# Model of first return stroke based on a bidirectional leader concept

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## Introduction

We present a new return stroke model which belongs to the category of electromagnetic models based on the solution of the whole set of Maxwell's equations. We assume a realistic thundercloud charge structure, which leads to the induced charges along a lightning channel in agreement with a bidirectional leader concept. At the moment when the channel attaches the ground, the current wave propagates upward due to the potential difference between the leader channel and the ground, and due to the flow of the charges from the corona sheath to the lightning core. We show that the modeled electromagnetic field waveforms agree well with observations.

## Model formulation

We numerically solve Maxwell's equations by the method of moment similar to [da Silva and Pasko, 2015], coupled with Poisson's equation solved by a successive over-relaxation method. We modify the continuity equation due to the presence of the corona sheath. We use Ohm's law to take into account a finite conductivity of a lightning channel.

$$U(z,t_i) = U_{amb} + \frac{1}{4\pi\epsilon}\int_{h_1}^{h_2}\left(\frac{q(z',t')}{R(z,z')}dz' + \frac{q_s(z',t'_s)}{R_s(z,z')}dz'\right),$$

$$A(z,t_i) = \frac{\mu_0}{4\pi}\int_{h1}^{h2}\frac{I(z',t')}{R(z,z')}dz',$$

$$\frac{\partial A}{\partial t_i} + \frac{\partial U}{\partial z} + \frac{I}{G} = 0,$$

$$\frac{dq}{dt_i} + \frac{dI}{dz} = -\frac{q}{\tau_{CS}},$$

$$\frac{dq_s}{dt_i} = \frac{q}{\tau_{CS}}.$$

$U(z,t_i)$ and $A(z,t_i)$ denote electric and magnetic potentials, $U_{amb}$ is the ambient potential, $q(z,t_i)$ and $q_s(z,t_i)$ are the linear charge densities at the channel core and the corona sheath, $I(z,t_i)$ is the current flowing along the lightning channel, and $G$ is the line conductivity. $\tau_{CS}$ is the characteristic time of the charge flow from the core into the corona sheath, $t'_{(s)}$ is the retarded time defined by $t'_{(s)} = t_i - R_{(s)}(z,z')/c$, with $R(z,z')$ and $R_s(z,z')$ defined by $R(z,z') = \sqrt{(z-z')^2 + a^2}$ and $R_s(z,z') = \sqrt{(z-z')^2 + b^2}$, with $a$ and $b$ being channel core and corona sheath radius.

We start a return stroke process at $t = 0$ by adding a fixed zero potential element to the bottom of the channel. We label by $t_i$ ($t$) time before (after) the attachment. The streamer to leader transition which inhibits instantaneous step in the channel base current waveform is simulated by the rise of the streamer's conductivity from a very small value up to the estimated of the leader conductivity. The corona sheath charges start to flow into the core at the time when the potential of the lightning channel decreases to 90% of its initial value. From this moment the continuity equations are changed to:

$$\frac{dq}{dt} + \frac{dI}{dz} = \frac{q_s}{\tau_{CS}},$$

$$\frac{dq_s}{dt} = -\frac{q_s}{\tau_{CS}}.$$

The line conductivity evolves according to a phenomenological arc model [Toepler, 1906] (free parameters $\alpha$ and $\beta$ give the rate of the line conductivity increase. They also determine a value of the initial line conductivity):

$$G(z,t) = \frac{\beta + \int_0^t I dt}{\alpha}.$$

Modeled magnetic field is computed by the formula:

