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Parallel Program Systems for the Analysis of Wave Processes in Elastic-Plastic, Granular, Porous and Multi-Blocky Media

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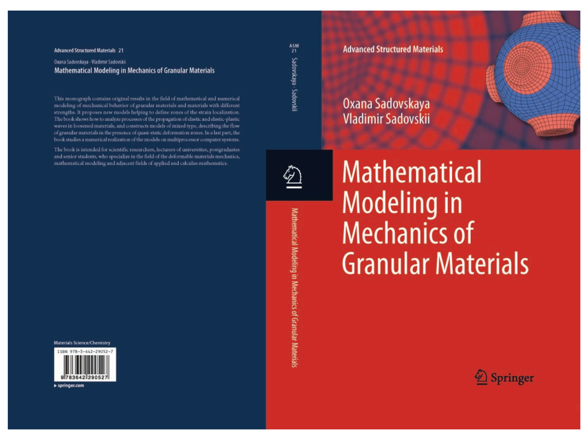
Constitutive relationships of a rigid contact

Springer, 2012



http://link.springer.com/book/10.1007/978-3-642-29053-4

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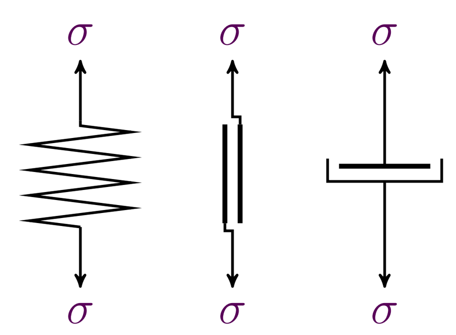
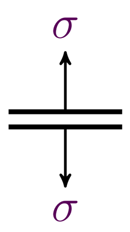


$$\sigma \leqslant 0, \quad \varepsilon \geqslant 0, \quad \sigma\,\varepsilon = 0 \quad \Leftrightarrow \quad \begin{cases} (\tilde\sigma - \sigma)\,\varepsilon \leqslant 0 & \sigma,\ \tilde\sigma \leqslant 0 \\ (\tilde\varepsilon - \varepsilon)\,\sigma \leqslant 0 & \varepsilon,\ \tilde\varepsilon \geqslant 0 \end{cases}$$



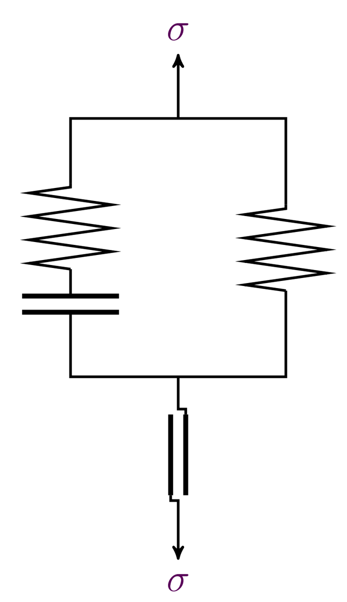
$$(\tilde{\boldsymbol\sigma} - \boldsymbol\sigma) : \boldsymbol\varepsilon \leqslant 0 \quad \boldsymbol\sigma,\ \tilde{\boldsymbol\sigma} \in \boldsymbol K \quad \Leftrightarrow \quad (\tilde{\boldsymbol\varepsilon} - \boldsymbol\varepsilon) : \boldsymbol\sigma \leqslant 0 \quad \boldsymbol\varepsilon,\ \tilde{\boldsymbol\varepsilon} \in \boldsymbol C$$

$$\boldsymbol K = \{\boldsymbol\sigma \mid \boldsymbol\sigma : \boldsymbol\varepsilon \leqslant 0 \quad \forall\, \boldsymbol\varepsilon \in \boldsymbol C\} \quad \Leftrightarrow \quad \boldsymbol C = \{\boldsymbol\varepsilon \mid \boldsymbol\sigma : \boldsymbol\varepsilon \leqslant 0 \quad \forall\, \boldsymbol\varepsilon \in \boldsymbol K\}$$

