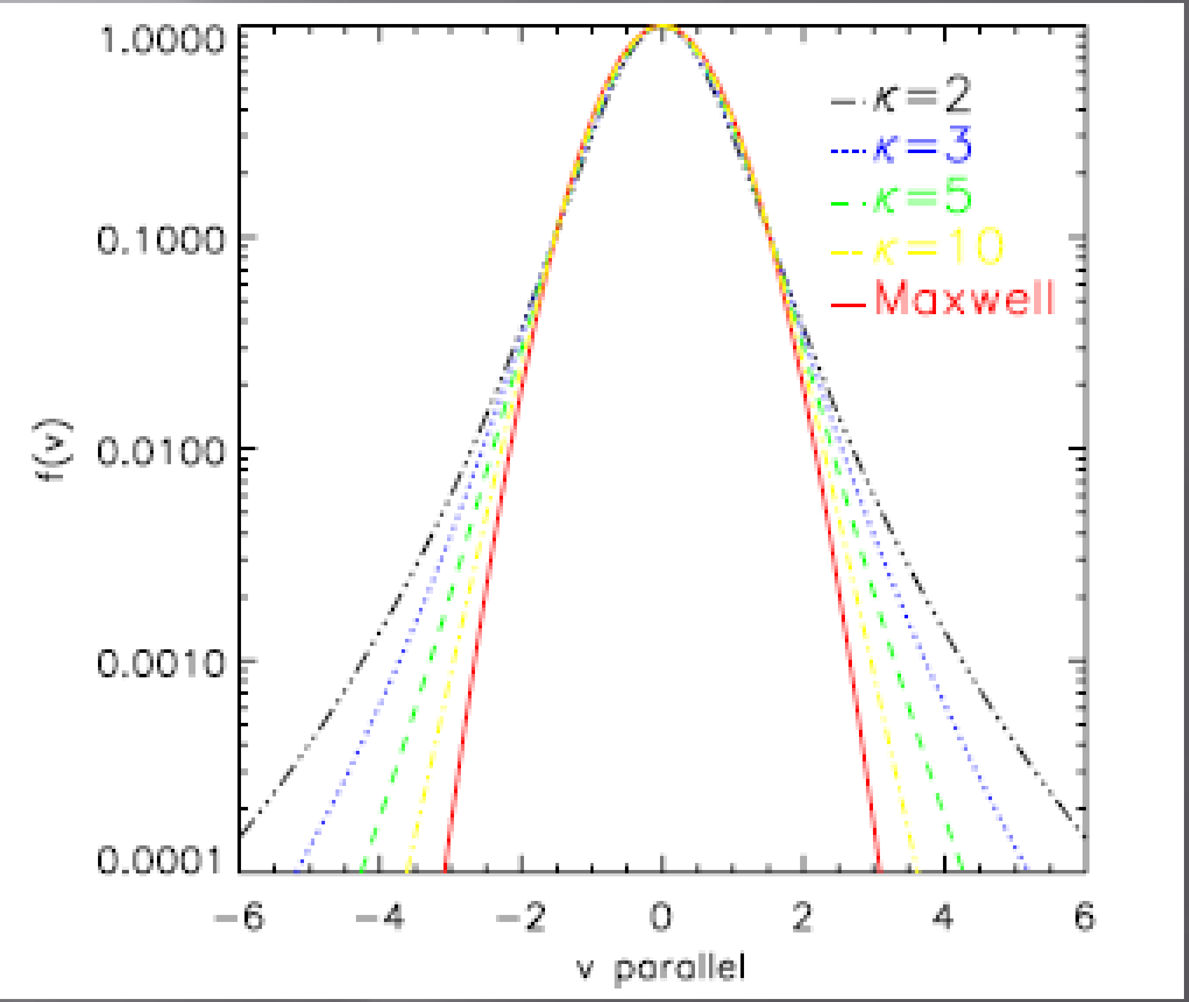
- The three dimensional kappa distribution function is defined as;

$$f_\kappa = \frac{1}{\pi^{3/2}\theta_\perp^2\,\theta_\parallel}\,\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{v_\parallel^2}{\kappa\,\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\,\theta_\perp^2}\right]^{-\kappa-1}$$

Where κ is the spectral index and Lorentzian thermal velocity is defined as;

$$\theta_\parallel^2 = \left(\frac{2\kappa-3}{\kappa}\right)v_{T_\parallel}^2; \quad \theta_\perp^2 = \left(\frac{2\kappa-3}{\kappa}\right)v_{T_\perp}^2$$

The kappa velocity distribution function for different kappa



Theoretical Modeling

- We will initiate our theoretical development with well known Linearized Vlasov equation, which is given by the following equation;

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}}\right) \widehat{f_\alpha} - q_\alpha \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{p}_\alpha} f_{\alpha 0} = 0,$$

- Where $\mathbf{p}_\alpha$ represents the momentum such that $\mathbf{p}_\alpha = \mathbf{m}_\alpha \mathbf{v}$ and **E** is the electric field which can be written in terms of gradient of some scalar potential i.e., $E = -\nabla\emptyset$

