Fully relativistic positioning for the Galileo for Science (G4S) project

GALILEO FOR SCIENCE

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The Galileo for Science project

G4S

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The full project is described in another presentation: POSTER X3.105

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Accurate orbit determination for Doresa and Milena

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Use of a Relativistic Positioning System (RPS) based on emitters fixed to the surface of the earth

This represents a Fully RPS based on the full exploit of Special and General Relativity and on the concept of space-time

In the case of GNSS satellites the positioning is traditional, very close to that of Nautical Astronomy, and the effects of Special and General Relativity are introduced as perturbations, i.e. as corrections to a Newtonian formulation

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The Principle of the measurement

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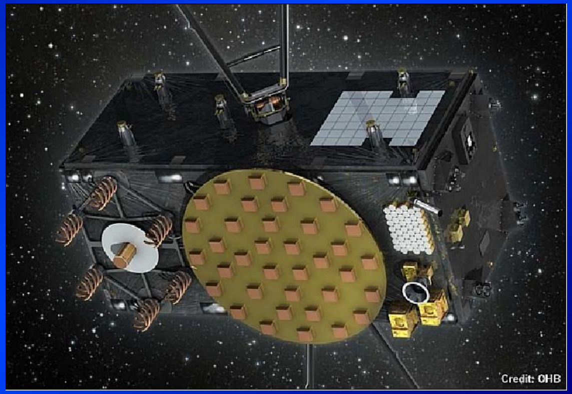
The counting of the pulses for a set of different emitters whose positions on the earth and periods are assumed to be known, is used to provide null emission, or light, coordinates for the receiver on a satellite



The measurement of the proper time intervals between successive arrivals of the signals from the various emitters is used to give the final localization of the receiver, within an accuracy controlled by the precision of the onboard clock

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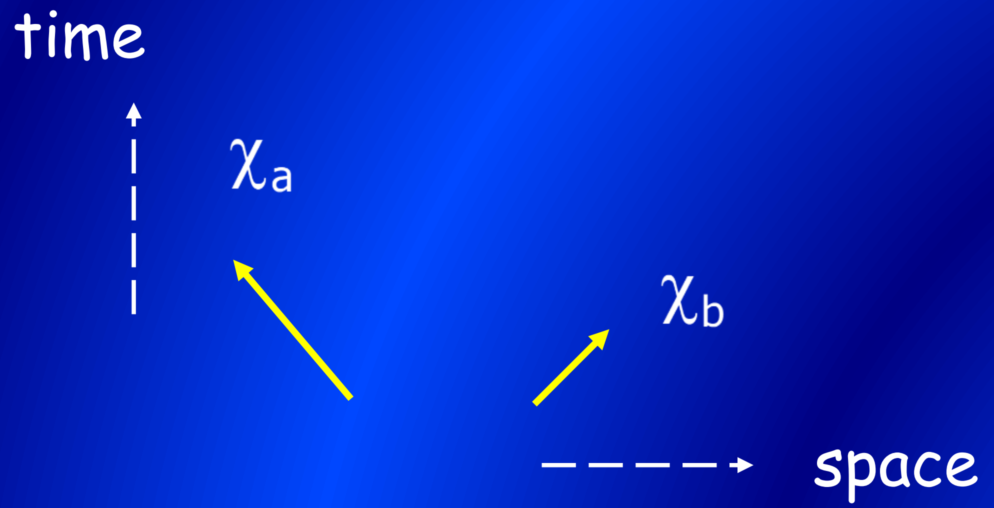
Null bases for space-time

$\chi_a, \quad \chi_b, \quad \chi_c, \quad \chi_d$

Null vectors corresponding to the space-time directions of electromagnetic signals emitted from four independent sources comprise a null base



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A position in space-time in terms of null wave-vectors