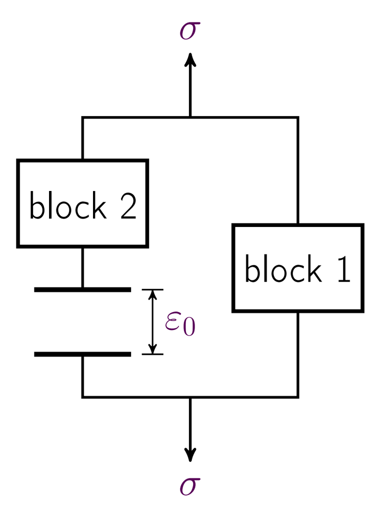
Rheological schemes

To make possible the constructing constitutive relationships of materials with different resistance to tension and compression we have completed the rheological method by a new element – rigid contact, which simulates an ideal granular material with rigid particles.

Granular medium



Porous medium



Mathematical model of a granular medium

Strain tensor $\quad \varepsilon = \varepsilon^e + \varepsilon^c + \varepsilon^p$

The inequality of Haar and Karman

$$(\tilde\sigma - \sigma) : \big(a : \sigma - \varepsilon^e - \varepsilon^c\big) \geqslant 0, \qquad \sigma,\ \tilde\sigma \in K$$

The Mises inequality $\quad (\tilde\sigma - \sigma) : \varepsilon^p \leqslant 0, \qquad \sigma,\ \tilde\sigma \in F$

Equations of motion $\quad \rho\,\dot v = \nabla\cdot\sigma + \rho\,g$

Kinematic equations $\quad 2\,\dot\varepsilon = \nabla v + (\nabla v)^*$

The set F of admissible variations is defined by the Mises yield condition.

As a convex cone K of stresses, allowed by the strength criterion, the Mises–Schleicher circular cone is used.

Mathematical model of a porous medium

Complete system of equations

$$\rho\,\dot v = \nabla\cdot\sigma + f$$
$$(\tilde s - s) : (a : \dot s - \nabla v) \geqslant 0, \qquad \tilde s,\ s \in F$$
$$b : \dot q = \frac{1}{2}(\nabla v + \nabla v^*), \qquad \sigma = s + \pi_K(q + q^0)$$

vector v, tensors s and q – unknown functions in this model

Constitutive relationships for dynamic problems of the theory of elasticity and plasticity

The process of deformation of elastic bodies is described by the system of equations

$$A\,\frac{\partial U}{\partial t} = \sum_{i=1}^{n} B^i\,\frac{\partial U}{\partial x_i} + Q\,U + G$$

When taking into account the plastic deformation of a material, the system is replaced by the variational inequality

$$(\tilde U - U)\Big(A\,\frac{\partial U}{\partial t} - \sum_{i=1}^{n} B^i\,\frac{\partial U}{\partial x_i} - Q\,U - G\Big) \geqslant 0, \qquad \tilde U,\ U \in F$$

$U(t,x)$ – m-dimensional unknown vector–function, $\tilde U$ – varied vector

A – symmetric positive definite matrix of coefficients under time derivatives

B^i – symmetric matrices of coefficients under derivatives with respect to spatial variables

Q – antisymmetric matrix, G – given vector

F – convex and closed set, determined by the criterion of plasticity

Constitutive relationships for problems of the dynamics of granular materials

In the problems of mechanics of granular media with plastic properties a more general variational inequality takes place

$$(\tilde V - V)\Big(A\,\frac{\partial U}{\partial t} - \sum_{i=1}^{n} B^i\,\frac{\partial V}{\partial x_i} - Q\,V - G\Big) \geqslant 0, \qquad \tilde V,\ V \in F$$

Vector–functions V and U are related by the equations

$$V = \varsigma\,U + (1-\varsigma)\,U^\pi, \qquad U = \frac{1}{\varsigma}\,V - \frac{1-\varsigma}{\varsigma}\,V^\pi$$

U^π – projection of the vector of solution onto the given convex cone K, by means of which different resistance of a material to tension and compression is described