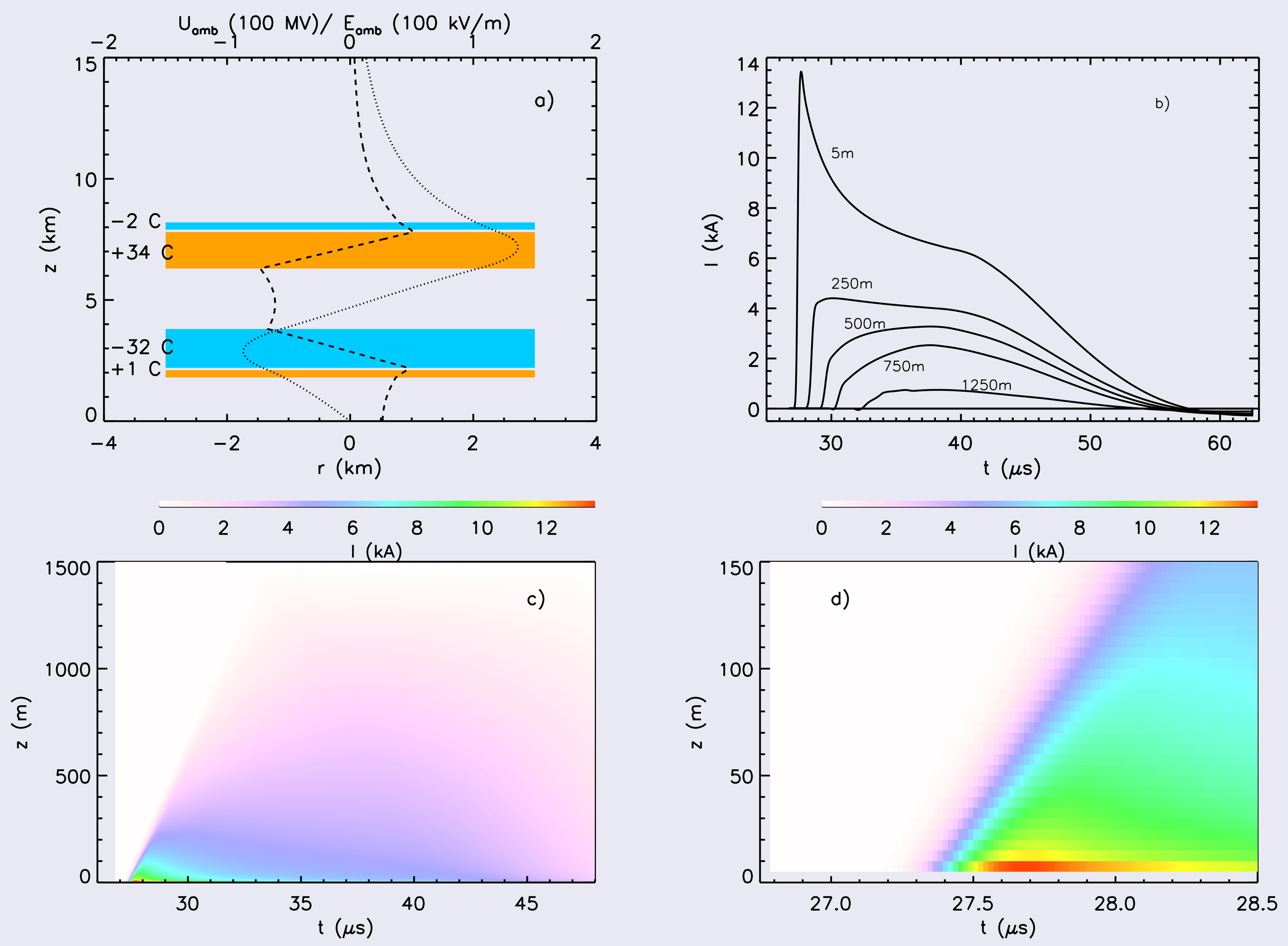
$$B_\phi(D,t) = \frac{\mu_0}{2\pi}\int_{h_1}^{h_2}\frac{\sin\theta}{R^2}I(z,t-R/c)dz + \frac{\mu_0}{2\pi}\int_{h_1}^{h_2}\frac{\sin\theta\,\partial I(z,t-R/c)}{cR\,\partial t}dz.$$

Finite conductivity of a ground is given by an approximate expression valid for return strokes at distances larger than a few km [Cooray, 2012]. $D$ is a horizontal distance to the observer, $R$ is a straight line distance $R = \sqrt{D^2 + z^2}$, $\theta = \arctan(D/z)$.