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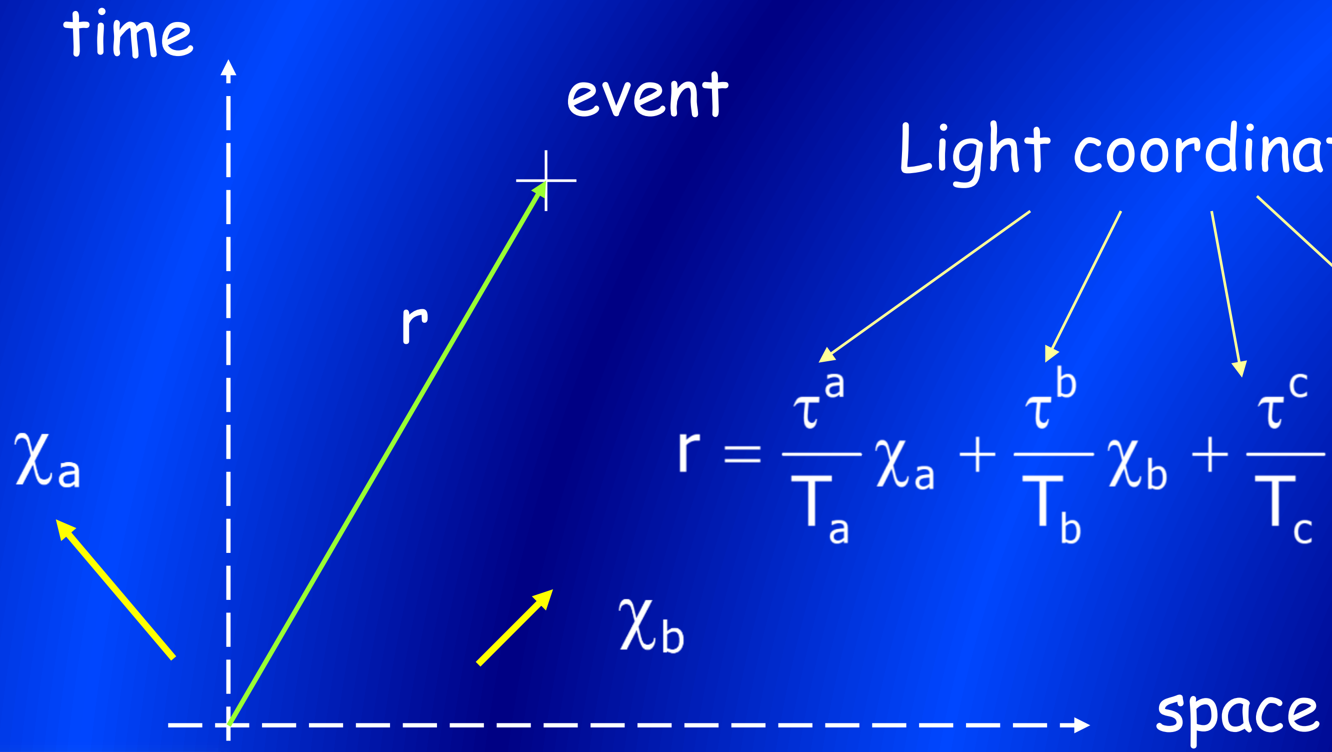

Light coordinates

$$r = \frac{\tau^a}{T_a}\chi_a + \frac{\tau^b}{T_b}\chi_b + \frac{\tau^c}{T_c}\chi_c + \frac{\tau^d}{T_d}\chi_d$$

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Wavevectors in terms of an ordinary timelike base

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Null geodesics have null tangent wave-vectors

$$\chi = cT(1, \cos\alpha, \cos\beta, \cos\gamma) = cT(1, \hat{n})$$

Direction cosines

Period of the signal

$$\chi^2 = 0$$

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Null wave fronts

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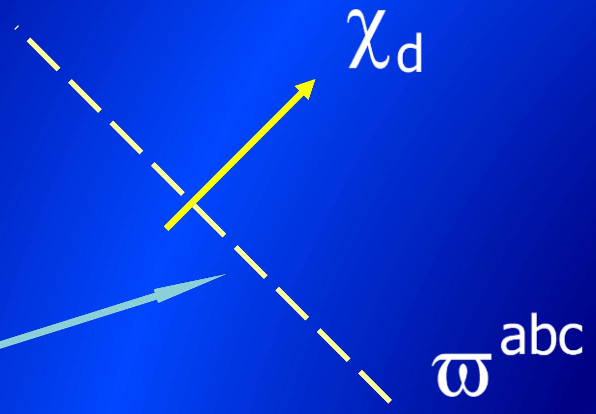
Null hyperplane
(in flat space-time
and source at infinity)

$$\varpi^{abc} = \varepsilon^{abcd}\chi_d$$



Null hypersurface in
three dimensions

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A null grid mapping space-time

