Bayesian inverse modeling for quantitative precipitation estimation

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Motivation

A key element of numerical weather prediction (NWP) is quantitative precipitation estimation. Polarimetric radar provides a wealth of information to this end.

- Exploit polarimetric observables to derive knowledge on underlying drop size distribution(DSD)

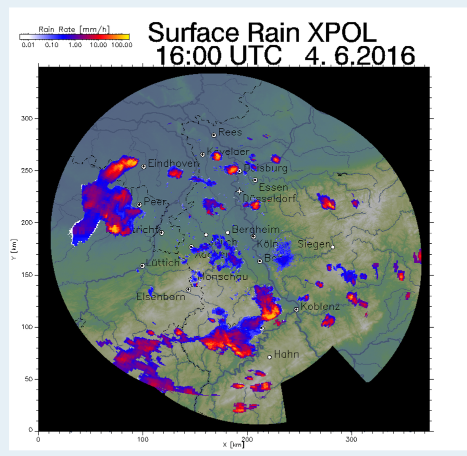
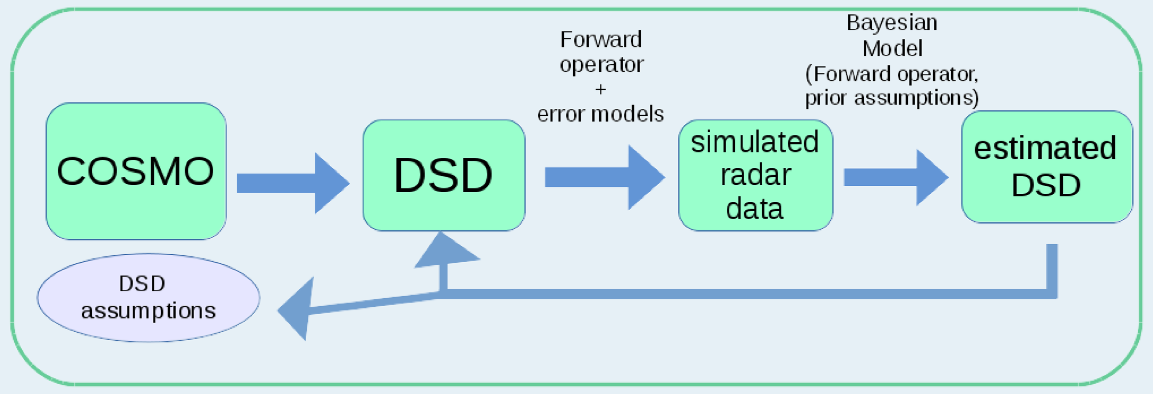


Fig. 1: Composite radar BoXPol and JuXPol.
www.meteo.uni-bonn.de/messdaten/

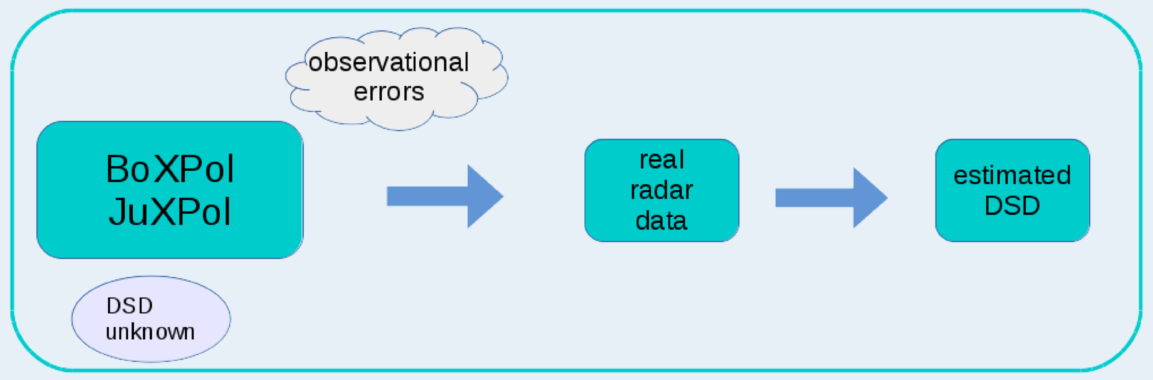
- Which further information on the system and its uncertainties can we gain from the estimation process?

