A proof of concept for scale-adaptive parameterizations: the case of the Lorenz '96 model

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Wouters-Lucarini Parameterization

Double dynamical system:

$$\frac{dX}{dt} = F_X(X) + \Psi_X(X,Y)$$

$$\frac{dY}{dt} = F_Y(Y) + \Psi_Y(X,Y)$$

- Coupling interpreted as a weak perturbation
- Application of Ruelle's response theory

Wouters, J. and Lucarini, V. (2012). Disentangling multi-level systems: averaging, correlations and memory. Journal of Statistical Mechanics: Theory and Experiment, 2012(03):P03003.

Wouters-Lucarini Parametrization

$$\frac{dX}{dt} = F_X(X) + D(X) + S(X) + M(X)$$

Second order parameterization:

- Deterministic term
- Stochastic term
- Memory term

Specific case:
Independent coupling

$$\frac{dX}{dt} = F_X(X) + \Psi_X(Y)$$

$$\frac{dY}{dt} = F_Y(Y) + \Psi_Y(X)$$

Deterministic term

$$D(X(t)) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \Psi_X(Y(t))dt$$

Stochastic term

$$\langle \sigma(t) \rangle = 0$$

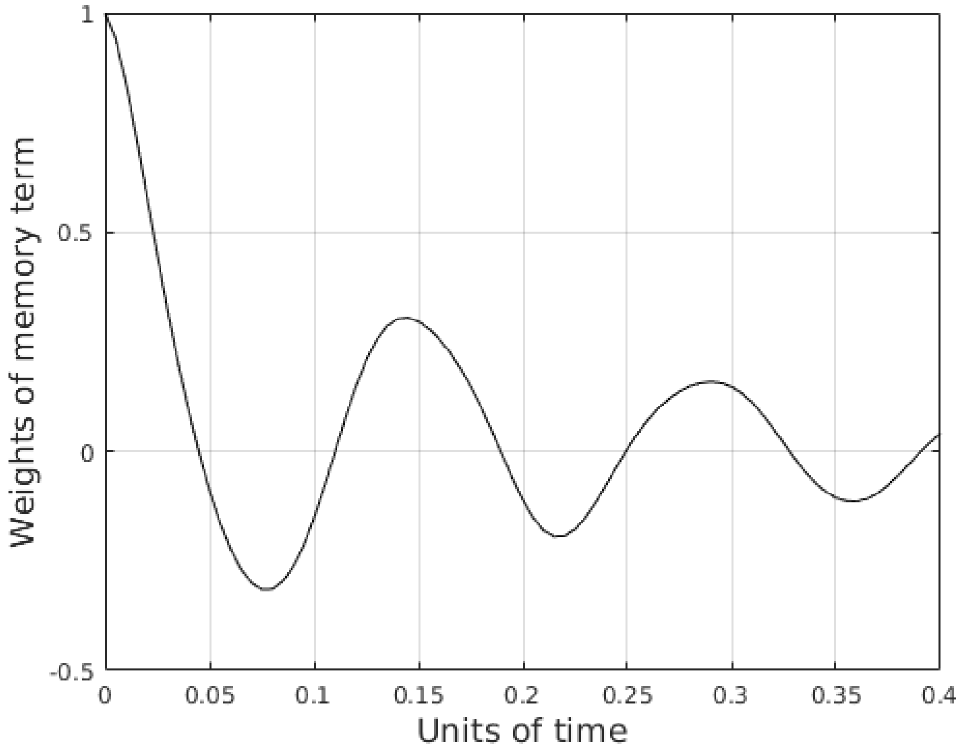
$$\langle \sigma(t_1)\sigma(t_2) \rangle = \langle [\Psi_X(Y) - D][\Psi_X(f^{t_2-t_1}(Y)) - D] \rangle_{\rho_0}$$

Memory term

$$M(X(t)) = \int_0^T d\tau \Psi_Y(X(t-\tau)) \lim_{T\to\infty} \frac{1}{T} \int_0^T \partial_Y \Psi_X(f^{\tau}(Y(t)))dt$$