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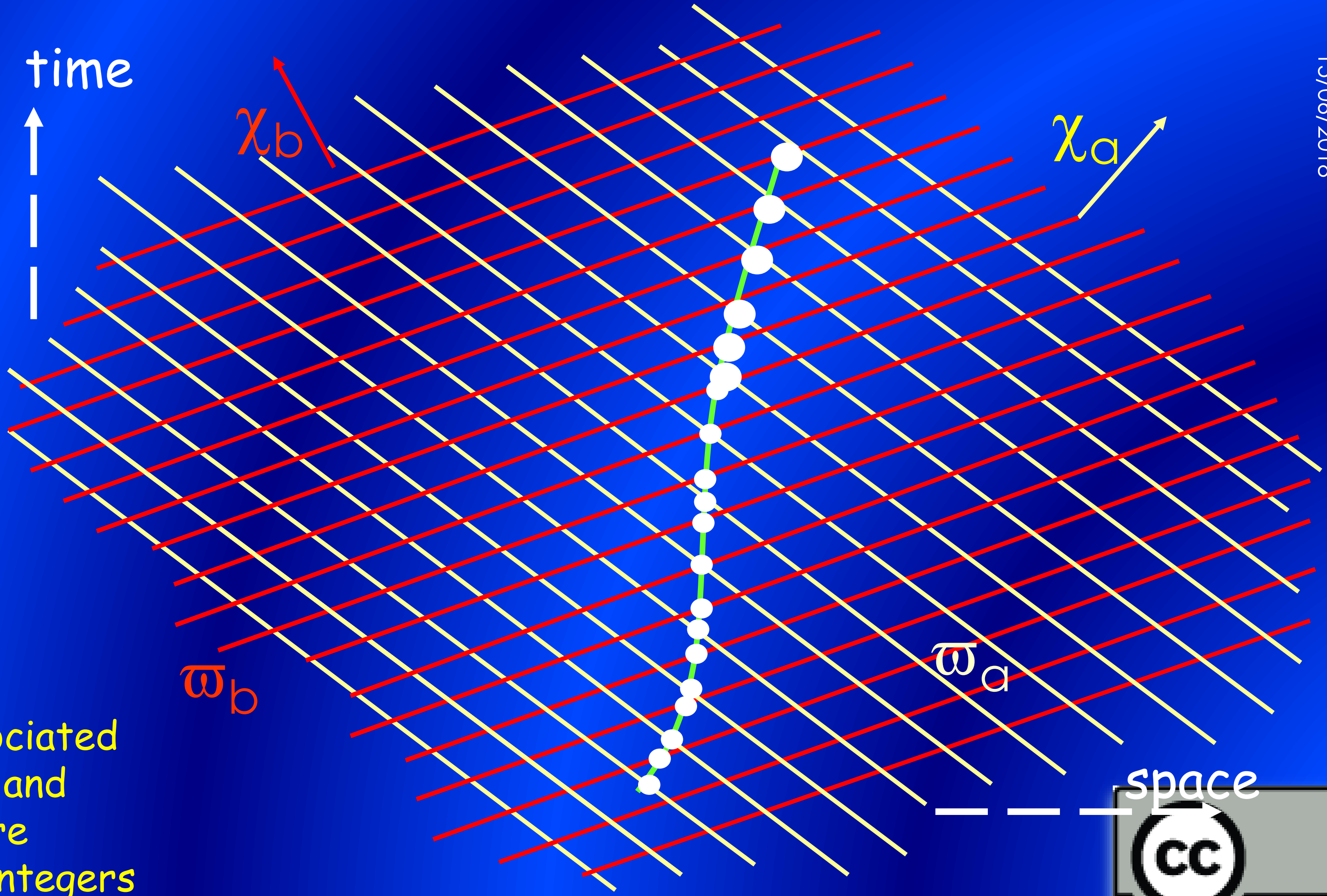
world-line of a receiver

arrival of a signal

Interval between arrivals measured by proper time of the receiver



Every wavefront is associated with an integer number and the knots of the grid are identified by a pair of integers