Simulated environment

We apply the Bayesian model in a **simulated environment** based on the NWP-model COSMO-DE.



simulated environment



real world

- COSMO-DE provides q_{nr} $\left[\mathrm{kg}^{-1}\right]$(number concentration of raindrops), q_r $\left[\mathrm{kg}\cdot\mathrm{kg}^{-1}\right]$ (specific rain content)
- DSD parameters are reconstructed from q_{nr} and q_r, using forward operators we generate polarimetric observables
- Perturb data with random error models to simulate measurement error
- Estimate posterior distribution and compare against – known – parameters

Forward problem

- 15-minutely outputs of q_{nr}, q_r from the two-moment scheme
- **Three-parameter gamma DSD**:

$$N\left(\vec{x},\vec{t},D\right) = N_0\left(\vec{x},\vec{t}\right) D^{\mu\left(\vec{x},\vec{t}\right)} \exp\left(-\Lambda\left(\vec{x},\vec{t}\right)D\right) \qquad \left[\mathrm{mm}^{-\mu(\vec{x},\vec{t})-1}\ \mathrm{m}^{-3}\right]$$

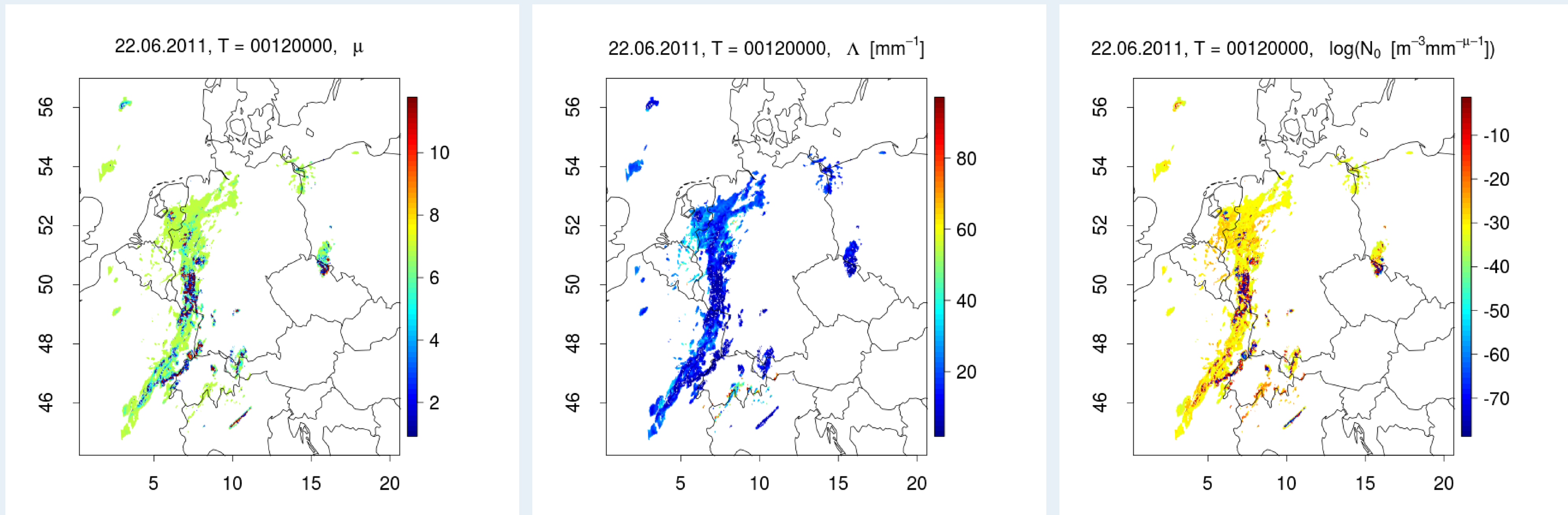


Fig. 3: Retrieved gamma DSD parameters.

- Rain rate and radar observables can be computed from the DSD
- Polarimetric radar gives information on particle orientation/shape and hydrometeor type
- **Polarimetric radar observables**:

$$Z_h\left(\vec{x},\vec{t}\right) = \frac{4\lambda^4}{\pi^4|K_w|^2}\int_{D_{min}}^{D_{max}} \left|f_{HH}\left(\pi,D,T\left(\vec{x},\vec{t}\right)\right)\right|^2 N\left(\vec{x},\vec{t},D\right)dD \qquad [\mathrm{mm}^6/\mathrm{m}^3]$$

$$Z_{DR}\left(\vec{x},\vec{t}\right) = 10\log\frac{Z_h}{Z_v} \qquad [\mathrm{dB}]$$

$$K_{DP}\left(\vec{x},\vec{t}\right) = \frac{180\lambda}{\pi}\int_{D_{min}}^{D_{max}} \mathrm{Re}\left(f_{HH}\left(0,D,T\left(\vec{x},\vec{t}\right)\right) - f_{VV}\left(0,D,T\left(\vec{x},\vec{t}\right)\right)\right) N\left(\vec{x},\vec{t},D\right)dD \ [\mathrm{deg}/\mathrm{km}]$$

- $f_{HH}\left(\pi,D,T\left(\vec{x},\vec{t}\right)\right)$, $f_{VV}\left(\pi,D,T\left(\vec{x},\vec{t}\right)\right)$ back-scattering amplitudes
- $f_{HH}\left(0,D,T\left(\vec{x},\vec{t}\right)\right)$, $f_{VV}\left(0,D,T\left(\vec{x},\vec{t}\right)\right)$ forward-scattering amplitudes
- λ radar wavelength in [cm], K_w dielectric constant of water
- T-matrix for numerical computation of scattering computationally expensive → lookup tables