Here we model a straight and vertical lightning channel. The effect of branches (not considered here) leads to the increased channel capacity and as a result the leader potential decreases. We take this effect into account by reducing ambient potential by hand.

## Current waveforms

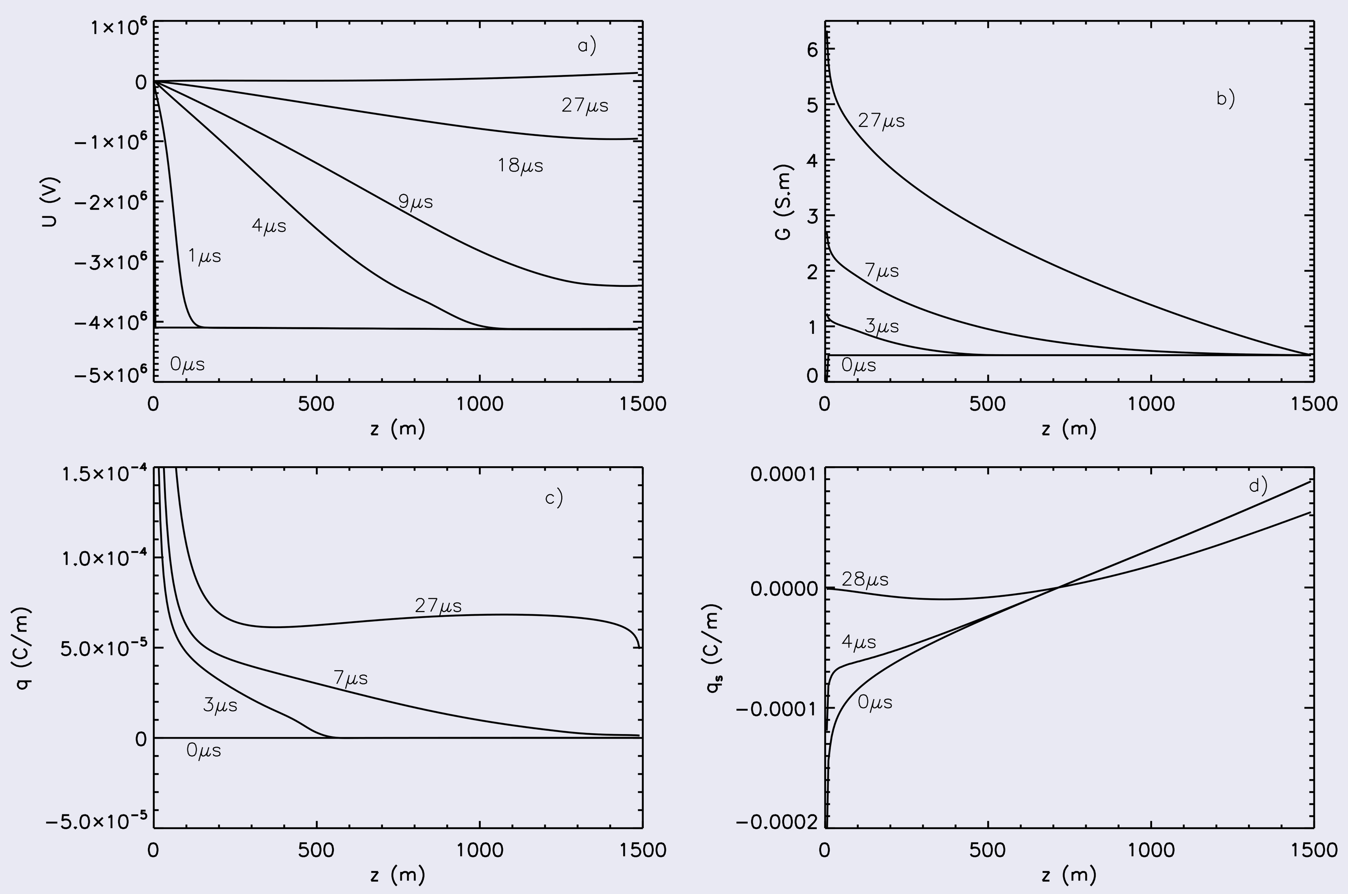
Figures b-d show how the current waveshapes evolve in time at different heights $z$ above the ground level. For a given thundercloud charge structure we show that the current peak amplitudes decrease with height and current rise-times increase with height. A channel base current is not a prescribed function but its waveshape is derived from our simulations.