$\varsigma \in (0,1]$ – parameter of regularization of the model characterizing the ratio of elastic moduli in tension and compression

Components of the vector of solution

In the model of elastic-plastic deformation

$U = \big(v_1, v_2, \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}\big)$ – two-dimensional case

$U = \big(v_1, v_2, v_3, \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\big)$ – three-dimensional case

When taking into account the rotational degrees of freedom of particles in the microstructure of a material in the Cosserat continuum model

$U = \big(v_1, v_2, \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{21}, \omega_3, m_{23}, m_{32}, m_{31}, m_{13}\big)$ – two-dimensional case

$U = \big(v_1, v_2, v_3, \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{32}, \sigma_{31}, \sigma_{13}, \sigma_{12}, \sigma_{21},$
$\omega_1, \omega_2, \omega_3, m_{11}, m_{22}, m_{33}, m_{23}, m_{32}, m_{31}, m_{13}, m_{12}, m_{21}\big)$ – three-dimensional case

v_i – components of the velocity vector, ω_i – components of the angular velocity vector

σ_{ij} – components of the stress tensor, m_{ij} – components of the moment stress tensor

In the most general model of a granular medium with different resistance to tension and compression, which takes into account rotation of the particles, the vector–function U contains components of v, s, ω and n, and the vector–function V is obtained from U by replacing the components of tensors s and n (conditional stress and couple stress tensors) onto σ and m

Mathematical model of the Cosserat continuum

Complete system of equations consists of the equations of translational and rotational motion, the kinematic relationships and the constitutive laws

$$\rho\,\dot v = \nabla\cdot\sigma + \rho\,g, \quad j\,\dot\omega = \nabla\cdot m - 2\,\sigma^a + j\,q$$
$$\dot\Lambda = \nabla v + \omega, \quad \dot M = \nabla\omega$$
$$\sigma = \lambda\,(\delta : \Lambda^s)\,\delta + 2\,\mu\,\Lambda^s + 2\,\alpha\,\Lambda^a$$
$$m = \beta\,(\delta : M^s)\,\delta + 2\,\gamma\,M^s + 2\,\eta\,M^a$$

The conditions of hyperbolicity $\quad 3\,\lambda + 2\,\mu > 0,\ \mu,\ \alpha > 0; \quad 3\,\beta + 2\,\gamma > 0,\ \gamma,\ \eta > 0$

Velocities of shock waves

$$c_p = \sqrt{\frac{\lambda + 2\,\mu}{\rho}}, \quad c_s = \sqrt{\frac{\mu + \alpha}{\rho}}, \quad c_m = \sqrt{\frac{\beta + 2\,\gamma}{j}}, \quad c_\omega = \sqrt{\frac{\gamma + \eta}{j}}$$

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- Sadovskii V.M., Sadovskaya O.V. **Analyzing the deformation of a porous medium with account for the collapse of pores**, *J. Applied Mechanics and Technical Physics*, 2016. V. 57, Iss. 5. P. 808–818.

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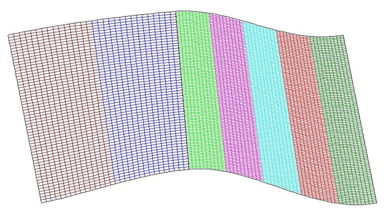
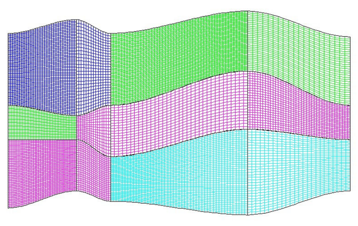
Uniform distribution of computational load

Blocky computational domain with curvilinear interfaces

2D case

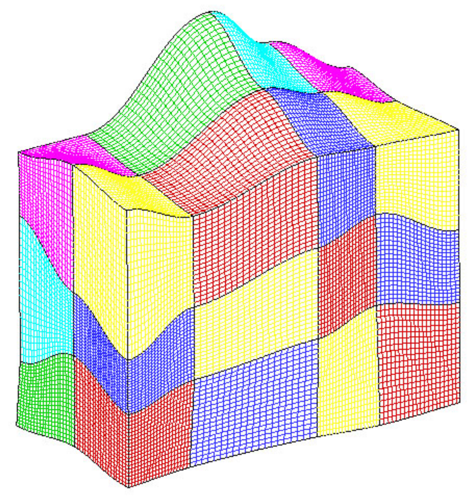


Decomposition of a medium body, consisting of 2 blocks, between 7 parallel processes