Theoretical Model

- The Laguerre Gaussian (LG) mode function can be defined as

$$F_{pl}(r,z) = C_{pl} X^{|l|} L_p^{|l|}(X) \exp\left(-X/2\right),$$

- Where the constant and Laguerre polynomial are defined as

$$C_{pl} = \sqrt{(l+p)!/4\pi p!},$$

- and

$$L_p^{|l|}(X) = \exp(X) d^p/dX^p \left[X^{l+p} \exp(-X)\right]/p!X.$$

Modified potential and distribution function

- Using the definition of the LG mode function, the modified potential can be written as

$$\phi(\mathbf{r},t) = \sum_{pl} \widetilde{\phi}_{pl} F_{pl}(r,z) e^{i\theta} e^{ikz-i\omega t}$$

- In order to obtain the solution of coupled Vlasov-Poisson equation, we will decompose the perturbed distribution function into LG modes, which requires

$$\widetilde{f}_{\alpha}(\mathbf{v}) = \sum_{pl} \widetilde{f}_{pl}(\mathbf{v}) F_{pl}(r,z) e^{i\theta} e^{ikz-i\omega t}.$$

Ion acoustic twisted wave

- The kinetic theory of Landau damping of ion acoustic twisted modes is developed in the presence of orbital angular momentum of the helical (twisted) electric field in plasmas with kappa distributed electrons and Maxwellian ions.
- The perturbed distribution function and helical electric field are considered to be decomposed by Laguerre-Gaussian mode function defined in cylindrical geometry.
- The Vlasov-Poisson equation is obtained and solved analytically to obtain the weak damping rates of the ion acoustic twisted waves in a non-thermal plasma.
- The strong damping effects of ion acoustic twisted waves at low values of temperature ratio of electrons and ions are also obtained by using exact numerical method and illustrated graphically, where the weak damping wave theory fails to explain the phenomenon properly.

Dielectric function of twisted ion acoustic wave

- The plasma dielectric function $\epsilon(\omega, k, lq_\theta)$

$$\epsilon(\omega, k, lq_\theta) = 1 + \chi_e + \chi_i$$

- of twisted ion acoustic wave can be expanded with respect to the small phase velocity limit of the twisted ion acoustic wave in comparison to Lorentzian thermal velocity of super-thermal electrons such that ξ_{ze}, $\xi_{\theta e} << 1$. So that

Contd.

$$\chi_e = \frac{1}{k^2\lambda_{De}^2}\frac{(\kappa-1/2)}{(\kappa-3/2)} - \frac{1}{\Upsilon^2}\frac{1}{k^2\lambda_{De}^2} + 2i\sqrt{\pi}\frac{\omega\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-1/2)}\frac{\omega_{pe}^2}{k^3\theta_{\|e}^3}\left[1+\Upsilon\left(1+\Upsilon^2\frac{\omega^2}{\kappa k^2\theta_{\|e}^2}\right)^{-\kappa-1}\right]$$

- and large phase velocity limit of the twisted ion acoustic wave in comparison to thermal velocity of Maxwellian distributed ion species i.e., ξ_{zi} , $\xi_{\theta i}$ >>1.

$$\chi_i = -\frac{\omega_{pi}^2}{\omega^2}\left[1+\frac{3k^2v_{T_{\|i}}^2}{\omega^2}\right] - \frac{\omega_{pi}^2}{\Upsilon^2\omega^2}\left[1+\frac{3k^2v_{T_{\|i}}^2}{\Upsilon^2\omega^2}\right] + 2i\sqrt{\pi}\frac{\omega\omega_{pi}^2}{k^3v_{T_{\|i}}^3}\left[\exp\left(-\frac{\omega^2}{2k^2v_{T_{\|i}}^2}\right)+\Upsilon\exp\left(-\Upsilon^2\frac{\omega^2}{2k^2v_{T_{\|i}}^2}\right)\right].$$

Damping rates with Lorentzian distributed electrons for non-planar cases (Υ=0.5, 1, 1.5) and planar cases (Υ=∞)

