Radiative-Convective Model with Precipitation

A One-dimensional Model of Atmosphere

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Abstract

Concerning with enormous computing cost, we establish a one-dimensional model rather than a three-dimensional one to reveal the most significant factors that change our climate in a long-term time scale.

For vertical momentum equation, hydrostatic approximation is applied , from which pressure is obtained as function of total mass of air above current layer. Vertical velocity is evaluated from continuity equation and equation of thermal conservation. For thermal balance equation, three kinds of heat sources are taken into account: radiative absorption of water vapor and carbon dioxide; latent heat flux; turbulent heat flux. To calculate radiation flux we solve the differential equations associated with upward irradiance and downward irradiance, respectively. Absorption coefficients of greenhouse gases in dependence of temperature and pressure are obtained from HITRAN database and bilinear interpolation is applied to utilize these coefficients in the differential equations. We consider uniform size distributed droplets at certain layer, with equation of diffusion of water vapor, equation of motion for droplets and phase change criterion we get the droplets distribution over height, and at the same time latent heat can be calculated. For calculation of turbulent heat transfer, we use modern theory to parameterize horizontal velocity profile, define the boundary layer height and calculate eddy viscosity, eddy conductivity and eddy diffusivity.

On the top layer of atmosphere density, pressure, concentration of water and gradient of temperature are set to zero, downward irradiance is set to input solar irradiance in dependence of time. On the surface boundary, ocean-atmosphere interface and land-atmosphere interface are considered separately, for both situations input and output of irradiance, thermal conductivity and latent heat are balanced on the interface.

With help of this model we can analyze the influences of atmospheric turbulence and surface boundary conditions on the long-term temperature distribution. How carbon dioxide effects global temperature is widely discussed, this model can also help to understand the role of greenhouse gases in global warming.

Main Objectives

1. Mathematical description with physical details in one-dimensional model.
2. Turbulent heat transfer studied by parameterization of turbulent eddy diffusivity.
3. Latent heat of water, simulation droplets.
4. Greenhouse effects of carbon dioxide and water vapor.
5. Interaction of different factors of influences on dynamics of atmosphere.

System of Equations

We are going to consider a vertical colum of air considered as **perfect gas without viscosity** . For the energy balance equation, heat source consists of three parts: turbulent eddy heat flux, absorption of greenhouse gases and latent heat of water. In order to study the effects of greenhouse gases in this system, we consider carbon dioxide and water vapor and calculate the absorption heat of them. In vertical momentum equation inertial part is not taken into account, i.e. quasi-static assumption is used. The equation of heat flux is written as follows:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{Q^*}{\rho C_v} + \frac{p}{\rho^2 C_v}\frac{\mathrm{d}\rho}{\mathrm{d}t} \qquad (1)$$

where

$$Q^* = -\frac{\partial q}{\partial z} + Q - J^{(e)} l^{(e)}$$

is heat source which consists of:

1. turbulent heat transfer, $q = -\lambda_d \cdot \frac{\partial \theta}{\partial z}$, where λ_d is turbulent thermal conductivity coefficient, θ is potential temperature;
2. absorbed heat by greenhouse gases Q;
3. latent heat of evaporation $J^{(e)} l^{(e)}$, or condensation (with minus sign).

Applying equation of state $p = R\rho T$ to equation of heat flux (1) we can rewrite the heat flux equation as the relations between derivatives of p and ρ with respect to t:

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\gamma - 1}{\gamma p} Q^* \qquad (2)$$

where $\gamma = C_p/C_v$ is heat capacity ratio of air, which can be considered as constant 1.4 .

From quasi-static assumption, vertical velocity does not occur in vertical momentum equation, thus pressure at certain altitude is defined by total mass above:

$$p = g\int_z^\infty \rho \mathrm{d}z' = gM \qquad (3)$$

where $M = \int_z^\infty \rho \mathrm{d}z'$ is the total mass above certain altitude. Pressure change rate is then defined by the change of mass:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = g\dot{M} = -g\int_z^\infty \left(\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y}\right)\mathrm{d}z' = 0 \qquad (4)$$

where

$$\dot{M} = \int_z^\infty \left(\frac{\rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y}\right)\mathrm{d}z'$$

is rate of total mass change.