Decomposition of a medium body, consisting of 12 blocks, between 4 processes

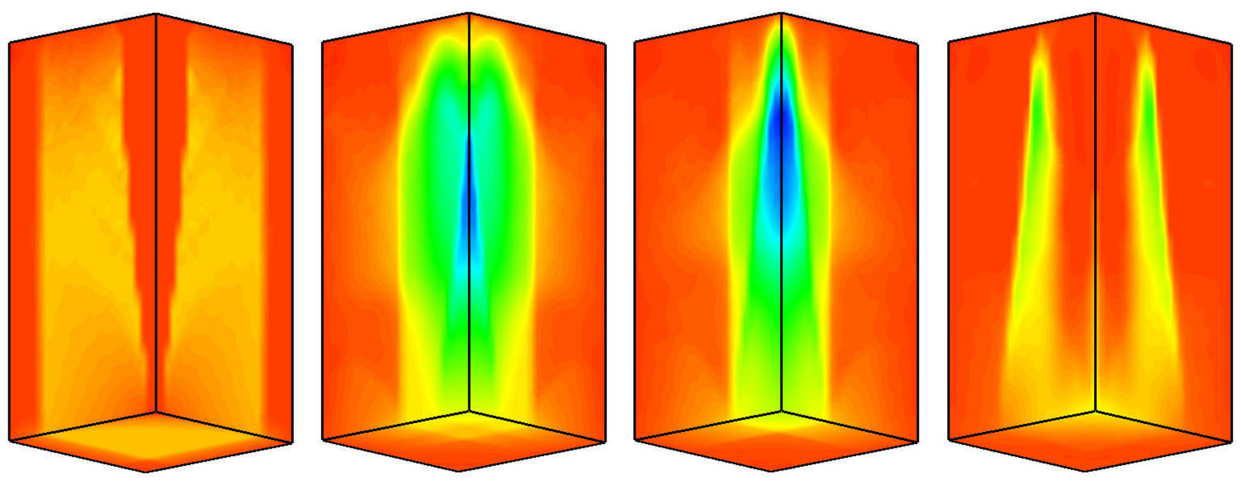
3D case



Decomposition of a medium body, consisting of 24 blocks, between 24 processes

Wave propagation in a loosened granular material

Cumulative interaction of signotons



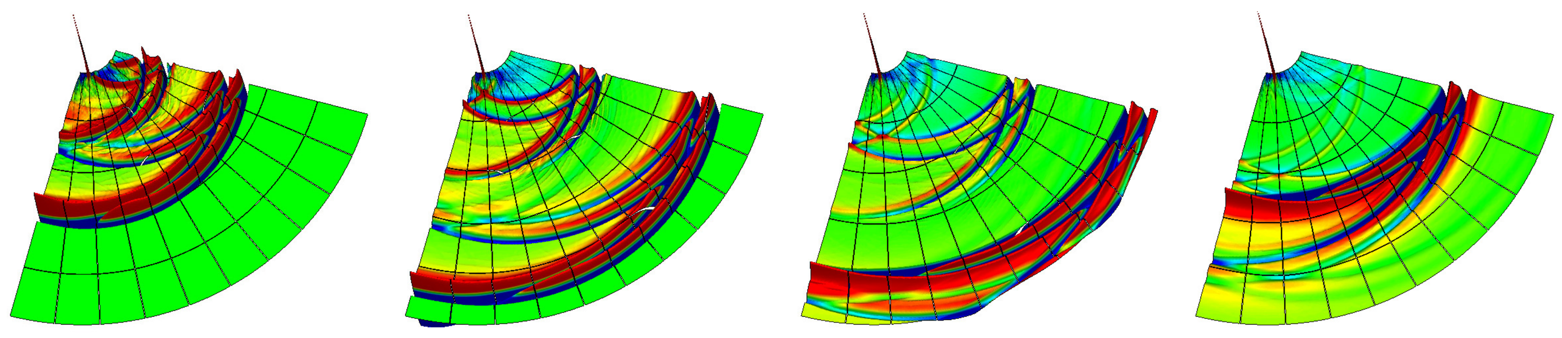
Level surfaces of the stress σ_{11} (different time moments)