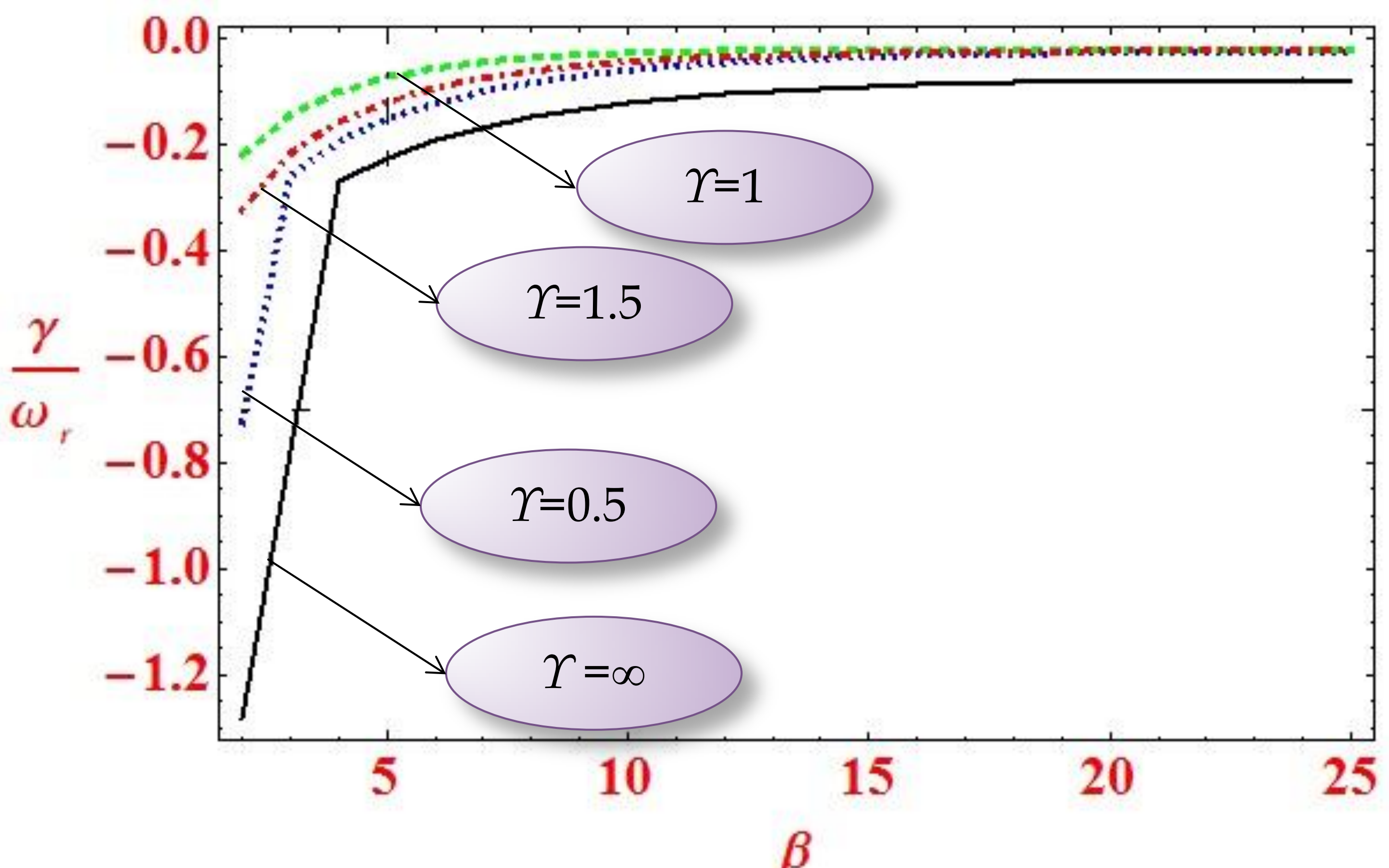


Fig. 2 (*b*)

Effect of normalized electron Debye length on the damping of twisted ion acoustic wave