Memory term

$$M(X(t)) = \int_0^T d\tau \Psi_Y(X(t-\tau)) \lim_{T\to\infty} \frac{1}{T} \int_0^T \partial_Y \Psi_X(f^\tau(Y(t)))dt$$



Modified Lorenz 96 model

$$\frac{dX_k}{dt} = -X_{k-1}(X_{k-2} - X_{k+1}) - X_k + F_1 - \frac{hc}{b}\sum_{j=1}^{J} Y_{j,k}$$

$$\frac{dY_{j,k}}{dt} = -cbY_{j+1,k}\left(Y_{j+2,k} - Y_{j-1,k}\right) - cY_{j,k} + \frac{hc}{b}X_k + \frac{c}{b}F_2$$

- Added a forcing in the second equation
- c: relative rapidity of the fluctuations
- b: relative amplitude of the fluctuations
- h: coupling strength

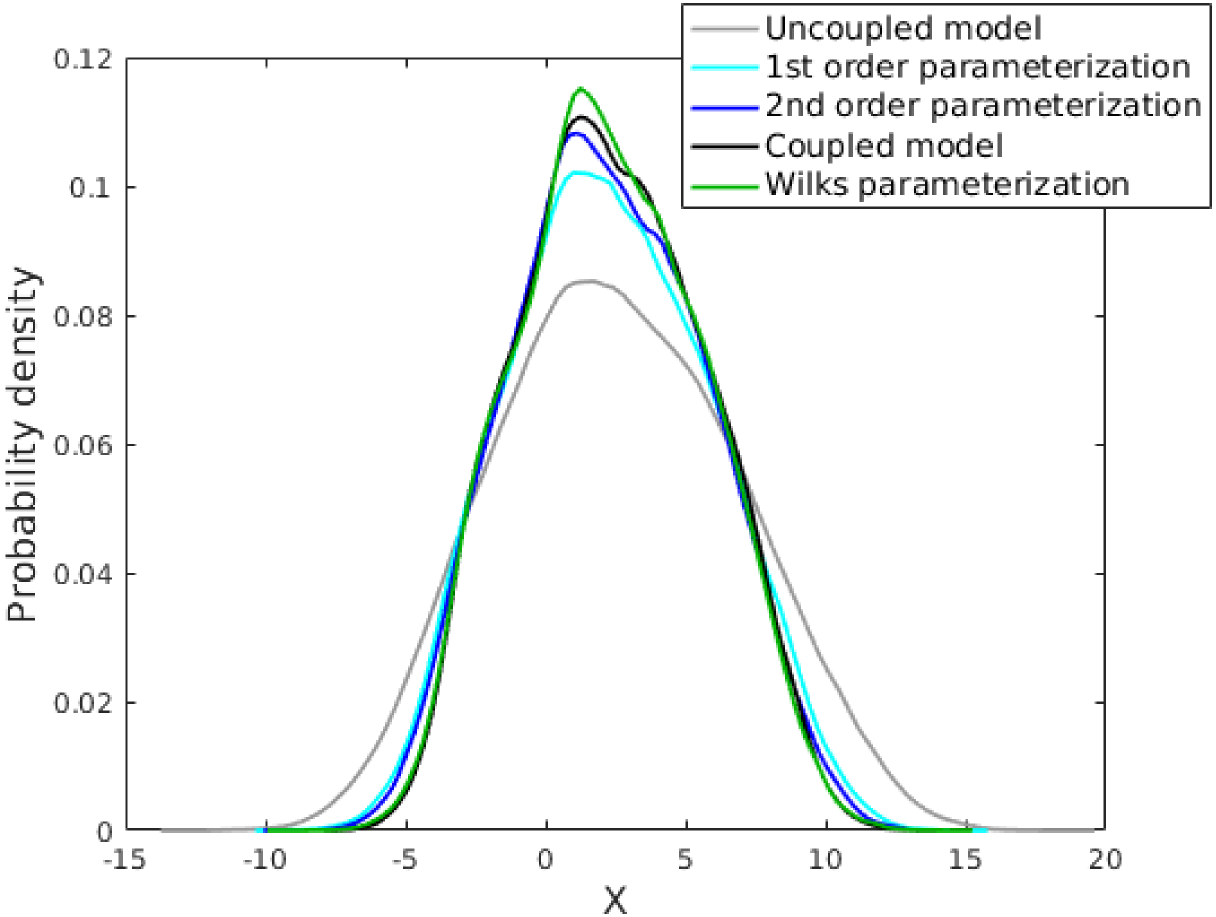
Lorenz, E. N. (1996). Predictability - a problem partly solved. In Palmer, T. and Hagedorn, R., editors, Predictability of Weather and Climate, pages 40-58. Cambridge University Press.