Return stroke parameters relate to the figure b in the section "comparison with observations": Channel length 1500m, 28% of the ambient potential due to branching effect, current wave speed $2.8\cdot10^8$ m/s, streamer to leader transition time 1$\mu$s, characteristic time of the corona sheath collapse range from 5$\mu$s at the ground to the 50$\mu$s at the upper end of a lightning channel, channel line conductivity parameters $\alpha = 25\cdot10^{-3}, \beta = 12\cdot10^{-3}$, $a$=1cm, $b$=1m and the ground conductivity of 7 mS/m.

## Return stroke channel characteristics

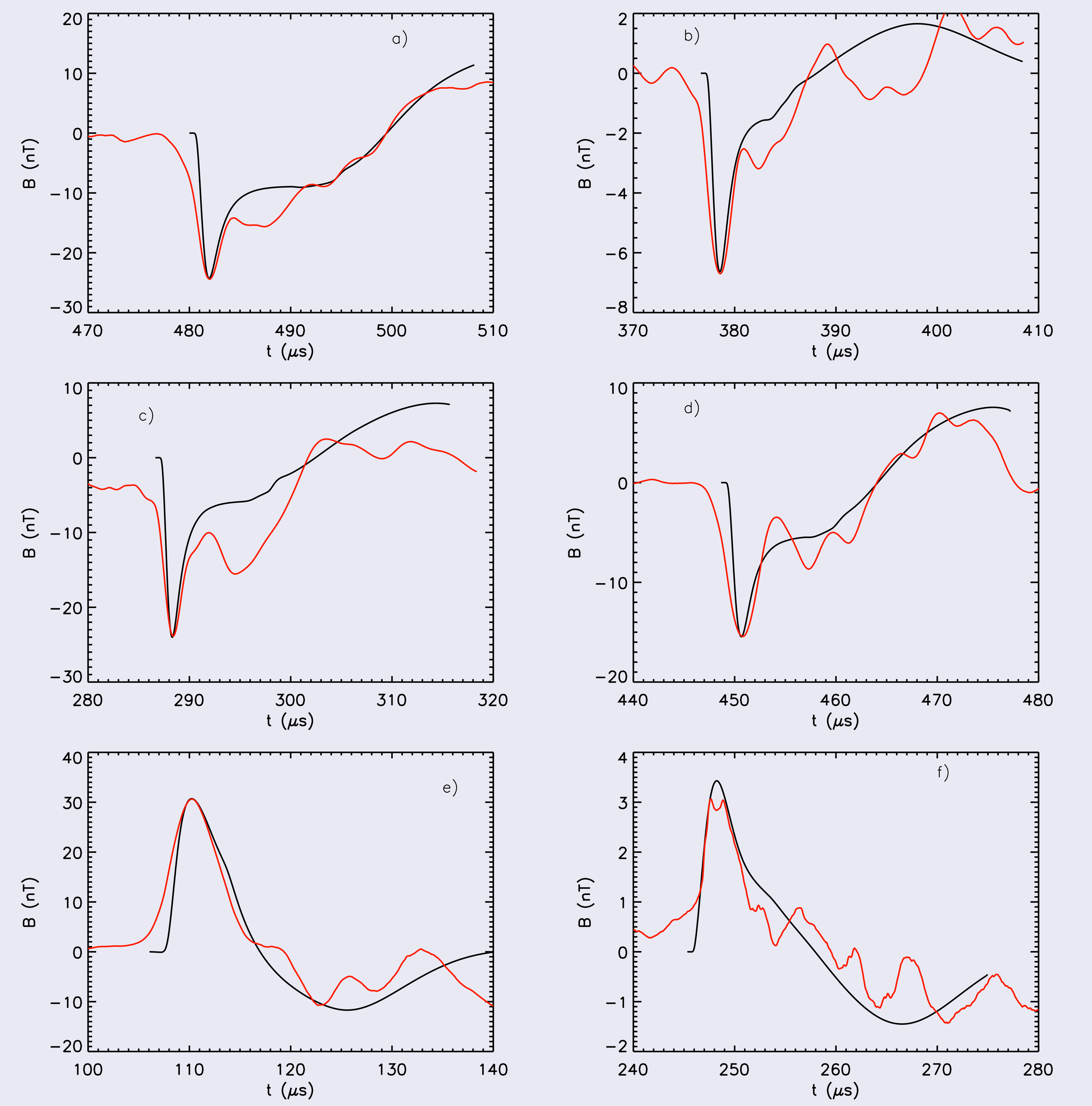
We show the evolution of a return stroke potential $U$ (a), line conductivity $G$ (b), lightning core charge $q$ (c), and corona sheath charge $q_s$ (d) depending on the height $z$.