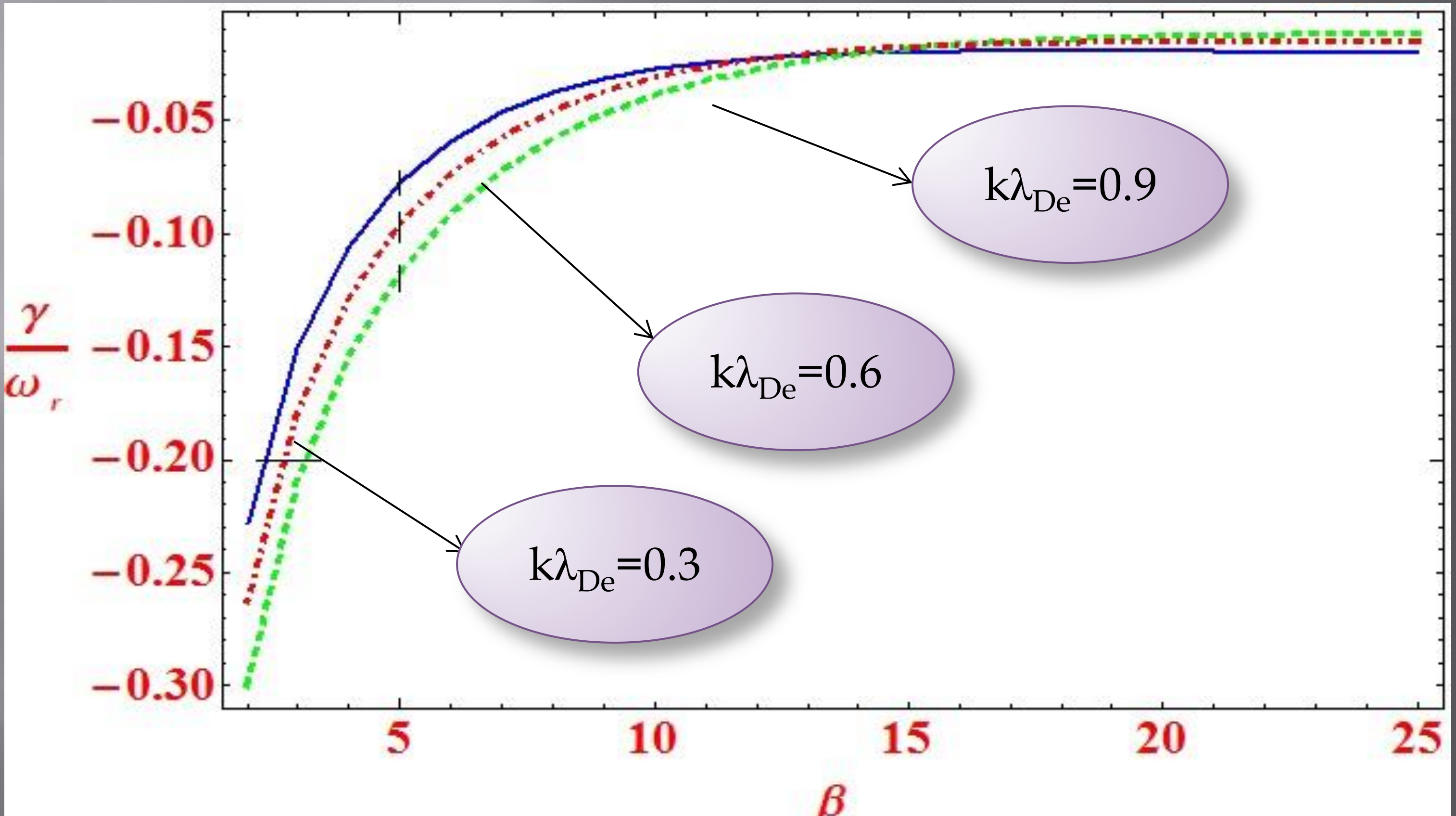


Fig. 3

Kinetic Study of electrostatic twisted instability in nonthermal dusty plasma.

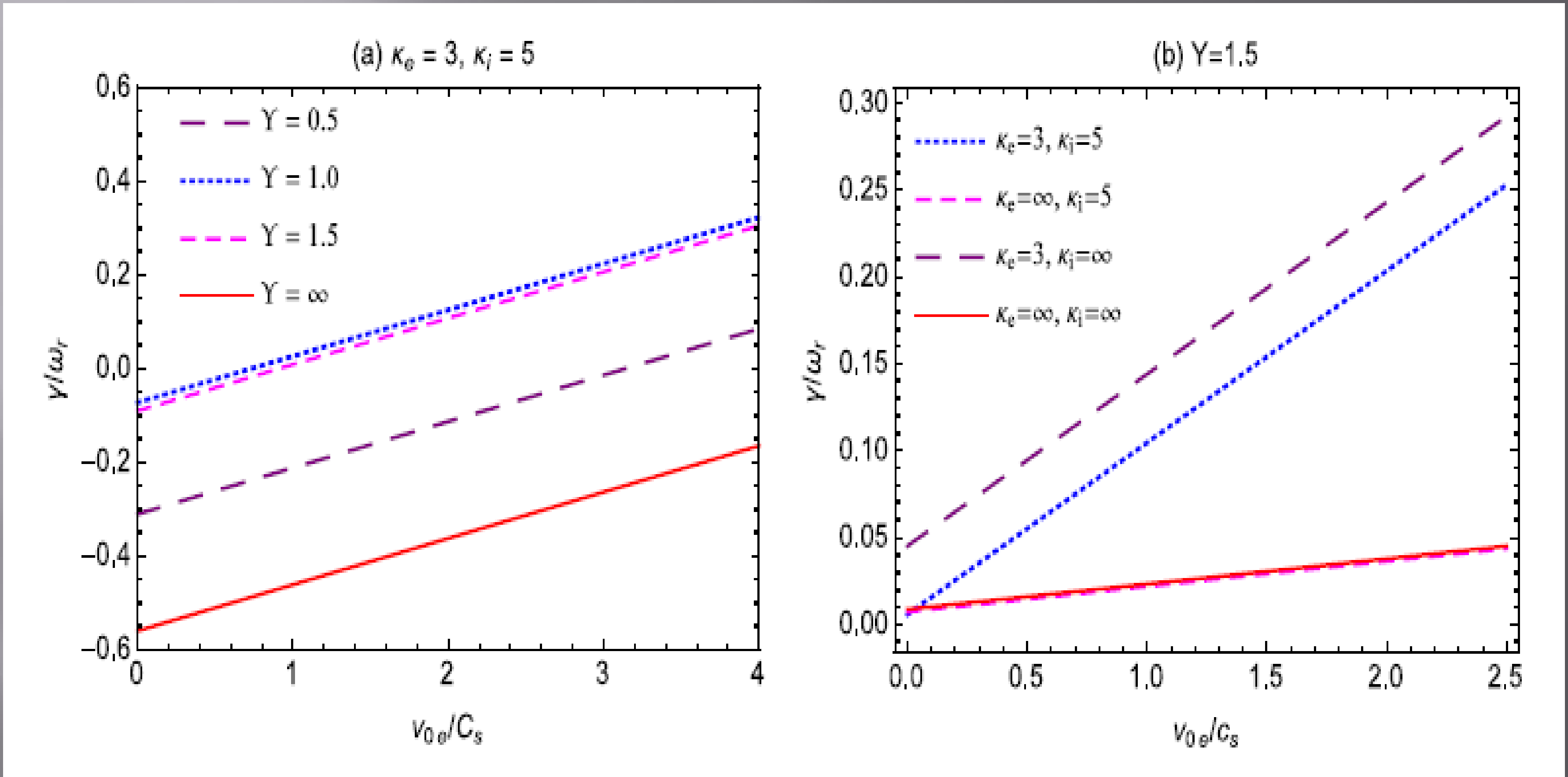
- The kinetic theory of dust ion acoustic twisted mode (DIATM) and dust acoustic twisted mode (DATM) is taken in consideration.
- The distribution of electrons are considered to be isotropic drifted kappa distribution while other species (i.e., ions and dust) follow isotropic kappa distribution.
- The drift of the electrons act as the source of the instability in DIATM and DATM.