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Light coordinates of an event

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$$\tau_{a,b,c,d} = [(n + x)T]_{a,b,c,d}$$

integer

fractional

$$r_i = X_{ai}\chi^a$$

$$X_{ai} = \frac{\tau_{ai}}{T_a} = n_{ai} + x_{ai}$$

$$r = \frac{\tau^a}{T_a}\chi_a + \frac{\tau^b}{T_b}\chi_b + \frac{\tau^c}{T_c}\chi_c + \frac{\tau^d}{T_d}\chi_d$$

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Simple Minkowski geometry

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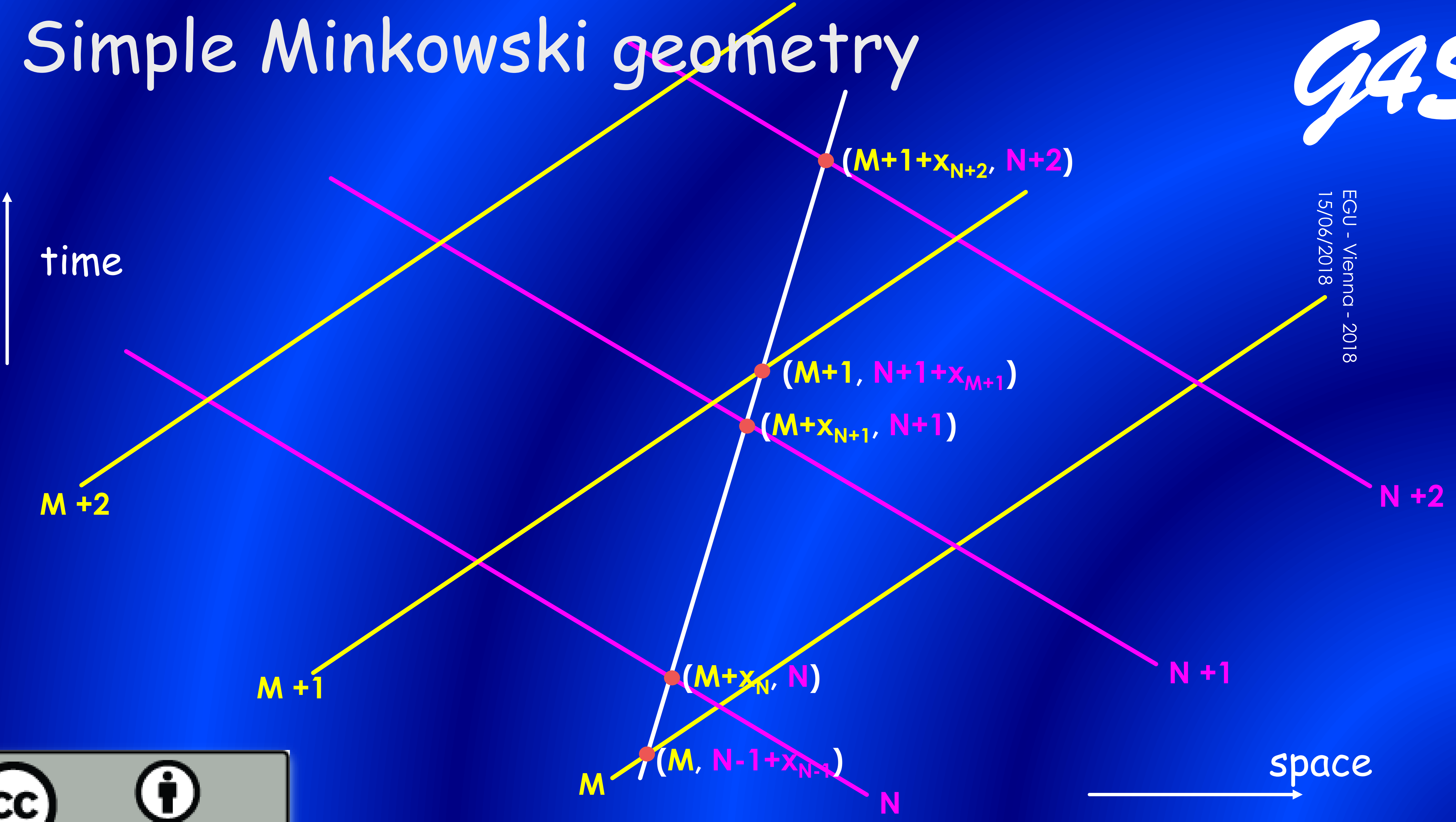