The statistical properties of the slow variable will be investigated to test the reliability of the approach.

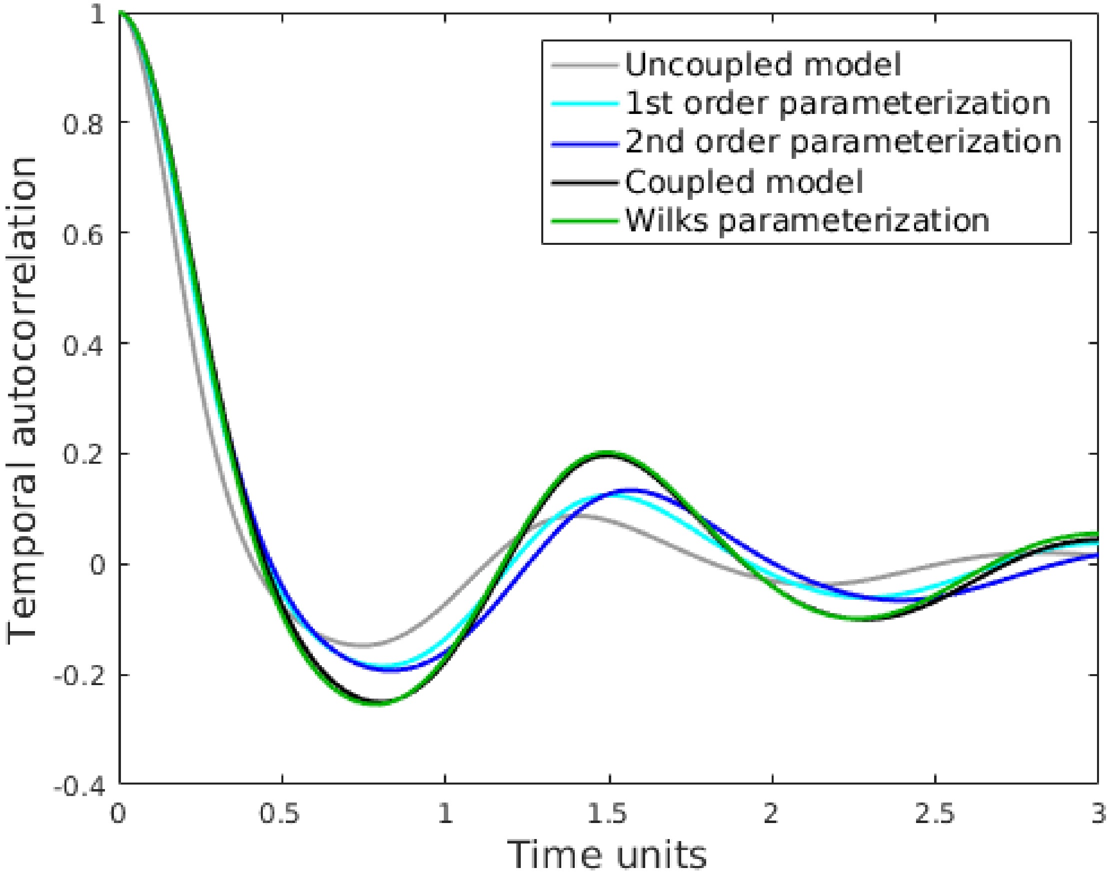
- Uncoupled model: X-dynamics without coupling
- Coupled model: full dynamics of Lorenz ‘96
- 1st order parameterization: WL with just deterministic term
- 2nd order parameterization: WL with deterministic, stochastic and memory terms
- Wilks parameterization: empirical parameterization composed by a fourth order polynomial and a markovian autoregressive term

Wilks, D. S. (2005). Effects of stochastic parametrizations in the Lorenz '96 system. Quarterly Journal of the Royal Meteorological Society, 131(606):389-407.