It can be seen that the RS potential approaches approximately zero potential as the time evolves. The positive charge along the RS channel is due to the transport of the negative charges to the ground.

## Comparison with Observations

Comparison of modeled return stroke waveforms (black lines) with broadband measurements (red lines) at two different measurement sites in France (see their coordinates in the table on the bottom). Our instrumentation consists of a broadband HF analyzer BLESKA (Broadband Lightning Electromagnetic Signal Keeper Analyzer; a clone of the IME-HF analyzer developed by the Institute of Atmospheric Physics for the TARANIS spacecraft) measuring in the frequency range from 5 kHz to 37 MHz, and a SLAVIA magnetic antenna (Shielded Loop Antenna with a Versatile Integrated Amplifier). A horizontal component of the magnetic field is sampled at a rate of 80 MHz. Events are triggered from intense impulsive signals. Absolute timing is provided by a GPS unit. The obtained difference between the modeled time $t$ and the observational time $t_0$ is given in the table below. The table also shows values of peak currents and distances for analyzed lightning discharges which were estimated by the French lightning detection network Météorage.




| Figure | Time (Météorage) | Date | RS position | $I$ [kA] | Measurement site | $D$ [km] | $t_0 - t$ [$\mu$s] |
|---|---|---|---|---|---|---|---|
| a | 04:03:59.069 UT | 19 June 2013 | 43.58N, 3.76E | -59.4 | 43.94N, 5.48E | 144 | 13.2 |
| b | 05:13:50.533 UT | 19 June 2013 | 43.77N, 4.10E | -12.4 | 43.94N, 5.48E | 113 | 2.5 |
| c | 05:54:29.869 UT | 19 June 2013 | 43.77N, 4.44E | -25.7 | 43.94N, 5.48E | 86 | 5.9 |
| d | 04:44:40.728 UT | 19 June 2013 | 43.68N, 3.86E | -25.0 | 43.94N, 5.48E | 134 | 3.4 |
| e | 07:19:58.370 UT | 20 September 2015 | 42.68N, 9.39E | -43.1 | 42.97N, 9.38E | 32 | 2.4 |
| f | 04:55:25.944 UT | 24 September 2015 | 42.45N, 9.92E | -12.1 | 42.97N, 9.38E | 73 | 0.8 |

## Conclusions

- We have developed a new return stroke model based on a bidirectional leader concept.
- We have modeled return stroke characteristics such as current waveforms, line conductivities, potential distributions, corona sheath charges, and core charges.
- At the excitation of a return stroke current we take into account both potential difference and corona sheath collapse.
- Channel base current waveforms are derived from our model.
- Modeled magnetic field waveforms agree well with observations.