Dielectric Function of DIATM and DATM

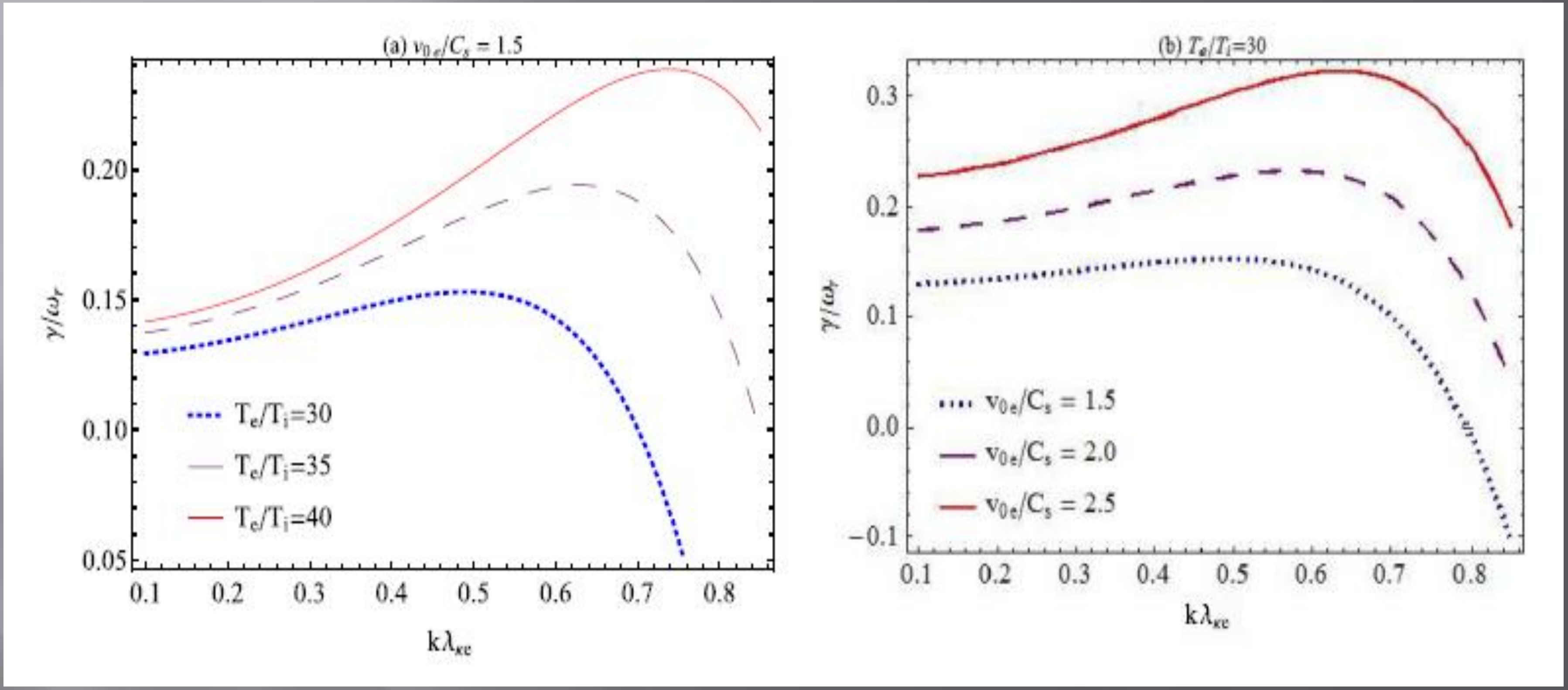
$$\epsilon(\omega,k,lq_\theta)=1+\frac{2\omega_{pe}^2}{k^2\theta_e^2}\left[\frac{2\kappa_e-1}{\kappa_e}+\xi_{z_e}Z(\xi_{z_e})+\xi_{\theta_e}Z(\xi_{\theta_e})\right]$$
$$+\frac{2\omega_{p_i}^2}{k^2\theta_i^2}\times\left[\frac{2\kappa_i-1}{\kappa_i}+\xi_{z_i}Z(\xi_{z_i})+\xi_{\theta_i}Z(\xi_{\theta_i})\right],$$

$$\epsilon(\omega,k,lq_\theta)=1+\frac{2\omega_{pe}^2}{k^2\theta_e^2}\left[\frac{2\kappa_e-1}{\kappa_e}+\xi_{z_e}Z(\xi_{z_e})+\xi_{\theta_e}Z(\xi_{\theta_e})\right]$$
$$+\frac{2\omega_{pi}^2}{k^2\theta_i^2}\left[\frac{2\kappa_i-1}{\kappa_i}+\xi_{z_i}Z(\xi_{z_i})+\xi_{\theta_i}Z(\xi_{\theta_i})\right]$$
$$+\frac{2\omega_{pd}^2}{k^2\theta_d^2}\left[\frac{2\kappa_d-1}{\kappa_d}+\xi_{z_d}Z(\xi_{z_d})+\xi_{\theta_d}Z(\xi_{\theta_d})\right]$$