Numerical solution of 3D problem of waves propagation in inhomogeneously loosened medium.
A quarter of rectangular block that is dissected by two planes of symmetry along the vertical axis is presented.
The upper boundary is free of stresses, the lower one is a non-reflecting interface, at the lateral faces uniformly distributed impulsive load acts. Due to the curvature of wavefronts a cumulative splash appears at the point of interaction.

Computations were performed for a compact ground.

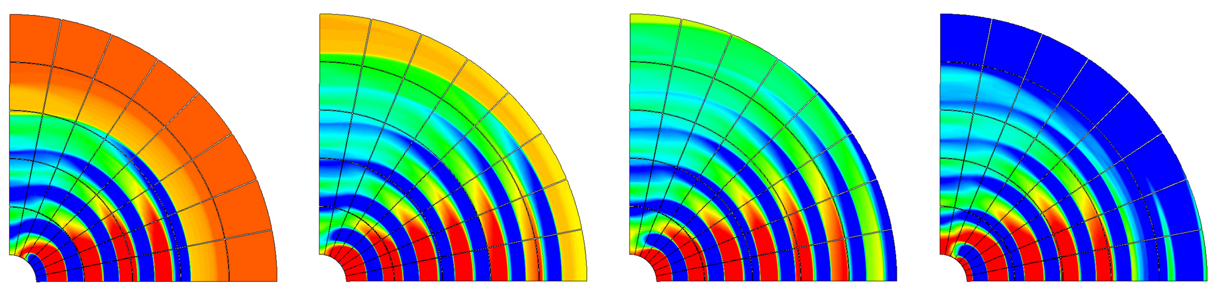
Wave propagation in a porous material

Cylindrical pipe expansion



Concentrated impulsive load (Lamb's problem): fields of radial stress

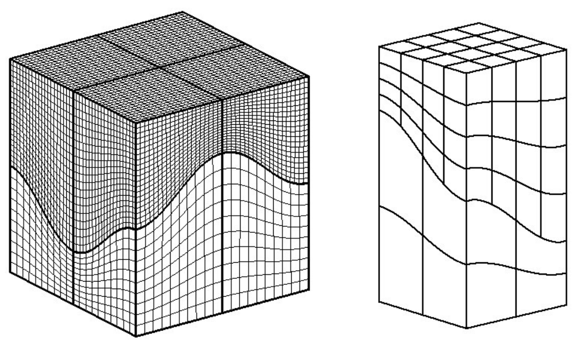
50 processors, 500 × 500 nodes



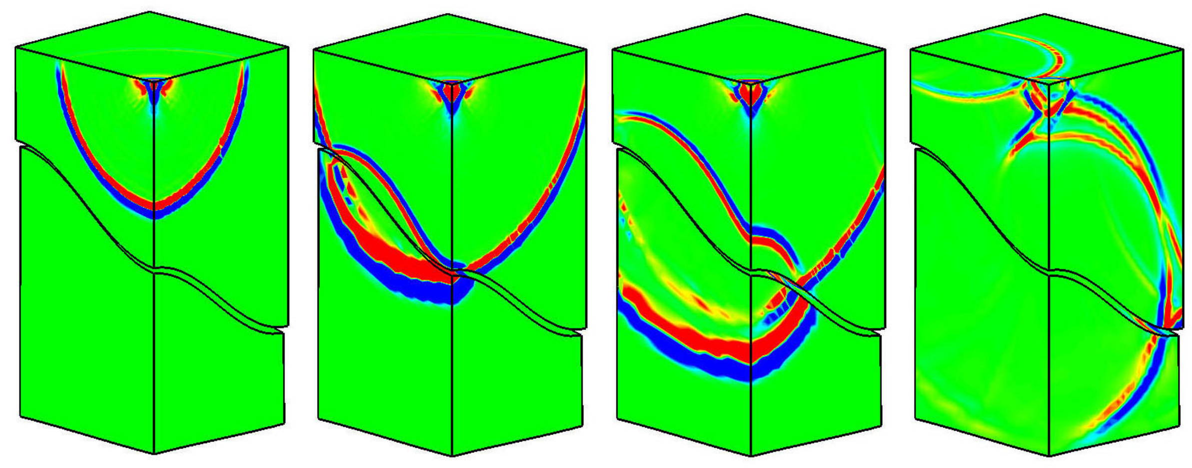
Periodic localized load: level curves of volumetric strain

