Results

4. Example of the inverse problem. Retrieval of $\text{[O}^3\text{P}\text{]}$ altitude profile using $O_b(b^{3}P, v=2)$ as proxy

In the altitude interval 90 - 140 km it is possible to get simple analytical expression for retrieval of $\text{[O}^3\text{P}\text{]}$ from solution of kinetic equation for the component $O_b(b^{3}P, v=2)$:

$$\rho^{\text{[O}^3\text{P}\text{]}}(v=2) = \frac{\rho_b(2)}{\rho_{b}^{\text{[O}^3\text{P}\text{]}(v=2)}} = \frac{\rho_{b}^{h}(2)}{\rho_{b}^{\text{[O}^3\text{P}\text{]}(v=2)}} (1 - \Delta g_{v} - \frac{N_r}{N_{\text{tot}}})$$

where $\rho^{\text{[O}^3\text{P}\text{]}}(v=2)$ is the target function, $O_b(b^{3}P, v=2)$ is the information source about $\text{[O}^3\text{P}\text{]}$. Formula (3) includes the YM-2011 - model’s parameters each of which has its own relative error, $\Delta g_{v}$. The values of the experimental errors of all parameters presented in Table 1 were taken from [1]. Let’s consider the parameter’s relative errors on the uncertainty of retrieved $\text{[O}^3\text{P}\text{]}$ from (3). We have used two approaches to estimate this effect (sections 5 and 6).

5. Sensitivity study

If the target function $\epsilon_{x_{i}}$ depends on the $n$ parameters $\epsilon_{i}$ then the sensitivity coefficient of the target function to variation of parameter $\epsilon_{i}$, $S_{i}(\epsilon_{x_{i}}, \epsilon_{j})$, is:

$$S_{i}(\epsilon_{x_{i}}, \epsilon_{j}) = \frac{\epsilon_{x_{i}}}{\epsilon_{i}}$$

The relative uncertainty of the target function, $\epsilon_{x_{i}}$, connects with the sensitivity coefficients, and also with the relative errors of all parameters which are included in (3), $\Delta \epsilon_{i}$:

$$\Delta \epsilon_{x_{i}} = \sum S_{i}(\epsilon_{x_{i}}, \epsilon_{j}) \Delta \epsilon_{j}$$

This approach is schematically presented in Fig. 2 by green lines with arrows. The results of using this approach are shown in Fig. 3a – 3e by a solid green line. It is important to notice that the sensitivity analysis for all of the expressions given below was firstly presented in [1].

6. Monte Carlo method

There is no established point of view on the form of the error distribution for the value of the rate constants for the most known reactions. We have tested two error distribution functions: the uniform distribution within $[-\epsilon, \epsilon]$ (Fig. 2, blue line) and the normal distribution, where the limits $[-\epsilon, \epsilon]$ correspond to the limits of 2σ (Fig. 2, red line). Given the errors of parameters, the formula (3) was transformed to:

$$\rho_{b}(2) = \frac{\rho^{\text{[O}^3\text{P}\text{]}}(v=2)}{\rho_{b}^{\text{[O}^3\text{P}\text{]}(v=2)}} (1 - \Delta g_{v} - \frac{N_r}{N_{\text{tot}}})$$

where $\Delta = (1 \pm \epsilon_{\text{RandomValue}})$ with RandomValue $\in (-1, 1)$. The results of the calculations were compared with the reference value of target function. For the reference value of target function, we have taken the values obtained from (3) (Fig. 3a). The similar investigation was carried out for the relative uncertainties of the $[O_b]$ and $[CO_2]$ altitude profiles retrievals at proxies $O_b(b^{3}P, v=2)$ (Fig. 3b – 3e).

7. Conclusions

- Estimates of the uncertainties of retrieval of $[O_b]$, $[O_a]$ and $[CO_2]$ altitude profiles have been performed with various types of proxies.
- The Monte Carlo method gives values of relative uncertainties that are about 1.5 times smaller than the sensitivity study. The sensitivity study overestimates the relative uncertainty values, since it is the result of the product of the sensitivity coefficient by the absolute magnitude of the error of each parameter.
- The values of relative uncertainties calculated by the Monte Carlo method insignificantly depend on the error distribution functions (the uniform distribution or the normal distribution).

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References


Fig. 3: Examples of the uncertainties of retrieval of the $\text{[O}^3\text{P}\text{]}$, $\text{[O}_b\text{]}$, $\text{[CO}_2\text{]}$ altitude profiles relative to the standard altitude profiles (dashed line) calculated using two approaches: (a) sensitivity study (green line), (b) Monte Carlo method (1000 runs) for the two error distribution functions: the uniform distribution (blue line), the normal distribution (red line).