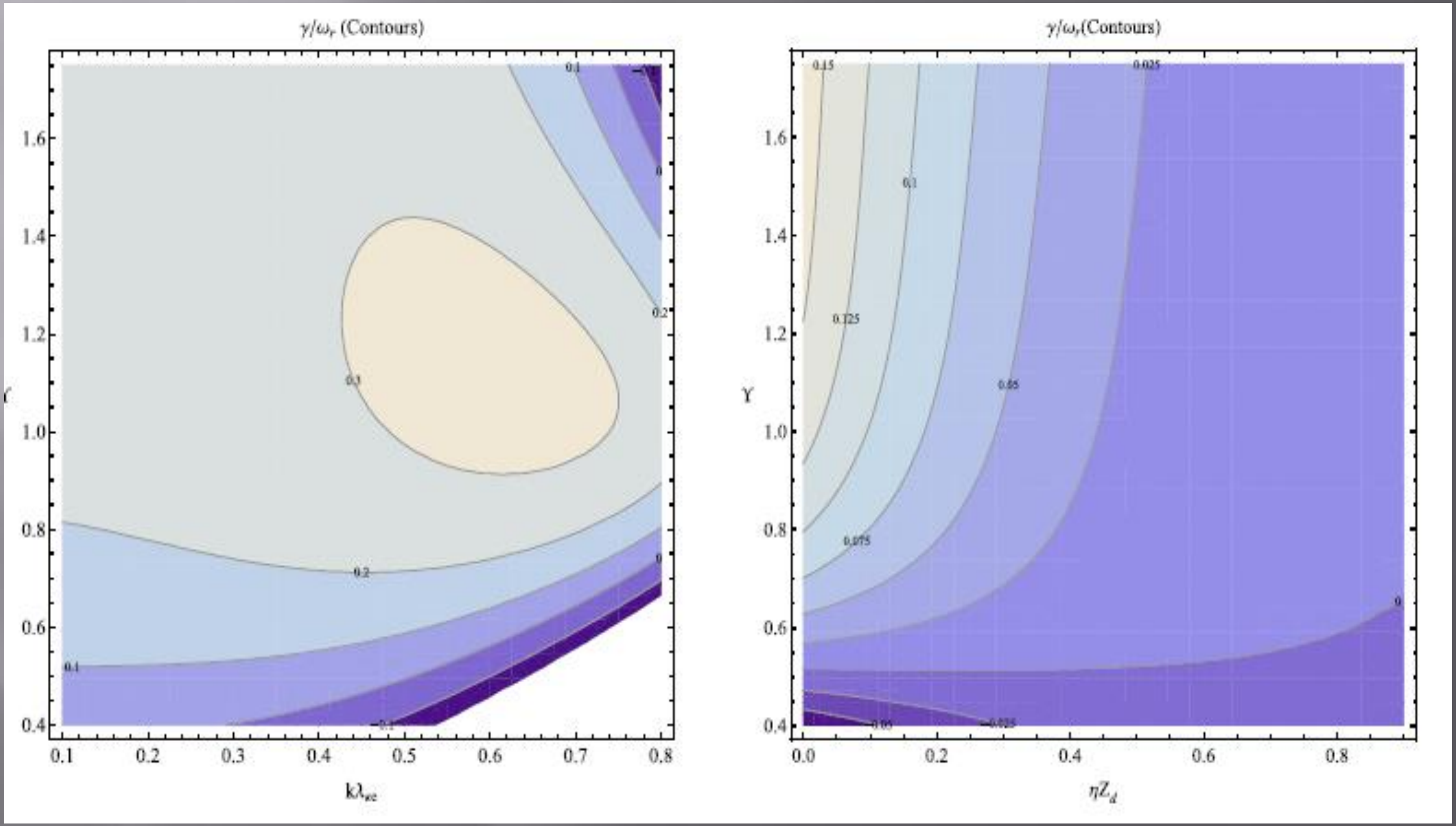
Growth rates of DIATM



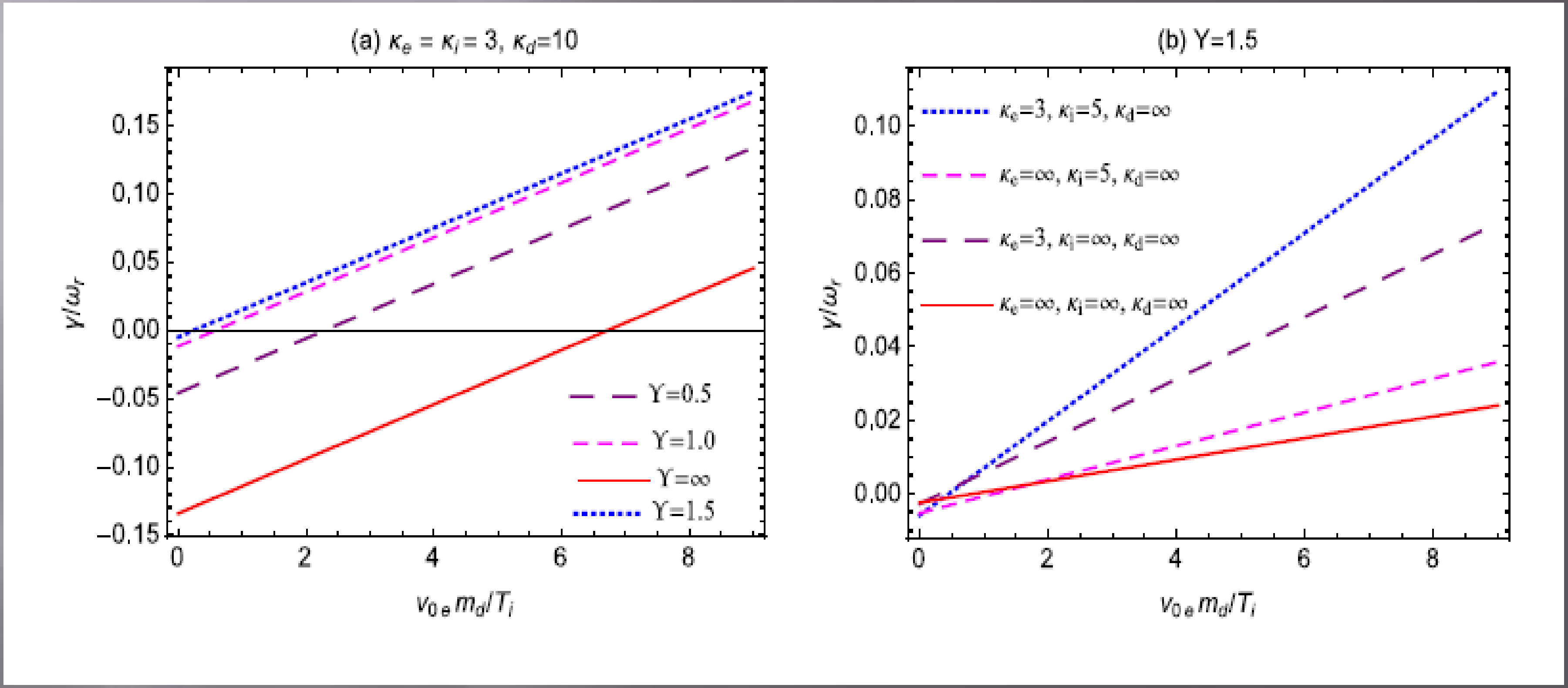
Growth rates of DIATM



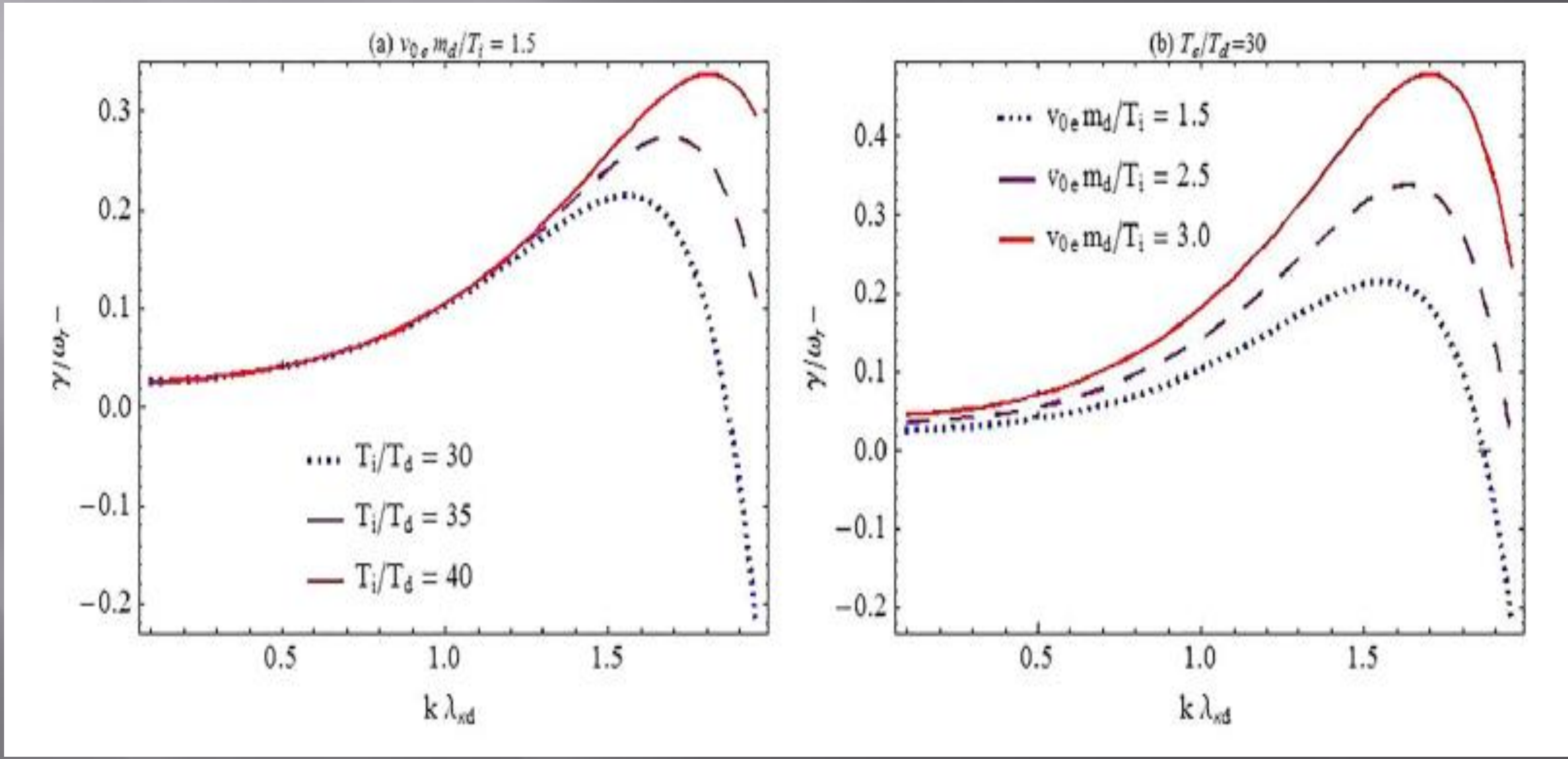
Growth rates of DIATM



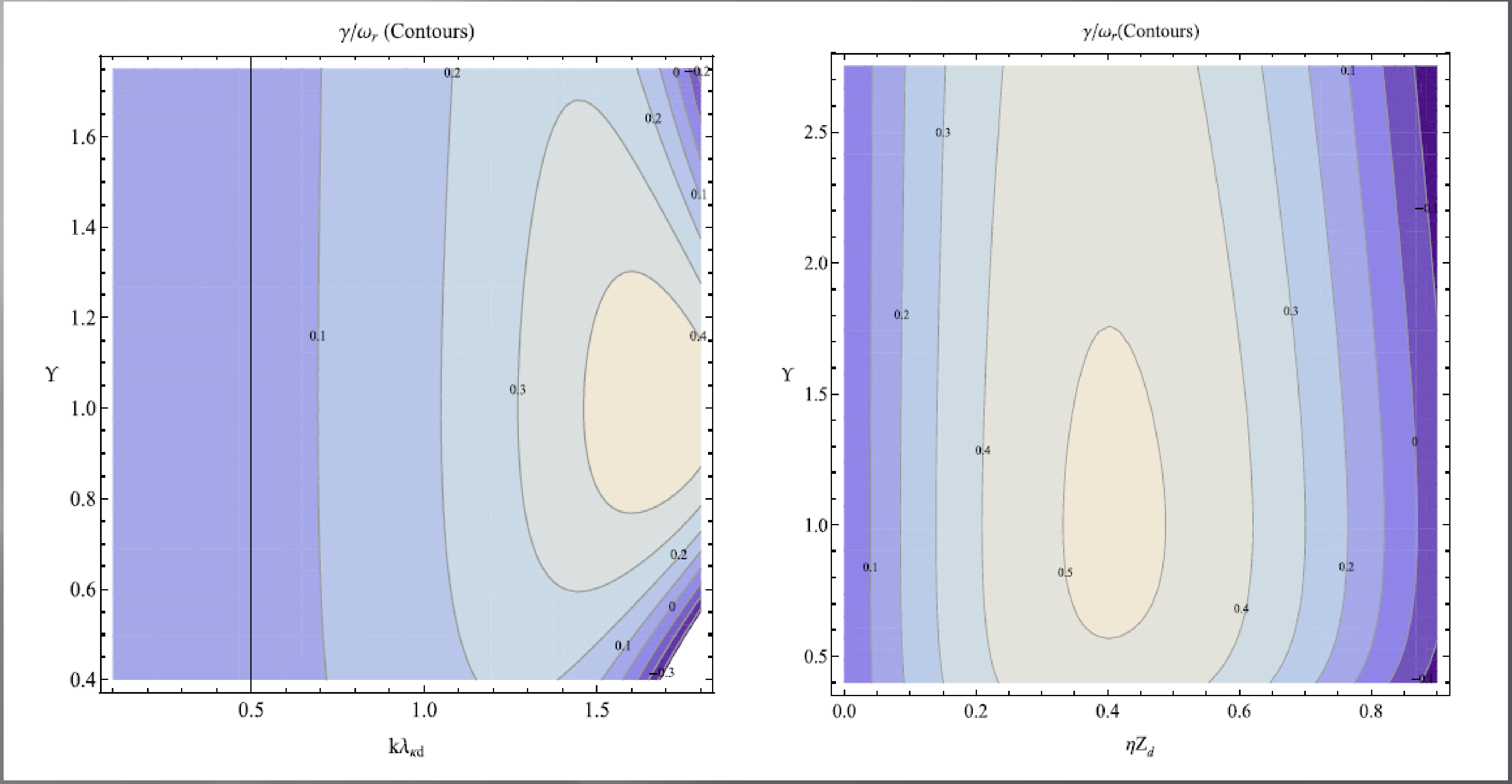
Growth rates of DATM



Growth rates of DATM



Growth rates of DATM



Conclusion

- This is a new advance kinetic modeling of space plasma in the presence of helical electric field carrying orbital angular momentum.

- We have studied the modes like Langmuir, ion acoustic, dust ion acoustic twisted modes and dust acoustic twisted modes in unmagnetized plasma: damping and growth rates are highly modified in the presence of orbital angular momentum.

- We have started to investigate the relevant modes in magnetized plasmas typically encounter in corona, solar wind and planetary magnetosphere, and preliminary results show similar major modifications.

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