Continuity equation is as follows:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0 \qquad (5)$$

Vertical velocity can be estimated by heat flux equation (2) and continuity equation (5):

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\left(\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) - \frac{1}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\gamma - 1}{\gamma p} Q^* \qquad (6)$$

In total, the final set of system is as follows:

$$\begin{cases} \frac{\partial T}{\partial t} = -v_z \frac{\partial T}{\partial z} + K_d \frac{\partial^2 T}{\partial z^2} + \frac{Q - Jl}{\rho C_v} - \frac{p}{\rho C_v}\frac{\partial v}{\partial z} \\ p(t,z) = p_h(t)\cdot \exp\left(-\frac{g}{R}\int_h^z \frac{\mathrm{d}z'}{T}\right) \\ \rho = \frac{p}{RT} \\ M = \int_z^\infty \rho \mathrm{d}z', \quad \dot{M} = -g\int_z^\infty \left(\frac{\rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y}\right)\mathrm{d}z' \\ \frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{gM} \end{cases} \qquad (7)$$

where $p_h(t)$ is the pressure on the surface, K_d is turbulent diffusivity coefficient, horizontal velocity is estimated by given horizontal velocity profile [2].

Boundary Conditions:

$$\begin{aligned} z = H: \quad & \rho = 0,\ p = 0,\ \dot{M} = 0,\ v_z = 0 \\ z = 0: \quad & v_z = \frac{\xi_0^{(e)}}{\rho_0}, \\ & p \equiv p_0, \quad \frac{\partial p_0}{\partial t} = \rho g v_{z,0}, \\ & q_{-0} - q_{+0} + (1 - A)G_{+0} - (1 - A)G_{-0} \\ & - U_{+0} + U_{-0} \quad = \quad \xi_0^{(e)} l^{(e)} \end{aligned}$$

where $\xi_0^{(e)}$ is flux of water evaporation on the interphase boundary, ρ_0 is the density of air on the lower boundary, G, U are downward and upward irradiance respectively, A is albedo. Here index (-0) means below the boundary, and (+0) means over the boundary. q_{-0}, q_{+0} are heat flux to the interface from below and over the boundary respectively. Since at height of 40 km, pressure and density of air is closed to zero, we take $H = 40$ km.

Heat Fluxes

Latent Heat of Droplets

Water in the air consists of two phases, gas (water vapor) and liquid: $\rho = \rho_{\mathrm{A}} + \rho_{\mathrm{W}}, \quad \rho_{\mathrm{W}} = \rho_{\mathrm{G}} + \rho_{\mathrm{L}}$, subscript L denotes liquid water and G denotes water vapor. Equations of mass conservation for water vapor and droplets are as follows:

$$\begin{cases} \frac{\partial \rho_{\mathrm{G}}}{\partial t} = -\frac{\partial \rho_{\mathrm{G}}(v_z + \omega_{\mathrm{G}})}{\partial z} + J^{(e)}, \quad \omega_{\mathrm{G}} = -\nu^{(d)}\frac{\partial}{\partial z}\left(\frac{\rho_{\mathrm{G}}}{\rho}\right) \\ \frac{\partial \rho_{\mathrm{L}}}{\partial t} = -\frac{\partial \rho_{\mathrm{L}}(v_z + \omega_{\mathrm{L}})}{\partial z} - J^{(e)}, \quad C_d \cdot \frac{\rho \omega_{\mathrm{L}}^2}{2}\frac{\pi d^2}{4} = \frac{2\pi d^3}{3}\rho_{\mathrm{L}}^0 g \end{cases} \qquad (8)$$

where $\omega_{\mathrm{G}}, \omega_{\mathrm{L}}$ are relative vertical velocity to air of water vapor and droplets, respectively; d is diameter of one droplet, $\rho_{\mathrm{L}}^0 = 998\,\mathrm{kg/m^3}$. $\nu^{(d)}$ is turbulent diffusivity of water vapor in air: $\nu^{(d)} = 0.22 + 0.0015 * (T - 273.15)$. C_d is drag coefficient, when Reynolds number is small:

$$C_d = C_d(\mathrm{Re}) = \frac{24}{\mathrm{Re}}, \quad \mathrm{Re} = \frac{\rho \omega_{\mathrm{L}} d}{\mu_m} \qquad (9)$$