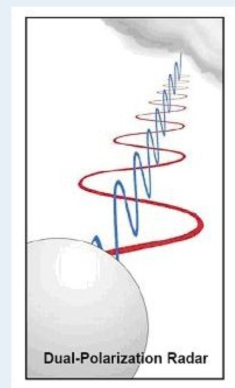


Fig. 4: Orientation of waves in polarimetric radar.
www.roc.noaa.gov/WSR88D/dualpol

Bayesian inverse problem

- Estimate posterior

$$\mathbb{P}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu}|\boldsymbol{Z_H^{obs}},\boldsymbol{Z_{DR}^{obs}},\boldsymbol{K_{DP}^{obs}}) \propto \mathbb{P}(\boldsymbol{Z_H^{obs}},\boldsymbol{Z_{DR}^{obs}},\boldsymbol{K_{DP}^{obs}}|\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu})\mathbb{P}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu})$$

- $\boldsymbol{\Lambda} := \Lambda\left(\vec{x}\right) \in \mathbb{R}^N$, $\boldsymbol{\mu} := \mu\left(\vec{x}\right) \in \mathbb{R}^N$, $\boldsymbol{N_0} := N_0\left(\vec{x}\right) \in \mathbb{R}^N$
- observations $\boldsymbol{Z_H^{obs}}, \boldsymbol{Z_{DR}^{obs}}, \boldsymbol{K_{DP}^{obs}} \in \mathbb{R}^M$
- **Forward operators**: ideal relations between parameters and data, e.g.

$$\boldsymbol{Z_H} := G_{Z_H}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu}), \quad G_{Z_H}: \mathbb{R}^N \to \mathbb{R}^M$$

- Observational errors play a big role → additive and multiplicative models

$$\begin{aligned}
\boldsymbol{Z_H^{obs}} &= \mathrm{diag}\boldsymbol{\epsilon}_{Z_H} G_{Z_H}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu}), \quad &\epsilon_{Z_H} \sim \mathcal{N}(\boldsymbol{1},\boldsymbol{\Sigma_{Z_H}})\\
\boldsymbol{Z_{DR}^{obs}} &= \mathrm{diag}\boldsymbol{\epsilon}_{Z_{DR}} G_{Z_{DR}}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu}), \quad &\epsilon_{Z_{DR}} \sim \mathcal{N}(\boldsymbol{1},\boldsymbol{\Sigma_{Z_{DR}}})\\
\boldsymbol{K_{DP}^{obs}} &= G_{K_{DP}}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu}) + \boldsymbol{\epsilon}_{K_{DP}}, \quad &\epsilon_{K_{DP}} \sim \mathcal{N}(\boldsymbol{0},\boldsymbol{\Sigma_{K_{DP}}})
\end{aligned}$$

- Prior: independence assumption

$$\mathbb{P}(\boldsymbol{N_0},\boldsymbol{\Lambda},\boldsymbol{\mu}) = \mathbb{P}(\boldsymbol{N_0})\mathbb{P}(\boldsymbol{\Lambda})\mathbb{P}(\boldsymbol{\mu})$$

- Markov Chain Monte Carlo (MCMC) to sample from posterior

Outlook

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We implement the Bayesian model in a simulated environment using COSMO-DE. To do so, we implemented an efficient lookup-technique.

- Characteristic cases to asses model capabilities

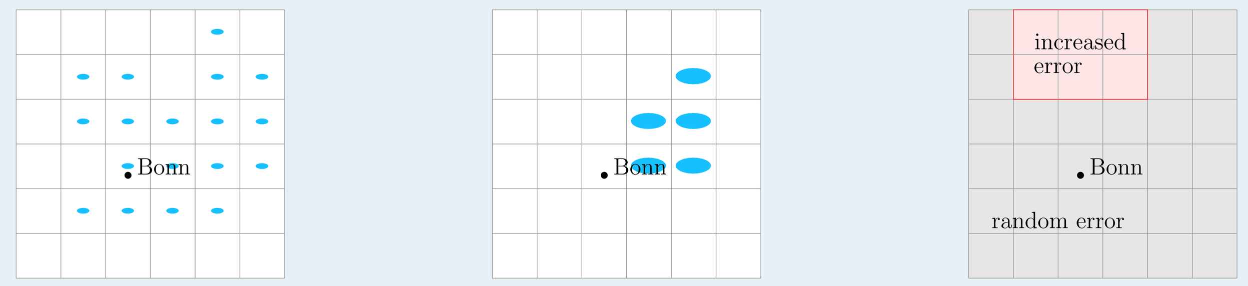

Fig. 5: Model testcases on small domain: light rain (left) and localized stronger rain (middle). Random error model with locally higher measurement error (right).

- Multivariate model: Gaussian processes for DSD parameters

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