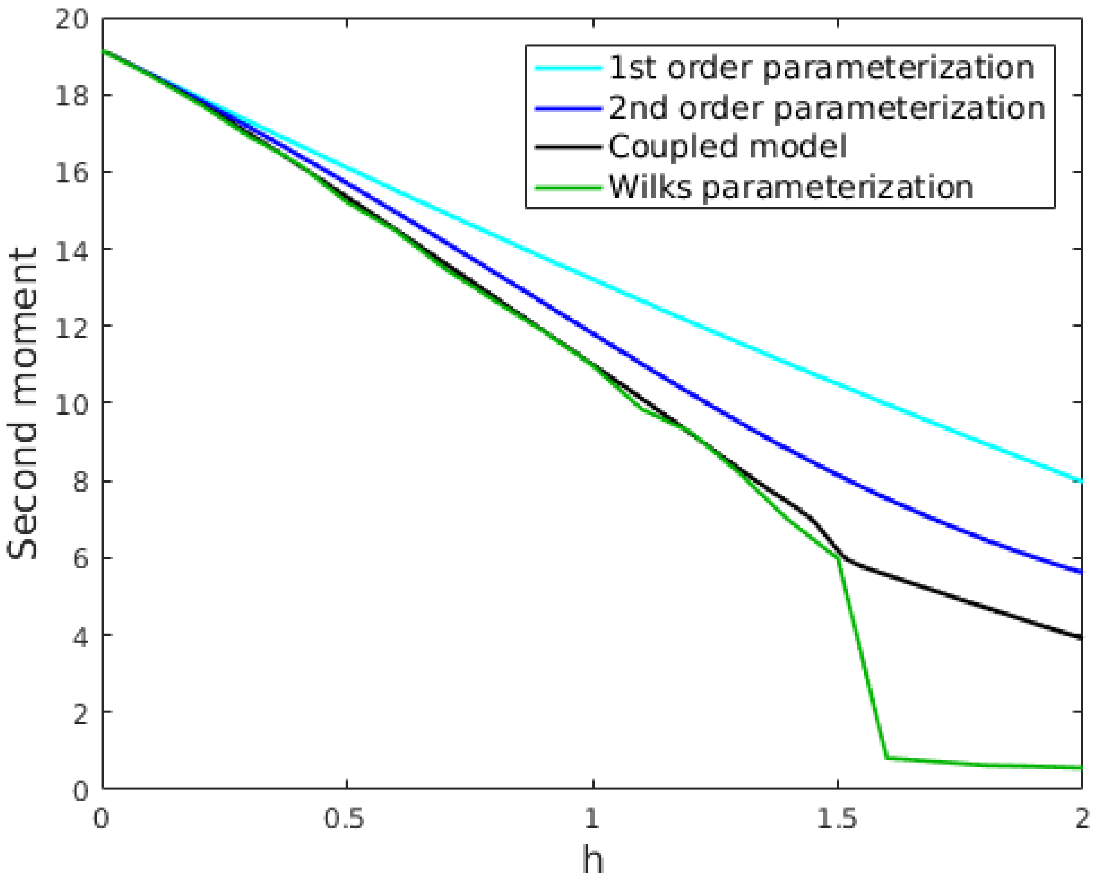
Probability density



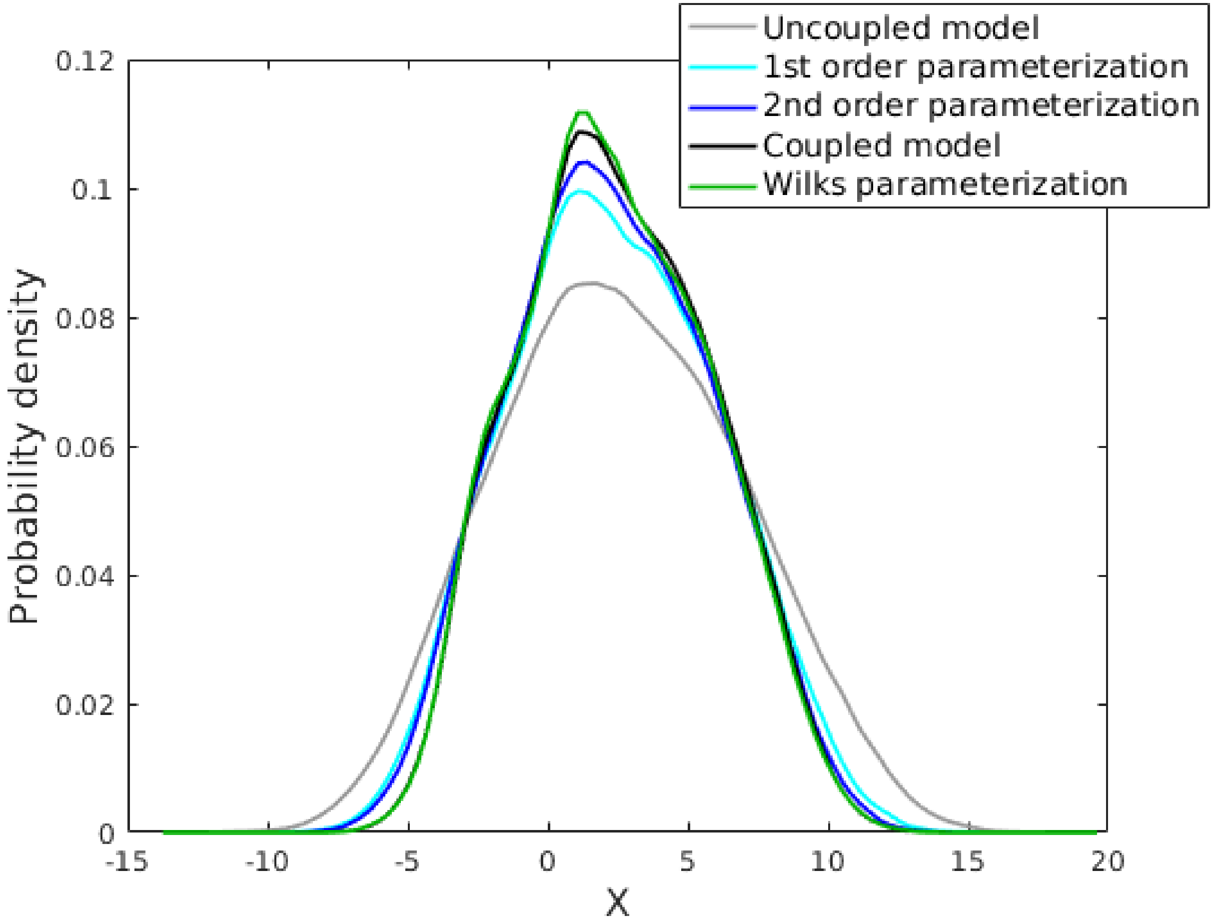
Temporal autocorrelation



Second centered moment vs coupling strength



Probability density



Conclusions

Computation of a stochastic parametrization with memory term and comparison with an existing empirical parametrization

- Good reproduction of the statistical properties of Lorenz 96 model
- Only one prerequisite: weak coupling
- Uncoupled equations needed for the calculation of the terms
- Scale adaptivity

G. Vissio and V. Lucarini, A proof of concept for scale-adaptive parameterizations: the case of the Lorenz '96 model, arXiv preprint arXiv: 1612.07223 (2016)