40 processors, 400 × 400 nodes

Lamb's problem for two-layered medium



Grid and a quarter of computational domain with decomposition on processes

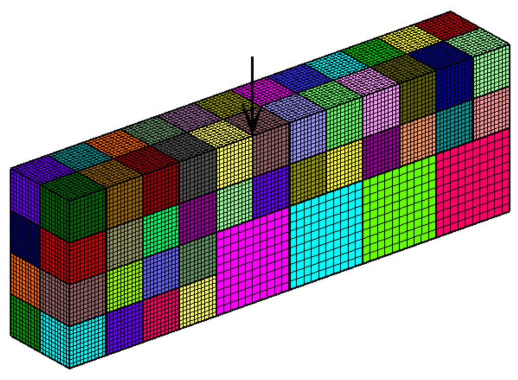


Level surfaces of the stress σ_{11} (different time moments)

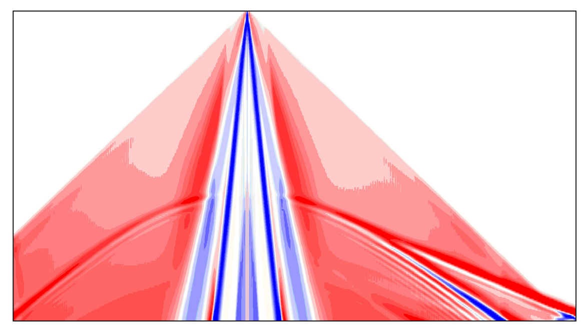
The elastic medium body consists of two heterogeneous layers, separated by curvilinear interface.
Upper layer is a compact ground, lower layer is a strong rock.

8 blocks, taking into account the symmetry of problem the computations are carried out only on a quarter of domain (2 blocks)
72 processors: 64 – in upper block, 8 – in lower block, grid dimension for each processor is $50 \times 50 \times 50$ nodes

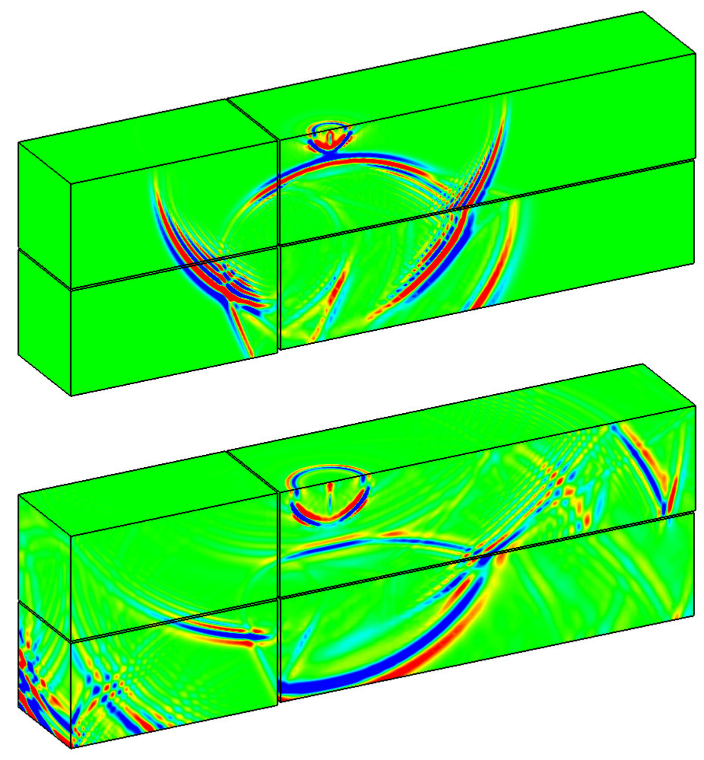
Lamb's problem for a medium with rigid inclusion