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A linear algorithm

i.e. a practical way to evaluate the fractional part x

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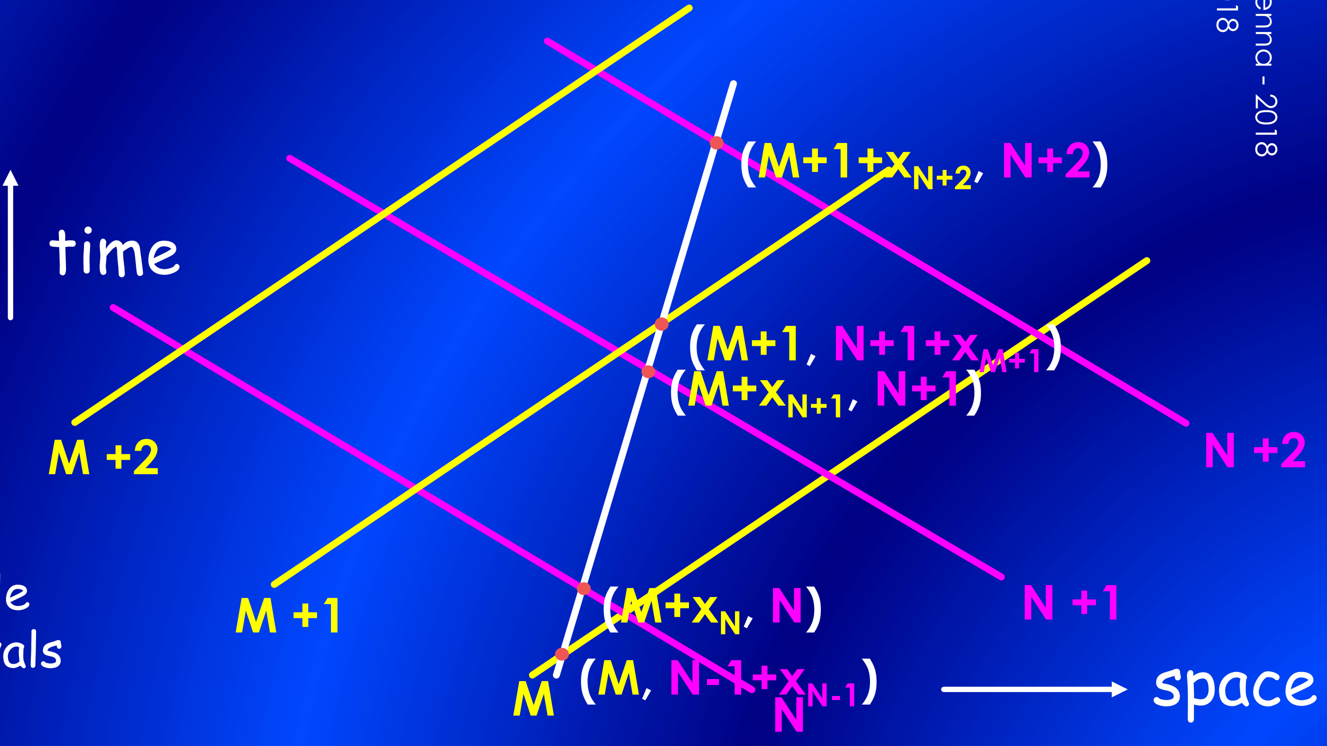
When the worldline of the user is straight, or may be thought to be approximately straight, the knowledge of the time span between the arrivals of the pulses is sufficient to uniquely localize the reception events, and then the position of the user



Given a sequence of eight arrivals, simple linear relations between the time intervals and the x's hold

Solving the corresponding system of linear equations leads to the complete definition of the positions in spacetime of the first four reception events

With a moving sequence of events, it is then possible to fully reconstruct the whole worldline of the receiver