So relative velocity can be expressed by diameter of droplet:

$$\omega_{\mathrm{L}} = \frac{2}{9}\frac{d^2 \rho_{\mathrm{L}}^0 g}{\mu_m}$$

According to Sutherland's formula,

$$\mu_m = \mu_0 \frac{T_0 + C}{T + C}\left(\frac{T}{T_0}\right)^{\frac{3}{2}}$$

for air, $C = 120\,\mathrm{K}$, $T_0 = 291.15\,\mathrm{K}$, $\mu_0 = 18.27\,\mu\mathrm{Pa}\cdot\mathrm{s}$. Mass transfer of water is proportional to droplet surface and to the difference of concentrations of water vapor between the droplet surface and the environment around it, for a particular droplet with diameter d:

$$J_d^{(e)} = n\pi d^2 \nu^{(d)} \mathrm{Sh}\frac{\rho_s - \rho_\infty}{d} \qquad (10)$$

n - number of droplets in unit volume ($1/\mathrm{m}^3$), ρ_s- mass concentration of water vapor on surface of droplet, which can be obtained by $\rho_s = \rho_{sat}(T_s)$, ρ_∞ - mass concentration of water vapor around droplets, which can be replaced by ρ_{G} above, Sh - Sherwood number, a dimensionless number representing the ratio of mass transfer to the rate of diffusive mass transport. Mass transfer coefficient Sherwood number (Sh) for a single sphere we take the following form:

$$\mathrm{Sh} = 2.0 + A \cdot \mathrm{Re}^m \mathrm{Sc}^n$$

where constants A, m, n can be different, for the first test we set $A = 0.552$, $m = 1/7$, $n = 1/3$.

Absorption of Solar Radiation

To calculate the absorpted heat flux by greenhouse gases, we get absorption coefficients of carbon dioxide and water vapor from database [1], the absorption coefficients are depended on pressure, temperature and wavelength of light. The wavelength range is $0.3 \sim 50\mu$m, the coefficients are interpolated in 2D (T, P) plane. The following is the equation for upward and downward irradiance for specific wavelength of light.

$$\begin{cases} \frac{\partial U_\lambda}{\partial z} = \beta\left(\alpha_1 \rho_1 + \alpha_2 \rho_2\right)\left(B_\lambda - U_\lambda\right), \\ \frac{\partial G_\lambda}{\partial z} = \beta\left(\alpha_1 \rho_1 + \alpha_2 \rho_2\right)\left(G_\lambda - B_\lambda\right) \end{cases} \qquad (11)$$

where U_λ, G_λ are upward and downward irradiance respectively, β is a constant from integration of all directions, taken as 1.67, B_λ is Planck function at specific wavelength. The heat flux is obtained by integrating the difference of downward irradiance and upward irradiance:

$$Q(z) = \int_\lambda \frac{\partial(G_\lambda - U_\lambda)}{\partial z}\mathrm{d}\lambda$$

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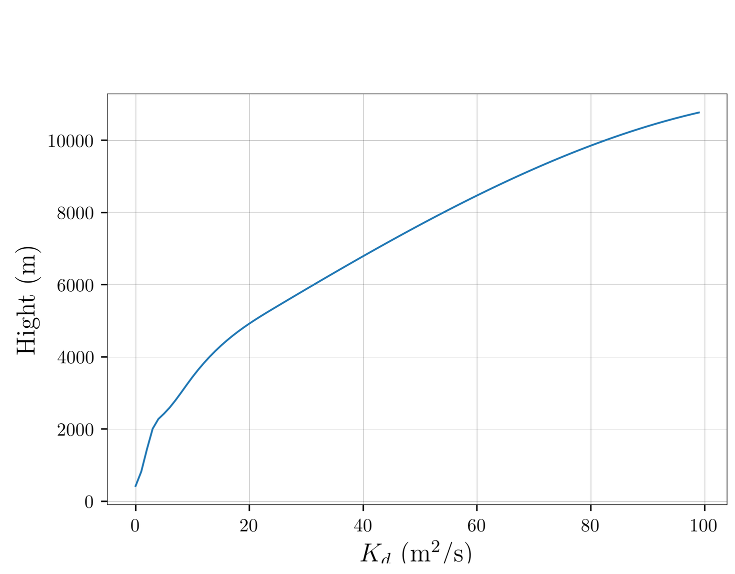
Turbulent Heat Transfer

Turbulent heat flux is estimated by studying diffusivity coefficient K_d, we first take a distribution of K_d over height, which does not change with time . Kumar et al. [3] gives a parameterization method of eddy diffusivity, which we will use in later study.

Figure 1: Eddy diffusivity coefficient



Forthcoming Research

1. Complete the code.
2. Different types of turbulent diffusivity coefficients.
3. Include other greenhouse gases.
4. Estimate distribution of droplet size.
5. Interaction of atmosphere with surface interface.
6. Parallelization of the program.

References

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