Distribution of computational domain between processors



Seismogram of the displacement u_1

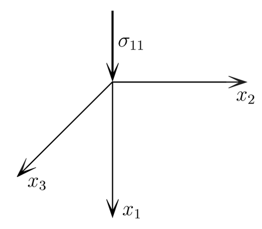


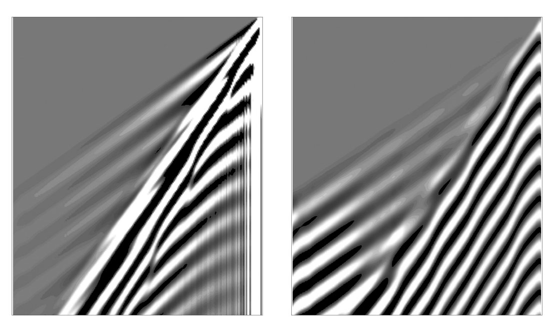
Level surfaces of the stress σ_{11}

The elastic medium body consists of two layers. Elasticity parameters of a compact ground are defined in upper layer and in part of lower layer, parameters of a strong rock are defined in remaining part of lower layer.

4 blocks, 68 processors: 64 – in a compact ground, 4 – in a strong rock, grid dimension for each processor is $50 \times 50 \times 50$ nodes

Natural resonance of the Cosserat medium



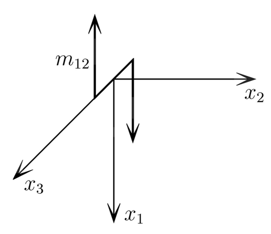


Seismograms of incident waves: impulsive loading (left) and periodic loading (right)

$\sigma_{11} = -p_1^* \delta(x)\, \sin(2\pi\nu t)$

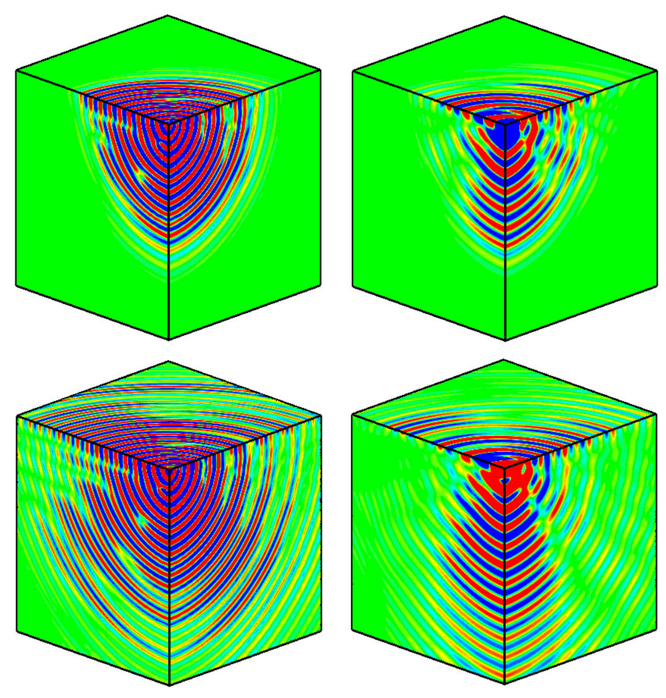
synthetic polyurethane

64 processors



$m_{12} = -q_2^* \delta(x)\, \sin(2\pi\nu t)$

Periodic loading: level surfaces of angular velocity ω_2 for nonresonance frequency (left) and resonance frequency (right)