A linear algorithm

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The arrival times of the signals in the sequence will correspond to the following space-time positions:

$$r_1 = (n_1 T)^a \mathcal{X}_a + [(n_1 + x_1)T]^b \mathcal{X}_b + [(n_1 + x_1)T]^c \mathcal{X}_c + [(n_1 + x_1)T]^d \mathcal{X}_d$$

$$r_2 = [(n_2 + x_2)T]^a \mathcal{X}_a + (n_2 T)^b \mathcal{X}_b + [(n_2 + x_2)T]^c \mathcal{X}_c + [(n_2 + x_2)T]^d \mathcal{X}_d$$

$$r_3 = [(n_3 + x_3)T]^a \mathcal{X}_a + [(n_3 + x_3)T]^b \mathcal{X}_b + (n_3 T)^c \mathcal{X}_c + [(n_3 + x_3)T]^d \mathcal{X}_d$$

$$r_4 = [(n_4 + x_4)T]^a \mathcal{X}_a + [(n_4 + x_4)T]^b \mathcal{X}_b + [(n_4 + x_4)T]^c \mathcal{X}_c + (n_4 T)^d \mathcal{X}_d$$

$$r_5 = [(n_1 + 1)T]^a \mathcal{X}_a + \cdots$$

$$r_6 = \cdots$$

$$r_7 = \cdots$$

$$r_8 = \cdots$$

Let us next consider a sequence of arrival times from respectively sources a,b,c,d and label the corresponding events as 1,2,3,4,5,6,7,8. We choose the first event as coinciding with the crossing of a hyper-surface belonging to the a family, so that xa1 = 0 . The second event will be at the next crossing of a b-hypersuface, the third will be at the encounter with the subsequent chypersurface, and so on

$$r_{ij} = r_j - r_i = \left(X_{aj} - X_{ai}\right)\chi^a = \Delta X_{aij}\chi^a$$

$$\frac{\tau_{ij}}{\tau_{jk}} = \frac{\Delta X_{1ij}}{\Delta X_{1jk}} = \frac{\Delta X_{2ij}}{\Delta X_{2jk}} = \frac{\Delta X_{3ij}}{\Delta X_{3jk}} = \frac{\Delta X_{4ij}}{\Delta X_{4jk}}$$

At least 8 events (two series of four)

Coordinates in the null frame

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We may easily count the n's but have no direct means to measure the x's. However if we suppose that the acceleration of the receiver is small enough to allow for the identification of the world line, in a couple of periods of the sources, with a straight line and if we carry on board the receiver a clock, we are able to measure the proper time intervals between the i-th and j-th arrival events, tij. With all this, trivial geometric considerations lead to the values of our interest. It is

Coordinates in the null frame

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$$x_{a1} = 0,\ x_{b1} = 1 - \frac{\tau_{12}}{\tau_{26}},\ x_{c1} = 1 - \frac{\tau_{13}}{\tau_{37}},\ x_{d1} = 1 - \frac{\tau_{14}}{\tau_{48}}$$

$$x_{a2} = \frac{\tau_{12}}{\tau_{15}},\ x_{b2} = 1,\ x_{c2} = 1 - \frac{\tau_{13}}{\tau_{37}} + \frac{\tau_{12}}{\tau_{37}},\ x_{d2} = 1 - \frac{\tau_{14}}{\tau_{48}} + \frac{\tau_{12}}{\tau_{48}}$$

....

$$\tau_{ij} = \tau_j - \tau_i$$

Difference of the proper arrival times between the j_{th} and the i_{th} event

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Uncertainties in the light coordinates

$$\left|\frac{\delta x}{x}\right| \leq 4\left(\frac{1}{\tau_{i,i+4k}} + \frac{\tau_{i,i+1}}{\tau_{i,i+4k}^2}\right)\delta\tau$$

$\delta\tau$: accuracy of the onboard clock (a few ns)

$$\left|\frac{\delta x}{x}\right| \sim 10^{-9}$$

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Conversion from null to practical coordinates

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Straighforward when sources at infinity (pulsars)

More elaborate when sources are at finite distance

In any case: very high accuracy when positioning is with respect to a starting point along the space-time trajectory of the receiver

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Double aim of the application of the RPS to G4S

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a) To offer an additional approach to the orbit determination

b) To test the RPS in inverse configuration, in view of other applications

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References

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3. Angelo Tartaglia, *Acta Futura*, issue 7, 111-124 (2013)