$\nu = 1.5\,\nu_*$ $\qquad$ $\nu = \nu_* = 1/T$

Parallel program systems for the solution of two-dimensional and three-dimensional elastic-plastic problems of the dynamics of granular media

Certificates of state registration of computer programs No. 2012613989 and No. 2012613990 from 28.04.2012 (Rospatent)

Programs: 2Dyn_Granular, 3Dyn_Granular

Parallel program systems for the solution of two-dimensional and three-dimensional dynamic problems of the Cosserat elasticity theory

Certificates of state registration of computer programs No. 2012614823 and No. 2012614824 from 30.05.2012 (Rospatent)

Programs: 2Dyn_Cosserat, 3Dyn_Cosserat

Parallel program system for numerical modeling of dynamic processes in multi-blocky media on cluster systems

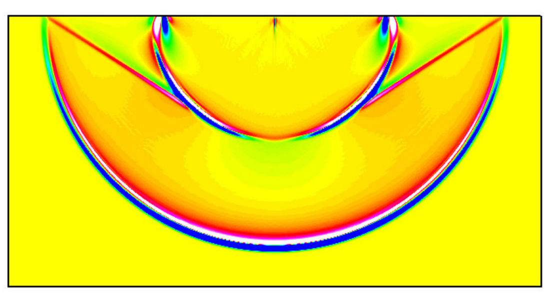
Certificate of state registration of computer program No. 2016615178 from 17.05.2016 (Rospatent)

Program 2Dyn_Blocks_MPI

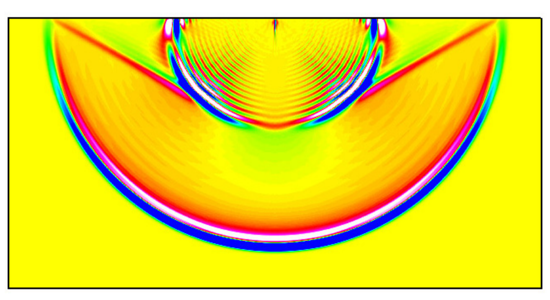
Comparison of numerical results in 2D case

Numerical results for 2D Lamb's problem on the normal action of a concentrated impulsive load on the boundary of an elastic block (programs **2Dyn_Granular and 2Dyn_Cosserat**)

Level curves of the normal stress σ_{11}



homogeneous elastic medium (classical theory of elasticity)



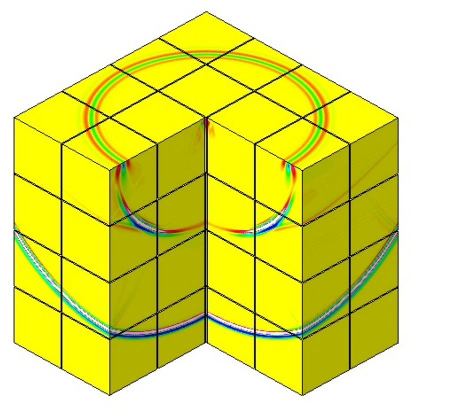
the Cosserat medium

10 processors, 1000 × 500 nodes

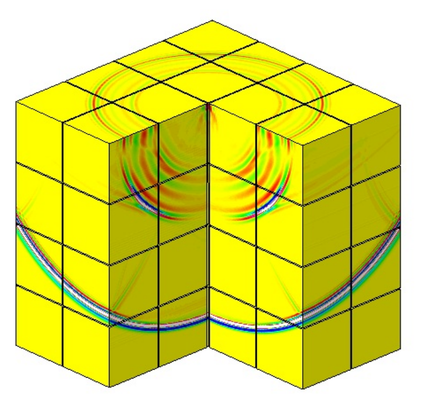
Comparison of numerical results in 3D case

3D Lamb's problem (programs **3Dyn_Granular and 3Dyn_Cosserat**)

Level surfaces of the normal stress σ_{11}



homogeneous elastic medium (classical theory of elasticity)



the Cosserat medium

64 processors, 400 × 400 × 400 nodes